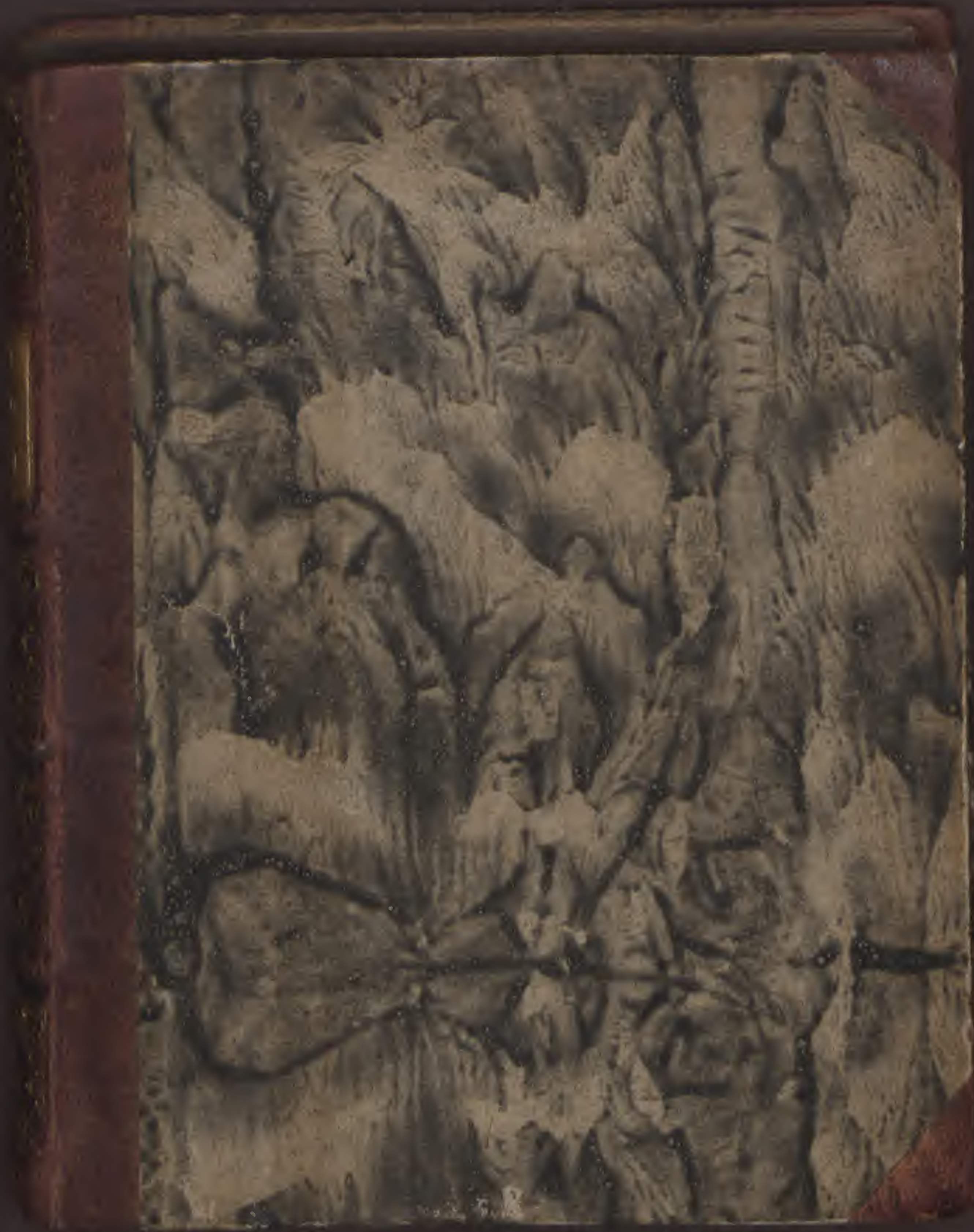



FINKII
GEOMETR.
ROTUNDI.

THOMAE FINKII
Flenſpurgenſis
GEOMETRIÆ RO-TVNDI Libri XIIII.

Ad

FRIDERICVM Secundum, Sereniſsimum Daniæ & Norvegiæ regem, &c.

Cum Gratia & Privileg. Cæſ. Maieſt.

BASILEÆ,

PER SEBASTIANVM

HENRICPETRI.

1775

POTENTISSIMO PRINCIPI AC DOMINO: FRIDERICO II:

Sereniſsimo Daniæ, Norvegiæ, Schlavorum Gothorumq; regi: illuſtriſsimo Sleſvici, Holſatiæ, Stormariæ ac Dithmarſiæ duci: Comiti in Oldenburgo & Delmenhorſt, &c. Domino ſuo clementiſsimo.

V*ERVM quidem eſt*, FRIDERICE *Rex, Princeps illuſtriſsime, tritum quod fertur ſermone proverbium: Vino Vendibili ſuſpenſa hedera opus non eſſe. Ea enim rerum bonarum eſt natura: ut tametſi à nullo laudentur: eos tamen, qui meliore mente præditi ſunt, ad ſe amandum excitent. Sed non minus quoq; Heſiodi, Vetuſtiſsimi poëtæ, querela vera eſt: ignorare homines quantum utilitatis atque commodi Malva & Albucum Vitæ humanæ conferant. ita enim pleriq; ſunt affecti: ut res etiam ſummè neceſſarias & fructuoſas, quia vel vulgares ſunt vel ad pompam & ſplendorem minus accommodatæ, negligant prorſus atq; contemnant.*

Videre id cum aliis in rebus, tum literis humanioribus licet: maximè verò iis in artibus: quæ ob certam ſuam, quam pariunt ſcientiam κατ᾽ ἐξοχὴν *mathematicæ ſolæ vocantur.*

Hæ enim cum dignitatis in vita parum habere videantur, ſor-

dent plerisq; & fastidiuntur: in amore contra & delitiis habentur: quæ ad quæstum sunt & magnificentiam comparatæ. Quotus enim quisq; est: qui non utilissimis hisce disciplinis vel neglectis prorsus, vel obiter saltem & quasi per transennam salutatis, illico ad ea provolat studia: quæ in vulgus probantur: quæ ampla proposita habere præmia putantur: quæ ad opes & dignitates viam videntur aditumq; patefacere.

At verò si rerum æstimatores paulò esse æquiores voluerimus: & dignitatem artes habere Mathematicas summam deprehendemus: & tantam in vita communi utilitatem: ut eas è societate humana qui tollunt, Solē ipsum è mundo tollere videantur.

Ecquis enim tam est ab omni historiarum cognitione instructus malè: ut Mathematum fuisse cultores Patriarchas sanctissimos, Rerumpub. gubernatores prudentissimos, naturæ indagatores solertissimos, non animadvertat? Abrahamum Iosephus testatur non tantum artibus Mathematicis excelluisse: sed easdem quoq; Aegyptios docuisse. Idem de sacerdotibus Aegyptiacis alii referunt historici.

De Regibus verò & Monarchis, qui hæc studia vel amarunt vel excoluerunt, quid dicam? Alexandrum illum Magnum Alexandriam Aegypti Mathematicis fundasse quis ignorat? Quis Iulii Cæsaris in his artibus ardorem nescit: qui

—— media inter prælia semper
Stellarum cœliq; plagis superisq; vacasse

se, gloriari apud Lucanum non veretur? Quid de Carolo Magno & eo, qui idem tecum, Rex serenissime, nomen obtinuit, Friderico Secundo imperatore referam? quorum alter tantum in Mathematicis disciplinis profecit: ut sui temporis artificibus annumerari queat: alter tantum eas amavit: ut ex Arabum monumentis in Latinum sermonem transfundi doctrinam Mathematicam, sumptibus non parvis curârit. Longum esset omnes & re-

ges & principes, qui hisce operam studiis dederũt, recensere. itaq; breviter saltem quosdam attingere volui: ut reliquorũ tibi memoriam, Rex serenissime, in mentem revocarem. inprimis tamen memorabile istud de Carolo Quinto est: quod etiam in thermis sedens, à studiis Mathematicis non quieverit: sed horologiis artificiosè describendis ocium fefellerit. Quin & in obsidione Vitebergensi, cum in castra ad illum principes venissent, brachiis suffultus tabulas inspexisse Geographicas dicitur. usq; adeo turpe sibi non duxit Imperator prudentissimus: etiam medios inter armorum strepitus ad studia Mathematica incumbere.

Jam verò ad Scholarum & Philosophorum umbracula descendere si placet: quis Thaletem, quis Pythagoram, Platonem, quis hujus discipulum Aristotelem, quis alios in omni scientiarum genere Clarissimos Viros, naturæq; speculatores acerrimos, plurimum in his temporis studiiq; posuisse negabit? Contra verò quis esse audebit tam insigniter impudens: ut tanta doctrina, virtute, prudentia præditos viros aut rem minus præstantem amasse & excoluisse: aut in inutilem eandemq; non necessariam, tantum studium tantamq; operam contulisse, affirmare non dubitet?

Certè de utilitate & necessitate harum artium si dicendum esset: reipsa videremus: non Argum illum, qui à Poëtis fingitur, vel decem oculorum millibus, tàm acutè tamq; latè unquam videre potuisse: quam longè lateq; duobus saltem, Arithmeticæ dico & Geometriæ, luminibus prospici queat.

Ut enim taceam: quantum ambæ hæ artes & ad Opticam & Astronomiam, cæteramq; Physicam, Medicinam, Politicam, Theologiam cognoscendam docendámve adjumenti adferant: quarum quæso in agendo, sive domi sive foris, seu publicè seu privatim, magis est & utilis & necessaria cognitio?

De Arithmetica fortassis alienũ est hoc in loco dicere: & alius aliquando dabitur de hac dicendi locus.

Ad Geometriam verò oculos convertere si velimus, quantus utilitatis sese quæso campus offert? quantus in Geodæsia? quantis in Mechanicis? in cuneo, vecte, libra, cochlea? quantus in fodinis? quantus in re militari? quantus in nautica? quantus in omnium artium conservatrice Typographia? Et quis est: qui apertam in omnibus Geometriæ necessitatem non videat? Quod stamus: quod sedemus: quod surgimus: quod ambulamus: Geometriæ usus est manifestissimus. Hinc domicilia extruuntur: hinc ædificantur urbes: hinc vestimenta nobis suppeditantur: hinc omnia tàm belli quàm pacis instrumenta parantur.

Tanti autem & tam multi utilitatis rivuli, cum ex omnibus universæ Geometriæ fontibus, in agros nostros scaturiant: fœcundissimus tamen esse videtur ille: qui ex Rotundi sese latebris effundit. Hinc enim exactissimo triangulorum calculo & machinarum instituitur secundum axes suos dispositio: & vix unquam elusæ deducuntur collimationis viæ: imò tota hinc hostilis illa in arces & oppida irruptio optimè perficitur. Turpi clade ipse sibi ad Meliteorum urbem Philippus nocuit: quòd scalas nimis breves muris admoveri curavit. idem ad Mediolanum Gallis obfuisse dicitur. Et nostri temporis plura recenseri exempla possent: odiosa ni essent. id igitur hac doctrina probè percepta vitari poterit. Hinc enim cum omnis generis magnitudines ita metiamur exactè: ut etiam in splendidissimis regum aulis notatu dignum nihil sit: quod hac dimetiri geodæsia nequeamus: quid facilius quam fossarum latitudines & profunditates capere: & anguli harum recti basin investigare?

Neque vero in bello saltem vim suam Geometria Rotundi exerit: sed in pace quoque: idque multo magis. pacis enim tempore Cœlestis civitatis vias exactissimè describit: sydera certis suis domiciliis distinguit: terram climatis, aliisq; ad cœli formam partibus, distribuit. Hinc eæ prodierunt tabulæ: in quas Alphonsus

phonsus Rex quadringenta aureorum millia, liberalitate sanè regia insumpsisse dicitur. Hinc illis veterascentibus novæ substitutæ fuerunt. Et hinc aliquando tandem, si ita Deo visum fuerit, nitori suo, omnium illa laudatissima scientiarum Astronomia, restituenda erit: non tàm fortassis tabularum inde confectis centuriis: quàm solidis calculi Astronomici axiomatis deductis: explosa insuper fictitiorum illorum circulorum & orbium myriade: quæ in ipsam est naturam naturæq; opificem injuria: & sine artificio optico luci artificiosæ contraria: sine Cœlesti numine corporibus illis nobilibus repugnans.

Tantam potentissime Rex, cum viderem ex hac unica Geometriæ parte, in omnes artes, utilitatem redundare: & vero non ita adhuc à quoquam traditam scirem: ut à studiosis hujus doctrinæ percipi commodè posset: rei literariæ hac in parte, pro ingenioli mei tenuitate, succurrendum esse duxi. Quæ igitur hac de re apud Regiomontanum notare, apud Copernicum perdiscere, è Ptolemæo atque Rheinholdo haurire, è Petro Nonio aliisq; viris eximiis tàm superioris quàm nostri seculi percipere, & pro quantulacunq; mea diligentia perscrutari potui: ea studiosè comportata, & pro tenui nostro judicio in præsens hoc volumen digesta, studiosis harum rerum communicare volui.

Neque verò hoc tanquam arrogantius à me dictum accipi volo. Novi ego meo me pede metiri: novi etiam quam curta mihi sit & accisa doctrinæ supellex. Veruntamen eo me consolor: quod qui non invidioso sunt ingenio præditi: quique publicarum sunt utilitatum studiosi: ii, si quid habent, quod commoditatem habere videtur, qualecunq; illud sit, in medium proferre solent: ut, quoad fieri possit, societati humanæ prodesse voluisse & etiam profuisse videantur. Ejusmodi voluntatem & ego in re fortassis levi & abjecta, utili tamen & pernecessaria declarare statui.

Sub

Sub tuæ autem, illustrißime Princeps, Majestatis nomine, tyrocinia mea in lucem prodire volui: non ut ignaviæ atq; inertiæ subsidium & patrocinium quærerem: sed ut quem Deus Opt. Max. patriæ meæ longè dulcißimæ præesse, vel hæreditario jure, illustrißimum Principem voluit: erga eum ego gratitudinis aliquod documentum relinquerem.

Quin & æquum esse arbitratus sum: ut grata Mathemata suos maximè Mecœnates & Patronos celebrarent. in quorum numero regiam T. M. consistere: vel insignis illa erga nobilissimum & in Mathematis excellentißimum virum Dn. Tychonem Brahe magnificentia docere satis poterit: Academia verò Hafniensis nunquam tacebit. Nimirum id verè regium est, ut regni, sic virtutum paternarum hæredem esse. Eodem enim animo M. T. parentem, laudatißimæ & serenißimæ memoriæ Principem, in Mathematis provehendis fuisse: & res testatur ipsa: & universa agnoscit hominum literatorum corona. uterq; igitur munere suo atq; officio regio functus est probè. Duo enim in rege bono divinus ille Plato requirit: potentiam regiam & animum philosophicum. Nullum enim regnum absq; potentia diuturnum: nullum absque literis præclarum. Hæc verò conjuncta cum sunt in rege: tum demum bene beateq; eos, qui subsunt, vivere necesse est. Hæc igitur duo, cum & tu, Princeps serenißime, hæreditario quasi jure, à parente tuo acceperis: sempiternam tibi laudem & felicitatem comparabis.

Itaq; hoc animo tuo philosophico, & hoc amore erga bonas literas earumq; cultores fretus, Rex clementißime: primitias hasce meas M. T. ea qua par est animi submißione dono dicoq;: & ut boni consulat etiam atq; etiam rogo. Scio equidem munusculum hoc tanto rege indignum esse. Sed tamen spero qua doctrina, sapientia, clementia & humanitate prædita est M. T. libenter hoc qualecunq; opusculum & me quoq; ipsum suam in clien-

telam recepturam. itaque me meaq́ʒ ſtudia M. T. commendo: eiq́ʒ omnia mea officia, operas & labores offero & mancipo. Etenim ſi hanc primam ſtudiorum meorum quaſi ſegetem non ſpreviſſe M. T. cognovero: frugem illi aliquando, ſi non optimam, aliquanto tamen quam hanc meliorem & maturiorem, Deo me bene juvante, offerre conabor.

Sed ne diutius M. T. regiam detineam: eam, cum tota illuſtriſſima Danica atque Holſatica domo, Deo ter Maximo commendo: qui eam, in utilitatem Reip. cum Chriſtianæ tum politicæ, ſalvam & incolumem, fortunatam & proſperam, annos deinceps plurimos, conſervet, tueatur, defendat. Amen. Baſileæ Rauracorum anni ſalutis humanæ M. D. LXXXIII. *Menſe Septembr.*

Majeſt. T.
addictiſs.

Thomas Finck
Flenſpurgenſis.

TH. FIN.

TH. FINKIVS FLENS. LEctori philomathi ac Mathematico

S. P. D.

CVM primum ego ad Mathematicas diſciplinas animum appuli, Lector benevole: id mihi negotii credidi ſolum dari: bona via ac methodo in iiſdem ut progrederer. Sed in Geometria dum verſor: copiam quidem in Euclidea ϛοιχειώσει propoſitionũ permagnam video: methodum vero in ubertate tanta nullam, aut vix ullam videre potui. Quam id me perturbarit: quam etiam cordi meo doluerit: & ego ſcio: & alii, familiariter mecum qui vixerunt, non ignorant. Alia itaq; ad perſcrutandam divinam hanc ſcientiam, via inſiſtendum mihi putabam: cum hac ſucceſſiſſet parum. itaque ad P. Rami me volumen Geometricum converti. inveni illicò quod in Euclide deſiderâram diu. Nam & methodi ſeſe clariſſima offerebant veſtigia: & ars quoq; ipſa copioſius aliquanto & luculentius inſtructa videbatur. Mirum, quam me res hæc delectârit: & ad penitus cognoſcendam hanc artem inflammârit. Quare unica lectione nõ contentus: aliquoties illud legi atq; relegi: artis nimirũ totius perdiſcendæ cupidiſſimus. Aperuit mihi vir hic mentis oculos: quòd Logices uſum in Mathematis egregiè monſtrare eſt viſus: Sed & ita judicium mihi, quod abſq; tamen oſtentatione dictum velim, acuit: ut tertia demum & quarta repetitione viderem: ipſum quoq; Ramum, cum cæteris penè ſatisfecerit omnibus, ſibi ipſi tamen nondum ſatisfeciſſe: ſed locos tantum ampliſſimæ hujus ſcientiæ comportaſſe atq; digeſſiſſe: quo certis eam limitibus incluſam haberet. itaque illius ego veſtigia ſequens: quos ille varios authores allegat: eos omnes, quotquot habere potui, cognoſcere diligenter cœpi: ex iiſque elementorum Geometricorum copiam conquirere: judicioque logico qualicunque digerere ſtudioſè ſum conatus. Sed hic jam Epipedometrica ſeſe offerebant: jam occurrebant ſtereometrica: quæ curſum laboris noſtri non mediocriter retardabant. Quamobrem unam mihi deligere partem, viſum eſt operæ precium: quam Geometriam ego Rotundi voco: uſus certè in omni Mathematicarum ac Phyſicarum diſciplinarum genere ampliſſimi. in qua cum mihi, hoc quidem tempore, ſatisfeciſſem: quòd amicis rectè judicantibus ſatisfeceram: diutius eam apud me latere nolui: ſed candide tecum, benevole Lector, communicare.

Ad οἰκοδόμησιν autem & ſtructuram ipſius operis quod attinet: id cum primis operam dare conati ſumus: ne quid ἀμέθοδον, hoc eſt ὕϛερον πρότερον eſſet: néve aliquid aut non ἀληθὲς aut non χρήσιμον aut non ἀκόλουθον, ut cum Galeno loquar,

in ele-

in elementis ipſis aſſumeretur.itaq; nec Euclidearum propoſitionum ordinem ſervare ſemper potuimus:nec Ramearum ubiq; connexionem retinere.

Data poteſtas hæc Hippocrati,data Leoni,Theudio,Hermotimo,Euclidi, Theoni,data aliis στοιχειώταις fuit:ut ordinem propoſitionum in anteceſſoribus ſuis ſine offenſione mutarent:eademq; alio quam ipſi modo traderēt. idem ſibi jus majores noſtri concedi voluerunt.Quotus enim quiſq; Arithmeticorum etiā eorum, qui authoritate in Scholis publica docentur, propoſitiones Euclideas ordine Euclideo diſpoſuit?Vogelinus certè,Cæſareus olim Mathematicus,in Geometricis idē attentavit:nec unius tantum libri,ſed varia variorum librorū elementa tranſpoſuit Idem ſibi licere arbitratus eſt Maurolycus,non in Euclide tantum: verum etiam in iis,qui poſteriores Euclide fuerunt:Menelao ſcilicet & Theodoſio.Neq; ante hos peccare ſeſe Regiomontanus putavit:ſi ſuo atque libero judicio veterum στοιχεῖα ſuum in librum digereret:& quidē alio modo quā à veteribus collecta invenerat digereret.

Non ego tantis me artificibus comparo : eandem tamen,quam illi,poteſtatem à Lectore candido conſecuturū me ſpero.Etenim in ea ſum ſententia:ut,ſi quid benè ſit ab iis qui arte præſtant:id ab aliis quoq; qui doctrina non ita fortaſſis excellunt, non arroganter annotari:& ad imitationem transferri poſſe exiſtimem.

Nec quicquam hac ratione aut Euclidi,aut Regiomontano,aut Vogelino,aut Ramo detractum arbitror. Damus alijs hanc veniam : petimusq; viciſſim:veniam tamen aureæ methodi doctrinæ conſonam.

Conſtructionis etiam hæc cura fuit:ut ipſa tractatio certis ſive capitibus ſive libris diſtingueretur.Libros tamen potiùs quam capita retinere placuit.utrumq; noluimus : ob citationem propoſitionum antecedentium in demonſtrationibus ſequentiū:quæ enumeratione elementi,capitis,libri nimis futura prolixa videbatur.

Cęterum in demonſtrationibus:eſſent'ne citanda elementa Euclidea an Ramea, nonnihil hæſitavi.Euclidis adducenda putabam: quod Scholarum ea ferè omnium authoritate eſſent munita.At Rami tandem citanda potiùs eſſe ſtatui: primum quidem quòd hæc ſtatim Euclidea monſtrarent:dein quod præ Euclide quædā haberet Ramus:quorum uſus mihi in demonſtrationibus fuit neceſſarius : tū quod Rameæ Geometriæ hanc meam qualemcunq; operam ita ſum accōmodare conatus: ut circulari ea ac Sphæricæ ſubjungi,aut cum eadem conjungi poſſit.Quapropter pleraq; ipſius elementa retinui:ſed aliorū ea Geometrarū & novis quibuſdam theorematis adauxi.itaq; ubi elementis quæ citantur additum videris R.Rami Geometriā intelliges:ubi vero abſq; R citata propoſitio fuerit:eā præſentis eſſe tractatus cōcludes.

Ex Menelao quoque elementa nonnulla adduco : ſuppeditavit etiam quædam Theodoſius.Sed ego omnia iſta Ptolemæo & Regiomontano noſtro aſcripſi : propterea quod Menelaus , tametſi Meſſanæ ſit excuſus , non facilè tamen reperiatur:

Theodosius vero à Vogelino aliter, aliter à Maurolyco, aliter demum à Pena, proinde q; variè sit editus. Atq; hæc constructionis ratio fuit.

Porro ad figurarum picturas & lineamenta quod attinet: nõ minor fortassis à me cura suscepta fuit. Sed earum elegantia penes me esse non potuit. Neq; vero Typographi nobis opera forsan defuisset: si sculptorem nancisci idoneum & suæ satis artis peritum potuissemus. Qua quidem re non offensum iri benevolum Lectorem arbitror: si istud cogitaverit: ut non aurum est semper quod aurum splendet: sic nec sordidum semper esse pallio sordido quod tegitur. Et minus forsan asinina leonem pellis deformat quam asinum leonina exornat.

in cæteris verò, an id, quod maxime nobis propositum fuit, assecuti simus, penes alios judicium esto. Vos enim ego Lectores Logicos & Mathematicos, Mathematicos, inquam, & Logicos Lectores rogo: ut candidè atq; syncerè de hac nostra qualicunque opella judicetis. Si assecutus sum, quod volui, gaudeo: sin minus, ignosci mihi cupio: sententiam emendare paratus: sive me ipse melius docuero: sive monitus quam aliis melius quid & elegantius didicero.

Datum id summis fuit ac clarissimis in omni scientiarum genere philosophis: ut absq; ulla nominis sui labe libros recognoscerent atq; corrigerent. Non igitur hæc tyroni occlusa esse via debet: & fortassis, quam illis, minus.

A' te itaq; Claris. David Vvolkensteini: quia princeps mihi & ad suscipiendam & ad ingrediendam rationem horum studiorum exstitisti: pro ea quæ inter nos est amicitiæ conjunctione contendo: ut hæc mea tyrocinia tuæ submittere censuræ velis: diligenterq; pro ingenii quo polles solertia, judiciiq; acumine examinare, ponderare, meq; ubi opus fuerit & tibi videbitur, pro candore tuo monere ne graveris. Nec de tua hac in re promptitudine quicquam dubito. Cum enim Mathematicis me multis, in primis verò tibi rem me facturum esse longè gratissimam scribas: planè confido futurum: ut id re ipsa sis declaraturus.

Quin & in eadem Argentoratensiũ Academia, nec te Conrade Dasypodi transire possum. vellem tibi amice ab Astrologicis cæterisq; domesticis tantum daretur ocii: ut hæc nostra perlegere atq; perpendere posses.

Sed & te Christiane Vrstisi & te Michaël Mæstline non rogatos relinquere non possum. peto igitur: ut quæ mihi humanitatis & benevolentiæ officia præstitistis: eadem è Musæo nostro jam avolanti metricæ ne denegetis Nota mihi vestra in studiis hisce eruditio est: nota in judicando fides: facite quæso: ne ignota mihi vestra sit in admonendo diligentia.

Nec te Lazare Schonere: qui vel ipso Schonerorum nomine Mathematicus es, difficilem mihi hoc in officio futurum arbitror.

Et quia semel hunc ego campum sum ingressus: ex eodem aliis egregiis artificibus non compellatis exire vix possum. qui si de facie mihi adhuc noti non sunt:

ex suis

AD LECTOREM.

ex ſuis tamen, quæ in hoc ſtudiorum genere ediderunt, ſcriptis innotuérunt.

Te itaq; nobiliſſime Dn. Tycho Brahe rogo: ut, ſi hæc mea etiam mare Balthicum ad te transvolarint: hoſpitium illis ne deneges: ſed benigno vultu excipias: itaq; traſtes: ut & quod boni ferãt nõ tegas: & quod reprehenſione in illis dignũ inveneris non diſſimules. Facies id ob cõmunia noſtra ſtudia: facies ob cõmunem regẽ: Principem atq; Dominum noſtrum clementiſſimum. Nuper in patria cũ verſarer, mare Balthicũ ſalutandi tui cauſa trajicere cõſtituerã. Sed ita illud tũ ſæviit: vt ei me propter inſtantẽ hyemẽ cõmittere non ſim auſus. Quæ igitur tũ præſenti mihi beneficia procul dubio exhibuiſſes: ea abſenti in Chartis hiſce meis ut præſtes etiã atq; etiã oro.

Pluribus hoc idem à te Dn. Thoma Diggeſee peterem: ſi de tuo ardore in his ſtudiis & induſtria dubitarem. E' ſcalis atq; alis Mathematicis notum mihi primum factum eſt nomen tuum. Fac quæſo: ut ſicuti ſcalas cœlo compendioſas admoviſti: ſic breviſſimos ad ſydera in calculo acceſſus eligas. Quod utinam Copernici problemata præſtarent. Ego enim quo pacto præſtent videre nondum poſſem. Tu itaq; mi Thoma judicabis: & calculum Gebri, Regiomontani, itemq; Copernici, cum eo, qui hiſce in libris ex Ptolemæo atq; Rheinholdo deducitur, conferes.

Quod ab hiſce, quos recenſui, obtinere ſum conatus: idem mihi, uti ſpero, cæteri quoq; artifices, ubicunq; locorum eſtis, tribuetis. Nominatim omnes appellare nequeo. A' te tamen Dn. Thaddęe Hageci, nec non Joan. Prætori, idem ſigillatim peto. Vos enim ego ob ſublimem illam in Meteorologicis eruditionem miratus ſum & amavi ſemper: ſed & deinceps colam & amabo perpetuò. Cæteros ego artifices nominatim omnes perſequi ſi vellem: & charta me & oratio deficeret. Tot enim excitavit Deus Opt. Max. ſingulari ſua bonitate hoc noſtro ſeculo inſignes & exercitatos in hoc ſtudiorum genere viros: vt percenſeri numerando vix poſſint. Quare cum difficile ſit non aliquẽ, nefas quenquã præterire: Vos ego ſingulos & univerſos una voce appello: veſtræ cenſuræ atq; judicio ſubmiſſè mea ſubjicio: examinãdaq; permitto. Quod ſi morem geſſeritis: libenterq; mihi hac in re gratificati fueritis nõ ego

Bina dabo argento perfecta, atq; aſpera ſignis
Pocula—
Nec tripodas geminos, auri duo magna talenta:
Cratera antiquum—

Sed, ſi eo contenti eſſe vultis, me ipſum vobis largiar: ad ſimile officiorum genus paratiſſimum ſemper habebitis: veſtri ſtudioſum experiemini ſemper: beneficii præſtiti immemorem nunquam deprehendetis.

Atq; hic præfationis hujus finis eſto: quam ipſe librorum contextus inſequetur: prius tamen præmiſſis elementis duobus: quæ tamen ad finem reſtituta ſunt. Ea ad quintum Rami librum referenda puto: & ſic deinceps citabo. Valete Lectores optimi: & ſtudiis noſtris juvenilib. benignè favete Baſileæ Raurac. M. D. LXXXIII.

- Rotundum est
  - Circulus: in cujus Geomet. est
    - Partitio Est.n.
      - Simplex in
        - Lineis
          - Rectis
            - Secantibus — 1
            - tangentibus
              - solis — 2
              - cum rectis — 2
          - Obliquus
            - solis — 2
            - cum rectis — 2
        - Segmento quod est
          - Sector in
            - centro — 3
            - peripheria — 3
          - Sectio cujus
            - Adjuncta
              - Fabricæ quædam — 3
              - comparatio in angulis — 3
            - Species
              - Semicirculus in quo sunt
                - 1
                  - Anguli — 4
                  - Latera — 4
                - 2
                  - Sinus
                    - primus — 5
                    - secundus — 5
                  - ei connexa
                    - Tangens — 5
                    - Secans — 5
              - inæqualis semicirculo.
      - Conjuncta in adscriptione
        - circuli — 6
        - rectilinei
          - trianguli — 6
          - triangulati ordinati
            - communi — 7
            - propria
              - Quadrati — 7
              - Quinquang &c. — 7
    - Usus in
      - tetragonismo — 8
      - triangulorum
        - Canone — 9
        - Calculo: theorematis & postulatis novo modo exposito: — 10
        - unde deducitur Geodæsia rectarum — 11

Sphæra

- *Sphæra ubi sunt*
  - Geodæsia
    - simplex
    - comparata
  - Circuli ubi considerantur
    - Communia
      - perpendiculum
      - axis, &c.
    - specialia in maximo
      - 1 cum aliis
        - Ratio
        - Similitudo
      - 2 Triangulum ubi est
        - Expositio
          - Communis
            - Trianguli
            - Triangulorum
          - Propria
            - Rectanguli
            - Obliquanguli
        - Calculus ex
          - Ptolemæo & Rheinholdo per
            - consectaria
              - 17 pro
                - Rectangulo 16. ad inventionem tertii datis duobus quibusvis terminis per solã multiplicationẽ
                - Rectangulis 1.
              - 6 pro obliquangulo
            - Postulatum pro reliquis obliquanguli quæsitis.
          - 2. p. 5. Regiom. unde est duplex inventio facilior quam in postulato fuerat.

12 (simplex … Similitudo)

13 (Trianguli … Obliquanguli)

14 (Rectangulo … fuerat)

Benevolo Lectori S.

Qui tibi, amice Lector, volumen integrum communicavimus: nec quæ commiſſa fuere ſphalmata graphica reticere debuimus: ut qui tecum cōmmunicare plura parati fuimus: in pauculis hiſce tibi ne defuiſſe videremur.

Prior autem & ſolitarius numerus paginam: poſterior paginæ verſum denotat.

20. 19 *lege* 7.e.6. 32 *in figura i & u locis commuta & e ad alterū terminum a conſigna.* 25 *in figura ſecūda pro s e e pone s e p. & a ac u ad s propius admoneantur ut angulos notent, tum perpendicularis ad diametrum notetur literis i ad diametrū, t ad peripheriam.* 33 *in ſecunda figura in recta a o in fine deſideratur e.* 33. 3 *pro e o lege a o.* 44 *in prima figura in peripheria deſideratur i in linea o i.* 47. 21 *ſubdupli.* 54. 14 *pro a o e, o a e.* 60. 25 *hoc eſt.* 60. 44 *in baſi æquiangulum.* 68 *in termino ſiniſtro diametri deſideratur a.* 87. 6 *l & r angulis.* 96. 1 *dele: ſegmentum majus.* 133. 12 *præcedentis pro ſequentis.* 281. 11 *è ſequenti linea poſt datæ: ponatur peripheriarum.* 285. *in linea i o deſideratur e.* 324. 27 *planus è* 44 *per* 14. 324. 28 *planus è* $\frac{6\ 1\ 6}{6}$. 334 *in figuræ recta in t q deſideratur in fine n.* 339. 6 *o i e.* 340. 31 *diſtant igitur e &c.* 341. 2 *anguli a i e. inq; figura ultima literæ ſic conſequuntur l s t m.* 345 *in ultima figura in arcus o m fine deeſt t.* 355 *in arcus e y altero fine deſideratur u.* 378. 11 *pro ſi, si.* 379 *totus ſecundus verſus proxime antecedere debet* 23.e. 405 *in recta i y omiſſum eſt u. præterea* 91. 13 & 274. 14. & 275 25 & 288. 5 *pro i lege* 1. 283. 16 &. 305. 5 *in particula* 15 & 335. 21 *pro* 5 *leges.* 287. 13 *poſt e u.* & 305. 5 *poſt vocem per.* & 333. 27 *poſt* 15.e. & 334. 28 *poſt* 15 e. & 339. 11 *poſt vocē per.* & 342. 14 *poſt vocē per.* & 378. 8 & 382. 13 *poſt ſyllab. ca pro s, s.*

Etymologica & orthographica facilè Lector ipſe reſtituet ut 29. 9 *contactum.* 32. 21 *ſemiſſæ.* 32. 27 & 44. 12 *æquatur* 60. 23 *dico.* 60. 27 *rectæ.* 61. 34 *latus.* 63. 15 *per.* 70. 12 *exteriori.* 74. 30 *ad.* 79. 13 *præmiſſa.* 87. 6 *circuli.* 115. 1 *partes.* 131. 20 *interior.* 312. 2 *radius.* 315. 4 *tum.* 317. 5 *habeas.* 324. 12 *cubo.* 337. 30 *dimittantur.* 338. 7. *totam.* 352. 18 *minoris.* 353. 9 *diſſeminata. & ſi quæ ſunt alia.*

In loco reſtitutionis depravatorum numerorum canonis triangulorum prior numerus gradum quadrantis in ſupero margine, ſecundus minutam in ſiniſtro, tertius gradum ſignificat periodi quæ quarto numero indicatur.

Locus reſtit. in canone ſinuum

Reſtit.

- 0. 64. 0. 1. 1. & 64. 2. 1. 2.
- 3. 14. 41. 2. 2 & 62. 18. 1. 2.
- 4. 63. 18. 1. 1.
- 5. 27. 4. 2. 2 & 49. 20. 1. 2. & 35. 41. 2. 2.
- 7. 23. 45. 1. 2.
- 8. 13. 21. 2. 1 & 31. 56. 2. 2.

Locus reſtit. in can. ſecant.

Reſtit.

- 0. 9.28.3.1 & 23.5.9.1.3. & 49. 5.3.1.
- 1. 68.35.3.1.
- 2. 70.56.1.2.
- 3. 21.18.1.2 & 32.24.1.2 & 39.43.1.3. & 44.8.2.2.
- 6. 68.39.2.2.
- 8. 63. 9.1.1.

Locus reſtitut. in canone tangent.

Reſtit.

- 0. 15.27.1.1 & 8. 0.2.2 & 19.20.2.2. & 56. 8.1.2 & 59.59.2.2 & 86.57.2.1. & 87.21.3.2.
- 4. 46.14.3.2.
- 5. 39.14.1.2 & 43.32.3.2.
- 6. 8.34.1.2 & 16. 0.22 & 38.40.2.1. & 44.56.1.1 & 47.43.1.2 & 87.32.1.2.
- 7. 27.56.2.1 & 47.7.1.2.
- 9. 8.55.3.1 & 17.7.1.2 & 34.10.2.1. & 35.35.2.1 & 41.1.3,1 & 46.19.2.1.

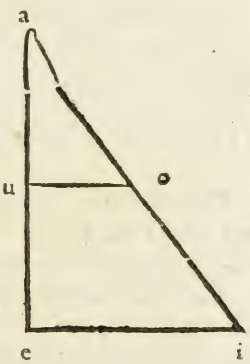
Præterea in canone ſinuum ad 21. 34. *locis commuta* 6 & 7.
in can. tang. ad 87.19. *muta* 6. *cū poſte.* 3.
in can. ſec. 37.22. *ult. period. ſic eſt* 291.

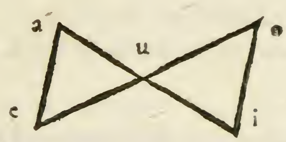
Reliqua ſi quæ ſunt in gradibus extremarum periodorum ex vicinis areis facilè reſtituentur.

ELEMENTA DVO PTO-LEMAICA INSERENDA AD finem libri quinti Rami.

14. *SI duæ rectæ se secantes parallelis pluribus intersecantur: parallelæ segmentis conterminis sunt proportionales.*

Esto sint duæ rectæ a e. a i. sectæ à se & parallelis o u. e i. Erit ut u o. ad e i, sic a o ad a i. Elementum veritatis suæ causam aliam non desiderat quam per species inductionem. Nec generalior demonstratio dari potest. Elementum enim cujus specialia exempla proximo genere differũt generali medio demonstrari nequit. Sufficit itaq; inductio, quod si parallelæ sint inæquales similiter contermina segmenta inæqualia existere: si sint æquales fore quoque ut segmenta habeantur æqualia: quemadmodum hic patet in intersectis verticaliter ad u.



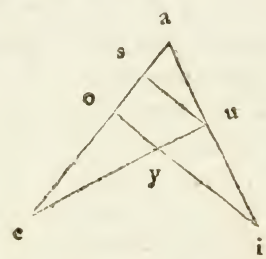


Nam ut a e & o i æquantur, ita & o u. u e. vel a u & u i.

15. *Si quatuor rectarum duæ faciant angulum, reliquæ ab harum terminis in se reflexæ priores secent: ratio unius ad segmentum suum, vel segmentorũ inter se fit è ratione ita conterminarum, ut prima facientium conterminetur*

antecedentis factæ principio, secunda huius consequentis fini contermina terminetur in finem consequentis factæ.

Sint enim quatuor rectæ a e. a i. e u. i o. quarum duæ priores faciant angulum ad a. reliquæ ab harum terminis reflexæ secent se in y. atq; duas priores in u & o.



Dico primò rationem i a ad a u esse factam è ratione i o ad o y. & y e ad e u.

Qui primus Ptolemæi casus est.

Secundò rationem i u ad u a esse factam è ratione i y ad y o. & o e ad e a.

Qui secundus est Ptolemæi casus.

Et sic deinceps plures colliges casus prout elementum habet.

Elementi autem ut & præcedentis generalis demonstratio nulla est. Ptolemæus casus suos duos speciales demonstrat speciatim, & primum quidem hoc modo:

Ducatur à termino u pararella ipsi o i. sitque u s. Erit tum per 14. e.

ut i a ad a u, sic i o ad u s.

Atqui ratio i o ad u s est facta è ratione i o ad o y & y e ad e u.

Ergo ratio i a ad a u ex iisdem erit facta Assumptionem hinc probat: ex axiomate illo logistico: Si numerus numeros multiplicet facti erunt multiplicatis proportionales: & vero assumpta deforis linea multiplicante o y. multiplicante inquam o i & u s. ergò erit ratio i o ad u s. facta è ratione o y ad o i. & o y ad s u. id est, per 14. e. y e ad e u.

Quod

Quod si hæc ad Arithmetica retuleris: prout ad multiplicationem proportionum referri debent omnia ea quæ de facta dicuntur ratione: facilis tibi in promptu erit demonstratio. Sint enim multiplicatæ proportiones, ut i o ad o y. sic i o ad o y. hoc est, utrinq; æqualitatis ratio: & rursus y e ad e u. ut o y ad u s. per 14. e.

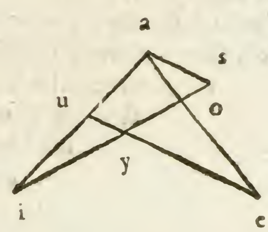
i o.	o y.	i o.	o y.
y e.	e u.	o y.	u s.

Hic quia secundarum rationum termini heterologi æquantur: erit per doctrinam multiplicationis proportionum, ut factus ab y e per i o ad factum ab e u per o y. sic i o ad u s. hoc est, è doctrina multiplicationis (quæ longè ab additione diverest) rationum ratio i o ad u s. hoc est, i a ad a u. fit è ratione i o ad o y. & y e ad e u.

Alter casus à Ptolemæo hoc modo demonstratur.

Ducatur rectæ e u per 3. c. 12. e. parallela a s.

Erit itaq; per 13. e. vt i u ad u a. sic y i ad y s. at assumpta deforis recta o y multiplicante ratio i y ad y s. fit è ratione i y ad y o. & o y ad y s. hoc est, o e ad e a per 13. e. Ergo ratio i u ad u a fit è ratione i y ad y o. & o e ad e a.



Poteris & eundē casum ex proportionum multiplicatione contexere, ut & primum. Sunt enim multiplicatæ proportiones

i y.	y o.	i y.	y o.
o e.	e a.	y o.	y s.

Ergo factus ab i y per o e ad factum ab y o per e a est ut i y ad y s. hoc est, i u ad u a.

Atq; hęc exempla hujus elementi Ptolemæus habet. Sunt ve-

 rò plura:

rò plura: ut & Theon ea collegit: quæ ut in promptu ſint hic accipe.

	Facientes.		Facta.	
Antec.	e a.	o i.		
Conſeq.	a o.	i y.	e u.	u y.
Antec.	a o.	e y.		
Conſeq.	o e.	y u.	a i.	i u.
Antec.	u a.	i o		
Conſeq.	a i.	o y.	u e.	e y.
Antec.	e o.	a i.	e y.	y u.
Conſeq.	a o.	i u.		

Quæ ſimiliter arithmeticè facile demonſtrantur: modo id, in demonſtrationibus contexendis, Theonis obſervetur: quod quæ deforis aſſumitur ſit vel parallela alteri duarum quibus interponitur, aut etiam adjungitur: vel pars alterius earundem: vel econtra ut altera harum ſit pars aſſumptæ deforis. Sunt & plures caſus de ſegmentis diverſarum rectarum, in quibus curioſi ſunt Arabes, Thebitius, Alchindus, etiam recentiores ſub nomine regulæ ſex quantitatum. Verum hi per doctrinam proportionum multiplicationis hinc deducti Arithmeticæ peritos non latebunt.

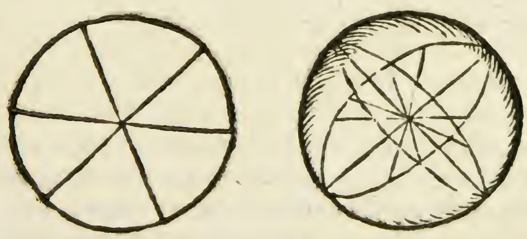
TH. FIN.

TH. FINKII
GEOMETRIÆ ROTVNDI,
LIBER PRIMVS.

De circuli rectis secantibus.

Rotundum est figura cujus radii omnes æquantur.

Geometria nobis proposita est rotundi: quæ benè metiatur rotundum. Hic itaque memor eorum de quibus in Præfatione dixi, necesse nō habeo plura repetere quàm ipsum subjectum mihi propositum requirit. Cum itaque Geometria proposita de rotundo sit: definiri rotundum oportuit: definitione collecta è 15. d. 1. & 14. d. 11. Eucl. Vocatur autem Rotundum pro figura rotunda: ut & vulgò planum, triangulum, quadrangulum pro plana figura, triangula, quadrangula dicimus.



Aequatio autem radiorum omnium peculiaris est rotundo. At æquantur (inquis) etiam radii parallelogrammi 3. c. 6. e. 10. R. Aequantur, inquam, sed non omnes: sed tantum ejusdem dia-

metri aut ſaltem plures diagoniorum, aut ex angulis ductorum, aut rectè ſe ſecantium ut in quadrato: ut in figuris ad dictum Rami elementum patet. Sic & in omni multangulo ordinato radii in angulos æquantur: interim tamen non omnes radii: multi enim ducuntur in latera proindeq; minores iis, qui in angulos tendunt. Et hæc ipſa æquatio è rotundo eſt: quia circulus in angulis circumſcribitur rectilineo.

Itaq;

2. Diametri in rotundo biſecantur radiis æqualibus.

Ratio hujus conſectarii in promptu eſt. Diameter enim eſt duplicatus radius: ſi ergo ex diametro auferas radium reſtabit radius, dimidius diametri.

Et

3. Rotunda diametrorum æqualium ſunt æqualia.

Euclides ſpeciatim de circulo proponit 1. d. 3. at generale eſt ad omne rotundum: ut nempe rotundum rotundo ſit æquale: ſi diameter diametro æquetur: aut radius radio.

4. Rotundum ex iſoperimetris ſui generis eſt maximum.

Sui generis intellige planum aut ſolidum: nempe planum plano ſolidum ſolido majus: Talis eſt circulus in planis, ſphæra in ſolidis.

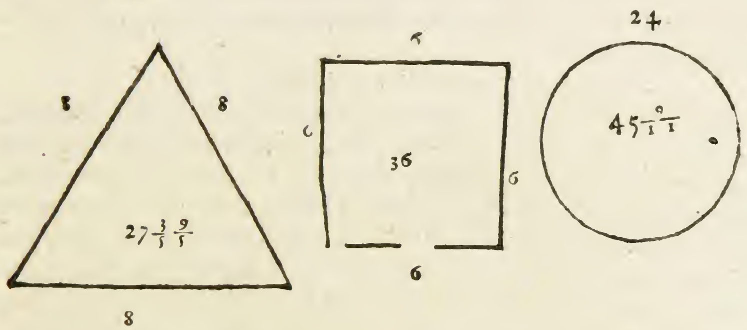
Elementi autem Veritas patet: Nam ut in planis circulus quolibet rectilineo ordinato, ſic in ſolidis ſphæra ordinatis quibuſlibet corporibus eſt ordinatior: Et ſic Ptolemæus rotundum πολυγωνίαν: Ariſtoteles ὁλυγωνίαν vocat: & Euclides etiam eſſe rotundum polygoniam multorum & infinitorum angulorum 2. p. 3. aſtruere videtur. Et vero axioma illud & principium Geometricum eſt: Ex iſoperimetris homogeneis ordinatius eſt majus, ex heterogeneis ordinatis terminatius. Principium hoc Theon à Zenodoro repetitum in Commentarijs ſuis in Ptolemæum, demonſtrat ad caput ὅτι σφαιροειδῶς ὁ οὐρανὸς φέρεται lib. 1. quod interpreti latino eſt ſecundum: & demonſtrat inductione ſpecierum demonſtratarum: nec poteſt ejus generalior

adduci

adduci demonſtratio: proximo enim genere inter ſe ſpecialia exempla diſsident: itaq; & nos exemplis poſt generale allatum principium contenti erimus ad hoc propoſitum elementum.

Sit itaque triangulum æquilaterum, quadratum circulus æqualis perimetri exempli gratia 24. erit hic latus trigoni 8. quadrati 6.

jam ex libris Rami per 9. e. 12. & 6. e. 11. area trianguli colligitur $27\frac{3}{5}\frac{9}{5}$. quadrati 36. circuli autem aream interius inveſtigabimus $45\frac{9}{11}$.



Sic & in ſolidis ſphæra iſoperimetro cubo, tetraëdro, icoſaëdro, dodecaëdro octaëdro major exiſtit.

Atque hinc Plato dixit, rotundam figuram omnium eſſe perfectiſsimam: ideoque Deum mundum Sphæricum figuraſſe: ut ſuo complexu cuncta contineret.

Ariſtoteles etiam in Mechanicis miraculorum omnium in rotundo cauſam & principium ponit.

Qui ſanè liber adoleſcentibus ſtudioſis debet eſſe commendatus. Aggregent huc ſtudioſi locum Quintiliani ex 10. c. 1. de ψευδογραφία illa: quæ ex ignoratione elementi hujus exiſtit. Qui vel propterea cognoſcendus eſt: ut conſtet quanto cum pudore philoſophos ſe jactent ii, qui elegantem hunc Geometriæ uſum ignorant: cum homini etiam rhetori non ſit ignotus. Et certè infiniti per omnia vitæ genera hinc errores & fraudes exiſtunt:

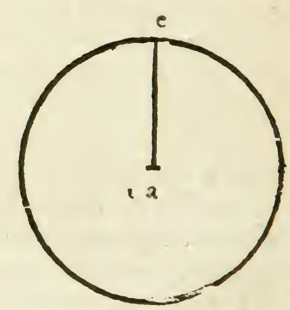
ſtunt: ſi ἰσοπερίμετρα concipiantur ἰσοχώρια. Hinc enim agrorum fraudulenta dimenſio: Geographia mendax: cum inſulæ numero dierum quibus circumeuntur aut circum navigantur, menſurantur. Menſurarum profectò cylindracea quam cubica plus capit: ut quotidiana etiam plebejos experientia docuit.

5. Rotundum eſt circulus aut ſphæra.

Dichotomia hæc eſt rotundi: qua eſſe contenti poſſumus. Nullum enim præterea rotundum eſt. Nam licet ſphæricum, hoc eſt ſphæræ ſuperficies etiam rotundum ſit: tamen hac diviſione à ſuo corpore non excluditur. Et nos tantùm hîc de duplici illo rotundo agemus.

6. Circulus eſt rotundum planum. *15. d. 1.*

Rectilineum Euclidi definitur planum rectis lineis comprehenſum: & ita circulus figura plana peripheria contenta ſive comprehenſa, cujus radii æquentur. Ea enim plurium verborum eſt ſententia. At hoc modo definitur brevius: Planum eſt quia æqualiter intra ſuos terminos interjacet. Nam ſi peripheriæ multas partes feceris: ex una in aliam ſuperficies ſpatio intra eas comprehenſo æquabitur. Et hoc habet commune cum triangulo quadrangulo &c. At rotundum eſſe: & ab ovata figura & reliquis planis omnibus ſejungit.

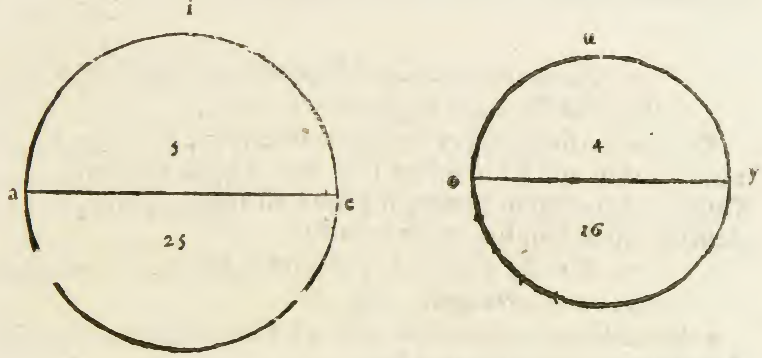


Deſcribendi circuli inſtrumentum idem eſt quod fuit peripheriæ: nempe circinus, ſive is crura recta habeat, ſive valga: ſive integra, ſive amputata. At ut illic motus & fluxus puncti in extremo radio peripheriam lineantis conſideratus eſt: ſic hic motus radii totam aream radiantis perpenditur. ut hîc, ſi a e à terminis e a manente a concipias circumvolvi, donec e ad e redeat area lineata eſt, quæ circulus vocatur.

Atq; hæc de circuli definitione: circulorũ cõparatio ſequitur.

7. *Circuli ſunt ut à diametris quadrata.* 2.p.12.

Circuli plana ſunt ſimilia: eorumque latera homologa ſunt diametri. At ſimilia plana habent duplicatam rationem homologorum laterum i.e. 6. R. Et quadrare diametros eſt duplicare rationem homologorum laterum. ut ex Arithmetica doctrina & ratione quadrandi diametros conſtat. Erit itaque ſic circulus ad circulum, ut quadratum diametri prioris ad quadratum diametri poſterioris. Sic circuli a i e diameter ſit 5. circuli verò o u y diameter ſit 4. Quadratum verò illius 25. hujus 16. erit ut 25 ad 16. ſic circulus a i e ad circulum o u y.

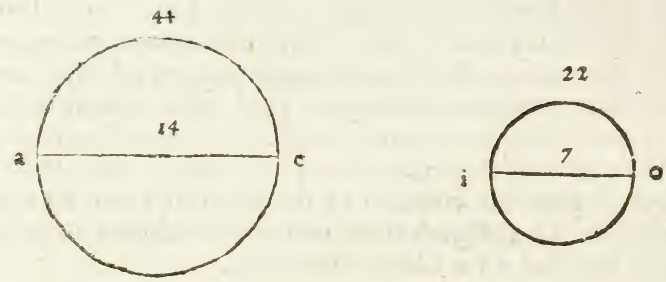


Quod ſi diameter eſſet a e quidem 14. o y verò 7. eſſetq; circulus major 154. minor hinc concluderetur eſſe $38\frac{1}{2}$. Nam quadratum de 14 eſt 196. de 7. eſt 49. jam ut 196 ad 49. ſic eſt 154 ad $38\frac{1}{2}$.

Itaq;

8. *Diametri ſunt ut peripheriæ.*

Eſt Pappi theorema 5.l.1.& 26.the.18.ut ſi ſint duo circuli a e. i o. Sitq; peripheria unius 44. diameter 14. alterius peripheria 22. diameter 7. erit, ut 44 ad 22. ſic 14 ad 7. per hoc elementum Peucerus inveſtigat in Geographia rationem arcuum Aequatoris ad parallelorum peripherias.

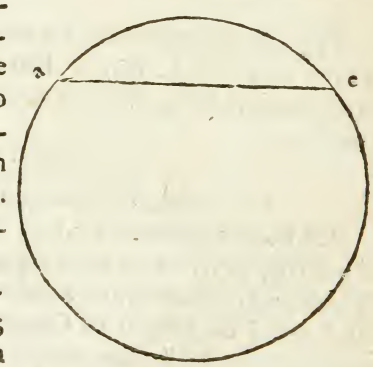


9. *Geometria circularis simplex aut cōjuncta erit. Simplex est in lineis aut in segmentis circuli.*

Distributio simplicis ex subjectis: assumpta propterea ut materia sparsim apud Euclidem fusa luce aliqua separari posśit. Consideratio autem linearum prima est inscriptarum, & quidem inscriptæ simpliciter, hoc modo.

10. *Si recta duobus in peripheria punctis terminetur: cadet intra circulum.* 2.p.3.

ut hic in subjecto schemate recta a e. Et in promptu causa est è definitione lineæ rectæ: quę brevisśima est intra eosdem terminos: brevior itaq; quam peripheria. Et certe postulatum est: & postulatur ab Archimede quod linea, quod superficies ad easdem partes obliqua non cadat extra. 1. & 2. d. 1. de sphæra. idem in isoropicis postulatur.



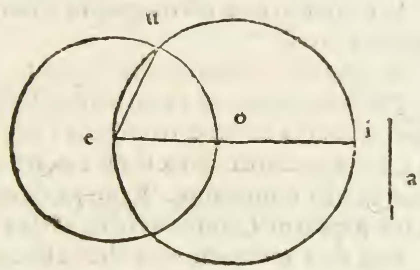
Theon vel Euclides imposśibile adhibet ex 16 & 8. p. 1. Atq; hoc elementum docet rationem

inſcribendæ circulo rectæ: ſumptis nempe duobus in peripheria punctis. Sequitur inſcriptio rectæ æqualis datæ.

11. Si à termino diametri ex eaq; radio æquante datam rectam peripheria deſcribatur: recta à dicto termino in concurſum peripheriarum inſcribetur dato circulo, æqualis datæ. 1.p.4.

Euclides mechanicè ſive proplematicè proponit: in datum circulum datæ rectæ, quæ non major ſit diametro circuli, æqualem rectam applicare: intellige tamen inſcribere: & cognita propoſitione præmiſſa conditio illa, quæ non ſit major diametro, eſt ſuperflua.

Sed eſto data recta a. datus circulus e u. jam e o æquetur datæ a. atq; termino e. radio e o deſcribatur peripheria, concurrens cum dato in u. atq; ab e. ſit recta e u. hæc eſt inſcripta æqualis datæ. Nam e u & e o hoc eſt ex fabrica recta a. ſunt radii ejuſdem peripheriæ. Ergo æquantur per 5. c. 5. e. 5 R.

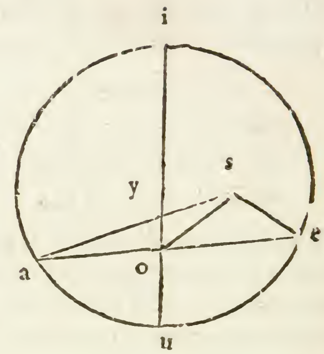


Sequitur jam inſcriptarum Coryphea diameter: cujus hæc eſt inventio.

12. Si inſcripta rectè biſecet inſcriptam: eſt diameter circuli, eiusq; medium centrum. 1.p.3.

Partes elementi duæ ſunt de diametro, de centro. Eſto inſcripta a e. eamq; rectè biſecet per 7. e. 5 R. recta i o u in o Erit i o u. circuli diameter. Nam inſcripti rectanguli diameter communis eſt circulo: At hanc inſcriptā biſecans eſt diameter rectan-

 guli



guli per 3. e. 11. R. Nam biſecta eſt pro latere rectanguli, & ſubtendit etiam biſectam peripheriam: quibus oppoſitæ & inſcripta & peripheria perinde biſecarentur. Atque ita cauſa communis eſt cum 3. e. 11. R.

Sequitur ſecunda pars de centro: quod in medio diametri patet eſſe per 3. c. 5. e. 5. R. & 2. e. in medio enim radii ſecantur æquales.

Euclides impoſsibile maluit, & ita cogit: quæ eſt deductio ad ſententiam abſurdam.

Si centrum non eſt in medio diametri, ſed alibi extra: pars æquatur toti.

At hoc abſurdum. Et illud igitur.

Sit enim centrum ex adverſarii propoſito extra y in s. ducātur itaq; s a. s o. s e. erunt triangula s a o. s o e æqualitera. radii enim s a. s e æquantur: & ex theſi a o. o e æquales ſunt: s o autem latus exiſtit commune. Ergo per 1. e. 7. R. angulus s o a. angulo s o e æquatur. Quare uterque rectus eſt per 8. e. 5. R.

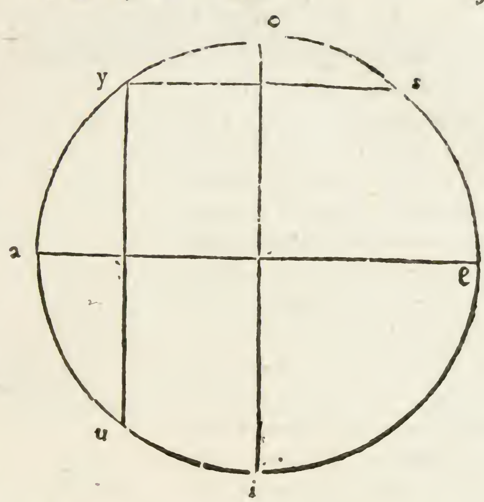
Sed & a o y rectus ex theſi eſt. anguli autem recticruri recti ſunt æquales: quare & angulus a o y. æquabitur angulo a o s. pars toti.

Et ſic de reliquis punctis: Verùm tamen ſi quis oppoſuerit eſſe quidem centrum in diametro, at nō in medio diametri: impoſsibile Euclideum de angulis ad o non ſufficit: quia tum anguli s o a. y o a revera æquabuntur. Verùm objectio eſt contra theſin: datur enim circulus: circulus eſt rotundum: rotundum autem biſecatur radiis æqualibus, & hic centrum eſt.

Itaq;

13 Si duæ rectæ duas inſcriptas non parallelas rectè biſecent: concurſus biſecantium erit centrum circuli. è 25. p. 3.

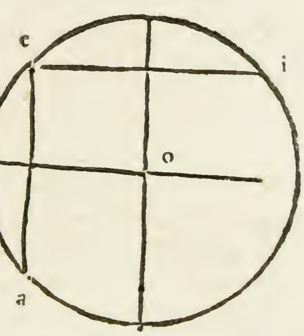
Sint



Sint inſcriptæ duę non parallelæ y u. & y s. Non parallelę inquam. Si enim parallelæ eſſent: jam una recta biſecás alteram, reliquam etiam biſecaret, ut in præmiſſa patuit, & ita concurſus non fieret: aut ſi concurſus dicendus, is fieri tùm poſsit extra medium licet ut diametro. Has itaq; inſcriptas biſecent recte a e. i o. itaq; erunt per 12. e. diametri: & proinde per 3. c. 5. e. 4 R. centrum in earum erit concurſu.

Et licet

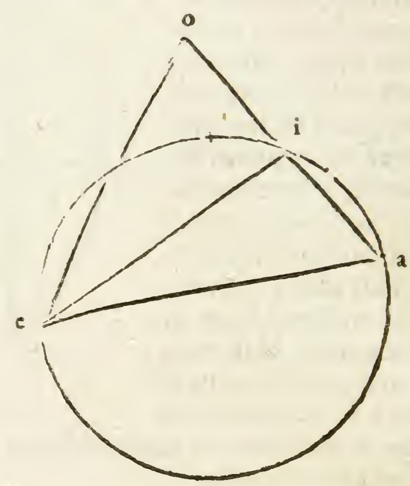
14. *Peripheriam ducere per tria puncta in rectam minimè cadentia.*



Poſtulati fabrica ex 13 e. facilis eſt. Rectas enim inter bina puncta inſcriptas biſecantes duæ in concurſu ſuo centrum habent. Radius eſt à concurſu in punctum. Dentur tria puncta a e i. quæ non cadant in rectam. per ea deſcribenda ſit peripheria. rectas itaq; e i. a e. biſecent duæ concurrentes in o. ubi centrum.

15. Si à termino diametri sit inscripta: recta à reliquo diametri termino in terminum inscriptæ, ei est perpendicularis. *è 31.p.3.*

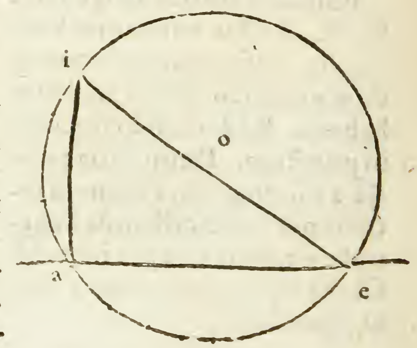
Sit enim diameter a e, & à termino a. inscripta a i. rursusq; ab altero termino inscripta e i in terminum prioris inscriptæ. Dico e i esse perpendicularē ad a i. ponatur enim angulo a e i. æqualis i e o æquicrurus. Aequabitur ergo per 2. e. 7. R. basis a i basi i o. itaque cum e i. æqualiter interjaceat, per 10. e. 2. R. erit perpendicularis.



Itaq;

16. Si recta infinita secetur à peripheria externi centri in punctis dato & contingente, & diameter sit à contingente: recta à dato puncto connectens diametrum erit perpendicularis super infinitam.

Fabrica perpēdicularis ad rectam datam exposita est 9. 10. e. 5. R. cui & hæc aggregari debet: quia facilis & expedita est. Hinc etiam Mechanici normam suam fabricantur. Sit itaq; infinita a e in eaque punctum datum a. ex qua erigenda sit perpendicularis ad datam. Centro itaq; quocunq; externo verbi gratia ad o. radio vero o a. describatur peri-



pheria

pheria ſecans rectam infinitam etiam in alio puncto quolibet e. jam ab e per o ſit diameter e i. rectaque ab i in a inſcribatur. atque hæc i a eſt perpendicularis ſuper infinitam. Nam eſt inſcripta à termino diametri in terminum inſcriptæ à reliquo diametri termino: & ita norma facta eſt i a e.

Et licet

17. *Per duo rectæ puncta deſcribere peripheriam.*

Nam recta connectens verticem perpendicularis (quam Ramea fabrica inventam huc adduces) ex uno puncto cum reliquo eſt diameter optatæ peripheriæ. ut ſi dentur in præmiſſo ſchemate a & e. perpendicularis ſit a i. recta connectens i & e eſt diameter, in cujus medio eſt centrum o. deſcribendæ peripheriæ.

Et

18. *Si recta à dato puncto faciens angulum cum infinita fiat diameter peripheriæ ſecantis infinitam: recta à dato puncto connectens ſegmentum infinitæ cum diametro erit perpendicularis ſuper infinitam.*

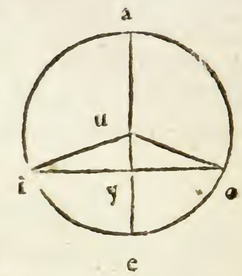
Alia hæc eſt perpendicularis fabrica. ante punctum datum fuit in recta infinita: hic extra datur. ut ſi detur in eodem præmiſſo ſchemate i. tumque recta i e faciens angulum cum infinita fiat diameter peripheriæ ſecantis infinitam in a. jam recta à dicto puncto i. connectens ſegmentum a e cum diametro eſt perpendicularis ſuper infinitam.

Sequitur inſcriptarum comparatio, & quidem in partibus ratio primo æqualitatis.

19. *Si diameter biſecet adiametrum: rectè ſecat: & contra.* 3.p.3.

Eſto, diameter a e ſecet adiametrum i o & quidem biſecet in y. dico etiam ſecare rectè. Ductis enim radiis u o. u i. Triangula

fiunt



fiunt æquilatera: ob æquales radios: æqualia ſegmenta ex theſi: commune ſegmentum diametri u y.

Si æquilatera per 1. e. 7. R. anguli ad y utrinque æquantur: Si æquantur, ſegmentum diametri u y. & proinde ipſa diameter recta ac perpendicularis eſt adiametro per 10. e. 2. R.

Converſa perinde facilis eſt: quod, ſi diameter rectè ſecet adiametrum, biſecet. Nam

Triangula æqua angulo æquicruro ſunt æquibaſia 2. e. 7. R.

At triangula i u y. y u o. æqua angulo æquicruro ad u exiſtunt. Nam anguli ad y ſunt recti ex theſi, & ideo ęquales per c. 8. e. 3. R. & rurſus anguli i & o. in triangulo i u o. per 10. e. 6. R. ſunt æquales. itaq; reliqui i u y y u o æquantur. & ſunt æquicruri quia i u. u o. radii ſunt, & u y commune crus exiſtit.

Ergo triangula i u y. y u o. æquantur baſibus i y. y o. biſegmentis nempe rectæ rectè per diametrum ſectæ.

Sequitur partium inſcriptarum ratio inæqualitatis.

20. Si adiametri interſecantur: ſegmenta ſunt inæqualia 4. p. 3.

Euclides negatione proponit non fore ſegmenta æqualia: & demonſtrat per impoſsibile è 19. e. Verùm res impoſsibili hoc non multum habet opus.

Nam

Si inſcriptæ ſunt biſectæ ſunt diametri. è 2. e. At inſcriptæ non ſunt ex theſi diametri.

Non ergo biſectæ, ſed inæqualiter ſectæ.

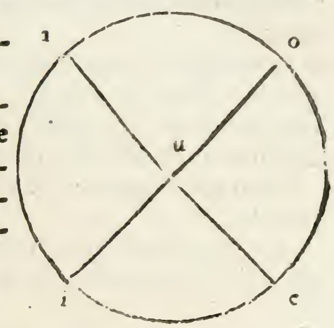
Et ſic ratiò in partibus inſcriptarum fuit: ſequitur in iiſdem proportio & quidem ſuo effecto ſignificata.

21. Si

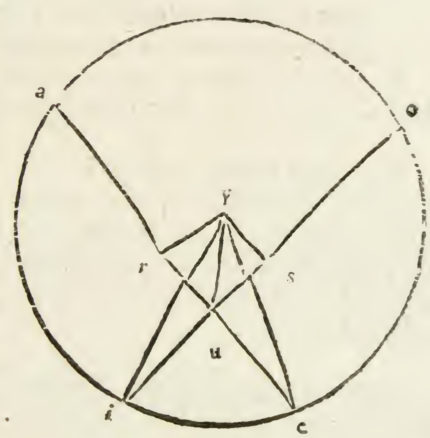
21. Si duæ inſcriptæ interſecantur: rectangulum è ſegmentis unius æquatur rectangulo è ſegmētis reliquæ. 35. p. 3.

Inſcriptæ enim a e. i o. interſecentu in u.

Erit rectangulum a u. u e rectangulo o u. u i. æquale. Nam interſectæ ſunt diametri aut adiametri. Si diametri jam radii erunt ęquales & proinde laterum æqualium ęqualia quadrata.



Sed ſi ſint adiametri. cadat diameter in communē ſectionem y u. Sintq́; perpendiculares y s. y r. Hic per 19. e. biſecantur a e. i o in r & s. jam ſyllogiſmus demonſtrationis hic eſt.



Aequalia eidem inter ſe æquantur:

At oblongum a u. u e. & oblongum o u. u i cum y u quadrato cõmuni eidem radii quadrato æquantur.

Ergo abjecto communi quadrato y u oblongum a u. u e æquatur oblongo o u. u i. Aſſumptio hinc patet:

Quadrata i s. s y æquantur quadrato radii i y. per 5. e. 12. R. At oblongum o u. u i. cum quadratis y s. u s hoc eſt cum quadrato y u.

to y u æquatur quadratis i s. s y. per 6. e. 13. R. & quadrati y s communis utrinq; additione.

Ergo oblongum o u. u i. cum quadrato y u æquatur quadrato radii.

2 Quadrata r e. r y. æquantur quadrato radii y e. per 5. e. 12. R. At oblongum a u u e cum quadratis r u. r y. hoc est per 5. e. 12. R. cum quadrato y u. æquantur quadratis r e. r y. per 6. e. 13. R. & communis quadrati r y utrinq; additione.

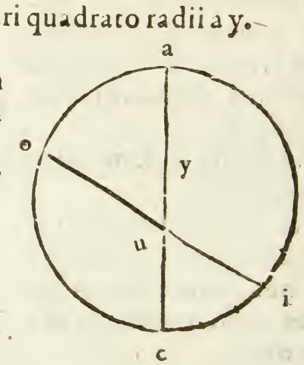
Ergo oblongum a u. u e cum quadrato y u æquatur quadrato radii.

Atq; hic significatur proportio partium inscriptarum ex effectis. Est enim effectum proportionis dicta æquatio.

Itaq;

22. Si diameter secet inscriptam: rectangulum segmentorum sectæ cum quadrato segmenti diametri è centro in sectam æquatur quadrato radii. 14. p. 3. Regio. Almag.

Sit enim inscripta o i. eam secet diameter a e. dico rectangulum o u. u i. cum quadrato y u æquari quadrato radii a y.



Nam quia recta a e est bisecta in y. per 2. e. secusq; in u. oblongum a u. u e. hoc est per præmissum o u. u i. cum quadrato y u æquabitur quadrato a y bisegmenti nempe radii per 6. e. 13. R.

Sequitur jam ratio integrarum inscriptarum quam sola diameter totam facit.

23. Inscriptæ æquidistant à centro, in quas à centro perpendiculares sunt æquales. 4. d. 3.

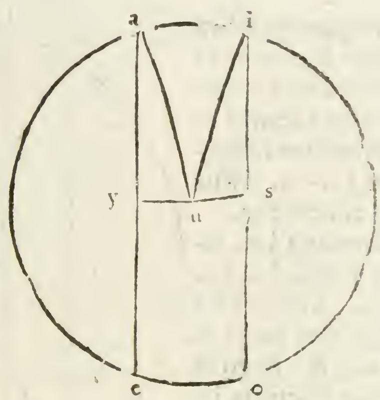
Sic

Sic a e. i o æquidistant à centro: quia perpendiculares in eas u y. u s sunt æquales.

24. Si inscriptæ sunt æquales: æquidistant à centro: & contra. 14.p.3.

Hoc Euclides æquè potuisset postulare benè ut superius: at demonstravit: hoc modo.

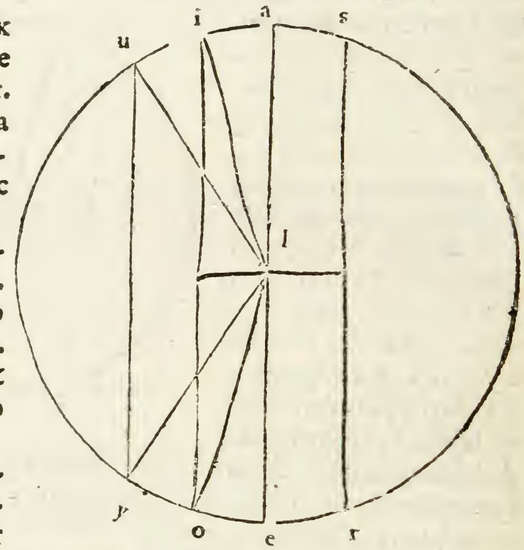
Sint inscriptæ æquales a e. i o. jam ducantur à centro u & radii u a. u i & perpendiculares u y. u s. Quare inscriptę bisecātur: ideoq; & dimidia æqualiū y a. i s æquabuntur. Nam proportionalia ad idem sive æquale æquantur. jam quadrata radiorum per 5. e.12.R. æquantur binis quadratis crurum a y. y u. & i s. s u. quæ bina ideo ęquantur: ablatis scilicet radiorū quadratis æqualibus: denuoque quadrata a y. i s. æqualia ex thesi, sunt enim laterum æqualium, tollantur. relinquuntur quadrata adeoque ipsa latera y u. u s per 2.c. 2. e. 12. R. æqualia. Latera autem hæc sunt perpendiculares à centro in inscriptas ex fabrica. Ergo per 23.e. inscriptæ æquidistant à centro.



Conversa eodem ratiocinio probari poterit. Nam quia æquidistant inscriptæ à centro. erunt perpendiculares y u. s u æquales. jam rursus ablatis quadratis radiorum & perpendicularium sigillatim æqualibus relinquuntur quadrata æqualia, & proinde latera a y. i s. proinde & horum dupla sunt æqualia, quæ sunt ipsæ inscriptæ.

25. *Inscriptarum inæqualium diameter est maxima: diametroq; propior major remotiore, remotissima minima: minimæq; propior minor remotiore, duæq; utrinq; à diametro solæ æquantur.* è 15.p.3.

Hic quinq; elementi partes habemus quas ut ordine demonstremus hoc nobis schema præmittatur, ubi diameter inscripta sit cum alijs inscriptis.



Prima pars quod ex inscriptis diameter a e sit maxima hinc patet.

Ducatur è centro radii in terminos inscriptarum l i. l o. Hinc jam ita concludes.

Diameter a l e. æquatur radijs l i. l o. per 2. e. At l i. & l o sunt majores quã i o. per 6. e. 6. R. Ergo & diameter inscripta i o major est.

Secunda quod propior maximæ exempli gratia i o. sit major remotiore u y patet per 4. e. 7. R. Nam Triangulum i l o. est æquicrurum triangulo u l y. quia radii æquantur: sed interim angulus i l o angulo æquicruro u l y est major tanquam totum parte. Ergo & basis i o superat basin u y.

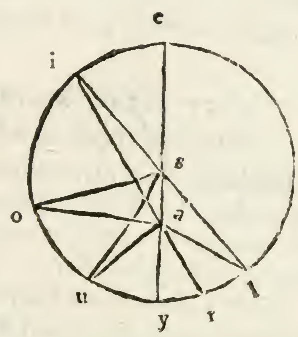
Tertia quod remotissima sit minima, & quarta quod propior minimæ minor sit remotiore consequuntur è prima & secunda. Nam si diameter est maxima: à diametro remotissima erit minima.

Quinta quod duæ solæ utrinq; à diametro æquentur per secundam

cundam patet. Nam si præter i o & s r statuatur tertia æqualis erit eadem inæqualis. Nam diametro propior aut remotior. At hoc est absurdum.

26. Rectarum à diametri puncto non centro, quæ per centrum est, maxima est: propiorq; maximæ major remotiore: reliqua maximæ minima: minimæq; propior minor remotiore: duæq; utrinque ad idem diametri segmentem è dicto puncto solæ æquantur. 7.p.3.

Propositionis hujus etiam plures partes sunt, quas etiam ordine in sequente schemate videbimus.



Prima pars quod rectarum è puncto diametri a. maxima sit e a. eodem modo patet quo prima præmissi. Nam i s. s a. hoc est e s a majora sunt quam i a. per 7.e.6.R.

Secunda quod i a propior maximæ major sit remotiore o a similiter constat. Nam triangulum i s a. æquicrurum triangulo o s a majus est angulo. Ergo & basis i a major est basi o a.

Tertia, quod reliqua maximæ a y sit minima patet. Nam Reliqua æqualis de inæqualibus, datis similiter directè sunt inæqualia. quod axioma Arithmeticum est.

At a y & a u sunt reliqua æqualis nempe communis s a de s a. a u. & s u hoc est s y inæqualibus. per 7. e. 6.R. Nam s a. a u majora sunt quam s u. Ergo similiter sunt inæqualia datis inæqualibus & a u. major quam a y.

Quarta pars, quod minimæ propior a u sit minor quam a o remotior patet per 4.e.7.R. Nam triangulum o s a. æquicrurum triangulo u s a. est majus angulo. Ergo & basis o a. major est quam basis u a.

Quinta quod duæ utrinque ad idem diametri segmentum è dato puncto æquentur patet, nam si angulo u s a sit æqualis a s r.

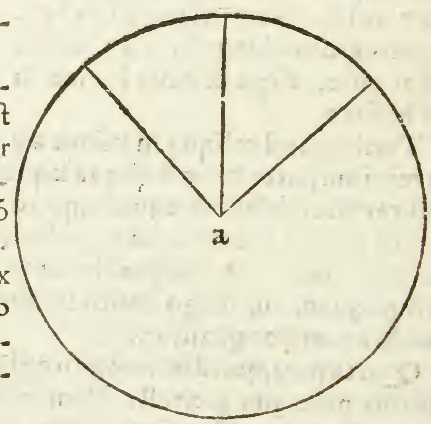
& latera u s. a s. & a s. s r ſunt æqualia: per 2. c. 7.R. u a. a r æquabuntur: Et quidem ſolę ęquantur, ut impoſsibili conſtat. Si enim nō ſolæ. Sit ergo a l a qualis tertia. ergo per 1.e.7.R. angulus l s a æquatur angulo r s a. totum videlicet parti quod eſt abſurdum. Euclides ait ad utramque partem minimæ ſolas duas æquari: at id tamen commune eſt ad utramq; partem etiam maximæ.

Elementum itaq; generale eſt de puncto quocunq; diametri extra centrū: ſive in diametro ſit, ſive in termino diametri (quod ſpeciatim Commandinus demonſtrat ad 8.p. Archimedis de lineis ſpiralibus) ſi hic notetur minimam in diametro tum non eſſe proindeq;, nec inde utrinq; æquales. Quæ per ſe patent.

Quinta autem pars ſuppeditat nobis conſectarium aliquod. Itaq;

27 *Si punctum in circulo eſt terminus trium rectarum in peripheriam æqualium: eſt centrum çirculi. 9.p.3.*

Euclidea demōſtratio hoc loco duplex eſt. at poſtrema expeditior eſt, & conſectarium è quinta parte.

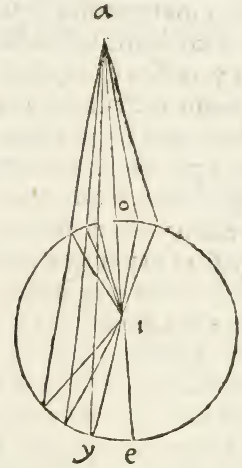


Nam ſi datū illud & propoſitum punctum a non eſt centrum, à diametri (quę per quodlibet in circulo punctum duci poteſt) puncto nō centro nō ſolum duæ utrinque æquantur: quia hic ex theſi dantur tres æquales ab eodem puncto. Quod tamen contra 26.e. cujus veritas conſtat, infertur.

28. *Rectarum à dato extra puncto in concavum peripheriæ quæ per centrum eſt, maxima eſt: propiorq́; maxima*

mæ ma or remotiore: in cõvexum ſegmentum maximæ eſt minima: minimáq; propior minor remotiore: duæq; utrinq; à maxima vel minima ſolæ æquantur. 8.p.3.

Elementi partes totidem ſunt quot fuerunt 26.e. & demonſtratio ſimillima.



Exempli gratia prima pars eſt, quod à dato extra puncto per centrum in concavũ peripheriç ſit maxima, nempe a o e ea patet. Nam recta à dicto pũct o in centrum a i. & radius in terminũ extra centrum inſcriptæ i y. ſunt maiores dicta inſcripta per 7.e.6.R.

Reliquas partes eodem modo contexes: quas tüæ diligentiæ relinquo.

Habuimus rationes rectarũ à puncto interno & externo inſcriptarum: Eæ angulos faciunt cum diametro: radiis: aut inter ſe: quorum rationes inter ſe hic conſiderare utile & proprium fuerit.

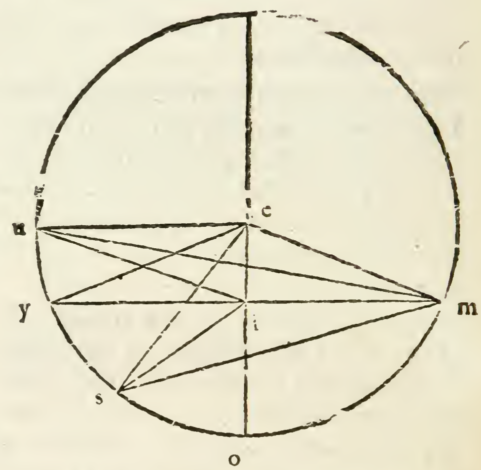
29. *Angulorum rectarum è diametri puncto non centro atque radiorum, maximus eſt rectæ perpendicularis ad diametrum: maximoq; propior maior remotiore: quaterniq; inſcriptarum per datum punctum æqualium duarum & cum radiis & cum diametro æquantur: biniq; utrinq; à diametro æquidiſtant à radiis & ſegmentis inſcriptarum æqualibus comprehenſi.*

Hoc elemento hypotheſium fabricatores nobiliſsimam illam aſtrorum ſcientiam penitus obruerunt: At nos ab exilio in patriam revocare ſtatuimus: nec ita miſerè ad proſthaphæreſes planetarias eſſe aſtringendum, ſed domum eſſe reducendum: quo aliis etiam ſuam operam communicare poſsit.

Elementi

Elementi igitur partes ordine videbimus: prima pars quod angulus maximus ſit è radio & perpendiculari ad diametrum in qua pũctum datum eſt.

Sit diameter e i o. ex ejusque puncto i egrediatur perpendicularis i y. & concurrat ei radius e y. rurſus è puncto i. cũ radio recta faciat angulum i u e. Dico angulum e y i. eſſe majorem angulo i u e. Continuetur enim y i. in m erit y i. æqualis i m per 19. e. ducatur radius e m. itaque per 10. e. 6. R. anguli ad y & m. æquãtur. ducatur & recta u m: erit rurſus



angulus e u m. æqualis angulo e m u. & quia u i major eſt quam y i. hoc eſt i m per 26. e. etiam angulus u m i major erit angulo i u m per 11. e. 6. R. jam ex Arithmetica proportione collige. Summæ ex inæqualibus atq; æqualibus ſunt inæqualibus datis ſimiliter inæquales.

Anguli autem e u i. e m i. hoc eſt e y i. ſunt ſummæ ex majori i m u & minori i u m. & æqualibus e u m & e m u. Ergo &c. atque e y i. major quam i u e. Et ſic de aliis angulis vel proximioribus ipſi y. Nam ſi remotiores ſint minores probabuntur ipſo angulo ad u. qui & ipſe minor eſt angulo ad y. Nec ex altera parte infra y. poterit dari major. Sit enim angulus infra y comprehenſus à rectis e s radio & i s. dico & ipſum eſſe minorem quam ſit angulus ad y. Ducatur recta s m. erit per 10. e. 6. R. angulus e s m. angulo e m s æqualis. Et rurſus in triangulo i s m. quia i m hoc eſt y i eſt major quam i s per 26. e. erit etiam angulus i s m major angulo i m s. per 11. e. 6. R. Hinc per axioma Arithmeticum concluditur.

Reliqua

Reliqua inæqualium de ęqualibus,inęqualibus ſunt reciprocè inæqualia.

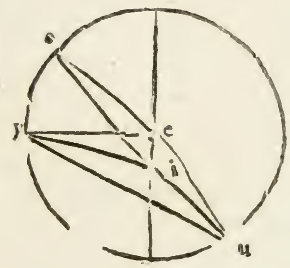
At anguli e s i & e m i hoc eſt e y i ſunt reliqui inæqualium majoris i s m.minoris i m s.de æqualibus e s m.e m s.

Ergo &c. nempe e s i minor quam e y i.

Atq; ſic prima conſtat propoſiti pars.

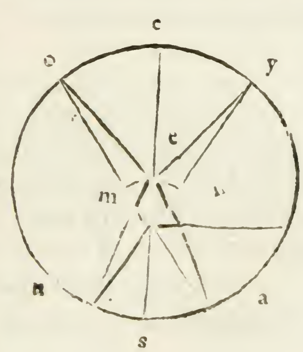
Secunda pars quod propior maximo ſit major remotiore.

Sit enim angulus e y i. propior maximo remotior vero e s i.jam ducta y u per 10. e. 6. R. Anguli e y u. e u y æquantur. Continuetur etiam recta s i recta i u.quæ quia minor eſt quam i y per 26. e. erit & angulus i u y angulo u y i major per 11. e. 6.R. Hinc ita colligitur. Anguli e y i. e u i.hoc eſt e s i.ſunt reliqui majoris i u y minoris i y u. de æqualibus e y u. e u y. Ergo reciproce ſunt inęquales,e y i major i s e minor.



Has partes brevius demonſtrant Regiomontanus , & poſt eum Nonius : Verum à poſteriori per doctrinam ſegmentorum circularium.Vide Epitomen Regiomont. & Annotationes Nonii ad theoriā ſolarem Purbachii.

Tertia pars quod quaterni à duabus per diametri punctum inſcriptis æqualibus & radiis comprehenſi ſint æquales nempe anguli a, y. o. u.



De oppoſitis a.o.& y. u. conſtat per 10.e.6.R.Nam o e.e a. item e y. e u ſunt æquales:quia radii.

De angulis autem o & y. vel u & a.probatur. Quia enim æquales

d ex

ex thesi sunt inscriptæ æquidistant à centro per 24. e. itaq; perpendiculares e m. e n. æquales per 23.e. secant in o. n y æquales: quia sunt dimidia per 19.e. æqualium, & radii e o. e y æquantur. Quare triangula e o m. e y n sunt æquilatera. itaq; per 1.e.7.R. angulus o æquatur angulo y. itemq; u & a. Sic de reliquis.

Quarta: quod quaterni à dictis rectis & diametro æquentur ad i. Nam in figura antecedente primo anguli verticales u i s. s i a. cum p i y. o i p æquantur. Deinde in triangulis e m i. e n i. quia e i æquepotest e n & i n. itemq; e m. m i per 5.e.12.R. ablata quadrata æqualia æqualium e m. e n. de æquali e i. relinquunt quadrata & proinde latera m i. n i. æqualia. Sunt itaq; æquilatera triangula. itaq; angulus m i e æquatur angulo e i n. & u i s &c.

Quinta: binorum crura præter radios, nempe segmenta inscriptarum alterna o i. i y & u i. i a. æquantur. Nam summæ æqualium ad æqualia æquantur. Et i o. i y sunt summæ ex n y. m o æqualibus & n i. & m i etiam æqualibus: ut in præmissis membris est demonstratum Constat tamen & per 26.e. hæc pars.

Sexta: bini æqualium dictorum angulorum o & y. item a & u utrinq; à diametro æquidistant. Nam triangula e i y. o i e sunt æquilatera. & ideo anguli y e i. o e i æquantur, atque reliqui ad duos rectos p e y. p e o. quibus anguli o & y à diametro absunt. Sic & de angulis u & a.

Ex hac autem parte Consectarium aliquod est: quo etiam Astrologi hypothetici utuntur.

Itaq;

30. Perpendicularis è puncto diametri non centro bisecat angulum inscriptarum per datum punctū æqualium.

ut si in præmisso schemate à puncto i. erigatur perpendicularis i t. ea bisecabit angulum y i a. Nam e i t. t i s anguli æquantur: quia recti & e i y. s i a æquantur. id enim jam demonstratum est. Ergo reliqui y i t. t i a æquantur quare totus y i a est bisectus.

31. Angulorū rectarum à dato extra puncto secantium peripherias æquales, maximus est cum recta per centrum: maximoq;

maximoq́; propior major remotiore · duoq́; utrinq́; à maximo soli æquantur.

Et hoc elementum Epicyclicũ est Astrologis.

Esto à dato extra puncto u rectæ u a. quæ centrum cõtinuata transiret, u e. u i secent peripherias æquales a e. e i. dico primo angulum a u e esse majorem angulo e u i.

Secus enim si non est major sit æqualis. si æqualis. fiant anguli u i r. u e s quoq; æquales. erũt ergo triangula æquiangula & per 9.e.7.R. ut u s ad u r sic u a ad u i. At u s major quam u r per 28.e. Ergo & u a major quam u i. Quod est contra 28.e. Ergo angulus i u e non est æqualis angulo i u e.

Potuisset demonstrari alio modo (Nam hæc ὕστερον πρότερον aliquid continere videtur) nempe si angulo e u a ponatur æqualis ad rectã u e. tum is æquatus continebit angulum i u e. Ergo eo est major & proin de a u e esset major eodem i u e.

Reliquæ partes per pręmissã absurdi deductionem aut ἐφαρμόσεως lucem per se patent.

Et perinde est sive rectæ in cõcavum secent, sive in convexum. res enim & rei demonstratio est eadem.

TH. FIN.

TH. FINKII GEOMETRIAE ROTVNDI,

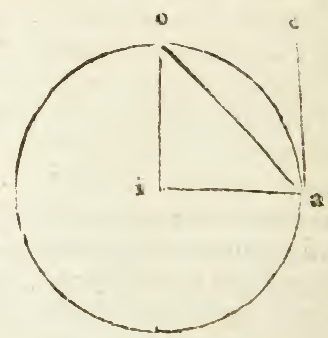
LIBER SECVNDVS.

De lineis circuli tangentibus sigillatim & coniunctim cum secantibus, atque obliquis.

SI recta est perpendicularis extremæ diametro: tangit peripheriam: & contra. è 16.p.3.

Postulatum hoc esset ex perpendiculi definitione: Nam si non tangeret, sed magis hac propenderet aut declinaret peripheriam secaret, nec esset perpendicularis. quod est contra thesin. Euclides tamen cogit.

Sit enim perpendicularis e a extremæ diametro a i. Quod si perpendicularis jam non cadit extra circulum, ut ait Euclides, nec tangit cadat sane intra ad punctum o. & connectatur o i. Tum in triangulo i o a. duo recti erunt i a o quidem ex thesi. i o a. vero ex 10. e. 6. R. At duos esse rectos in triangulo impossibile est per 9.e.6.R. Quare & impossibile est perpendicularem extremæ diametro peripheriam non tangere.

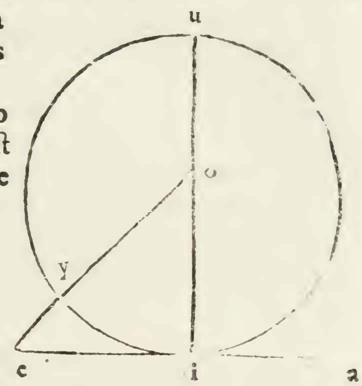


Conversæ demonstratio antecedenti simillima est.

Nam si a e tangit & tamen non est perpendicularis diametro

tro i o u. ducatur à centro o. alia perpendicularis o e. erit angulus o e i rectus,& o i e acutus.

jam quia acutus minor est recto erit per 11. e. 6. R. latus o i. hoc est o y. majus latere o e. pars nempe toto: quod est absurdum.



Itaq;

2. *Si recta est per centrum & contractum: est perpendicularis tangenti: & contra. 18.19.p.3.*

Est è conversa præmissi. Nam recta à centro in contactum vel è cõtactu in centrum est pars diametri. Ideoq; ei tangens est perpendicularis &c.

Et

3. *Punctum contactus est, quo à centro perpendicularis tangenti incidit.*

Contactus fit vnico puncto: & à centro in contactu recta est perpendicularis tangenti: ergo illic punctum contactus esse necesse est. Et

4. *Tangens est singularis eadem parte. è 16.p.3.*

Consequitur ex 1.e.& 10.e.2.R.

Nam perpendicularis est ex eadem parte singularis. Et tangens est perpendicularis, ut jam patuit. Euclides proponit: quod nulla recta cadere possit inter peripheriam & perpendicularem.

Et

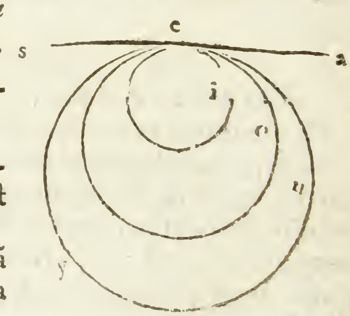
5. Angulus contactus eſt minor quovis acuto rectilineo. è16.p.3.

Angulum contactus vulgo vocant angulum contingentiæ: eum ſcilicet qui è tangente & peripheria continetur. Proclo κερατοειδὴς dicitur cornicularis: quia inſtar cornu ex peripheria & recta efficiatur. is minor Euclidi eſt quovis acuto rectilineo. Nam ſi minor non eſſet: caderet recta inter peripheriam & tangentem: quæ cum tangente rectilineum acutum comprehenderet, & proinde corniculari ut toto minorem: At per 4. e. hoc fieri nequit.

Sunt qui de hoc angulo quæſtionem movent: cur ſi angulus contactus magnitudo ſit, ea dividi nequeat: & cum dividi nequeat: angulum hunc magnitudinem eſſe negant. ideoq; ſemicirculi angulum recto rectilineo eſſe æqualem: imo quorumlibet ſemicirculorum angulos æquari. Sed Geometria convincit angulum contactus eſſe minorem acuto rectilineo quovis. Non tamen propterea minimum poſtulamus. ſecari poteſt & dividi Geometricè & quidem infinite: ſed in diviſione ad angulum rectilineum non pervenietur. Poſtulat Euclides in opticis angulum quidem minimum at non geometricum: ſed opticum: ſed phyſicum.

Et

6. Anguli contactus in æqualibus peripheriis ſunt æquales: inæqualium autē minoris cornicularis eſt major.



Hæc ἐφάρμοσις in ſequenti figura ſatis docebit.

Nam angulus a e u & s e y æquantur: & angulus a e u minor eſt angulo o e a minoris circuli.

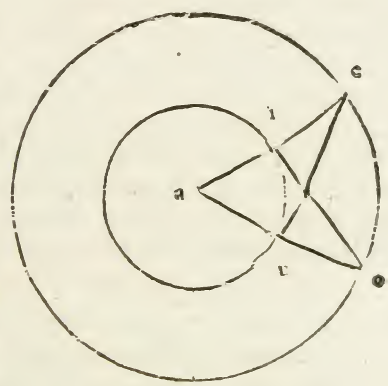
Quod multo ſanè mirabilius quā 5. e. & verum interim Geometria convincit.

Fallitur

Fallitur itaque Peletarius qui angulos semicirculorum putat esse æquales. Nam si anguli contactuum sunt inæquales: ablati de rectis rectilineis qui fiunt per tangentem & diametrum relinquunt inæquales. Extat Peletarii scriptum hac de re: quod perlegi potest: notari etiam ea debent quæ ab aliis ipsi sunt opposita: & omnia tamen norma Geometrica expendenda.

7. Si à radio ex datæ peripheriæ centro ad datum extra punctum peripheria describatur, & à concursu datæ radii, radio ipsi perpendicularis in descriptam connectatur cum dicto centro recta à dato puncto in concursum datæ & connectentis tanget datam peripheriam. 17.p.3.

Sequitur jam fabrica tangentis & quidem Euclidea hoc elemento exposita. ut data sit peripheria i u. & datum extra punctum e. à quo ducenda sit tangens. jam radio a e. à datæ peripheriæ centro ad datũ extra punctum describatur peripheria e o. & sit in descriptam radio a e. perpendicularis i o. qua connexa ad centrum per rectam a o. ducatur e u. Dico ductam e u tangere datam peripheriam. Nam est perpẽdicularis extremę diametro. Sunt enim triangula e a u. o a i. æqua angulo communi æquicruro. itaq; per 2. e. 7. R. sunt æquilatera & per 1. e. 7. R. angulus a u e. angulo a i o, æquatur: & cum ille ex fabrica rectus sit: erit etiam angulus a u e rectus. Quare cum e u in a o incidens æqualiter interjaceat per 10. e. 2. R. est perpendicularis.



Hæc Euclidea est & Ramea fabrica: & quidem ut apparuit satis laboriosa. Quare faciliorem sic accipe.

8. Si

8. Si recta à centro datæ peripheriæ ad datum extra punctum fiat diameter peripheriæ: recta à dato puncto in sectionem datæ & descriptæ tangit datam peripheriam.

Sit data peripheria u i & datum extra punctum e. jam à centro a ducatur recta a e. quæ fiat diameter describendę peripheriæ e u i. sumpto nempe ejus medio pro centro.

Tum à dato puncto e ducatur recta in sectionem. e u. dico rectam e u esse tangentem quæsitam. Est enim perpendicularis ad u a per 15. e. 1. at u a est diametri datæ peripheriæ pars. Quare u e est perpendicularis extremæ diametro &c.

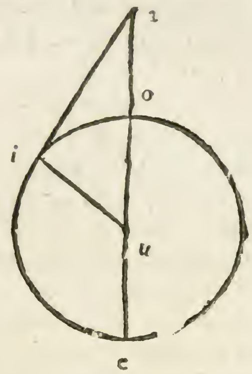
Hanc fabricã esse faciliorem facile intelligitur collatione. Nam Euclidea rectam à centro ducit: describit peripheriã: perpendicularem fabricat: perpendicularis terminum connectit cum centro: à dato puncto rectam ducit. At hæc rectam ducit: rectæ semissæ radio describit peripheriam: rectam ducit. utere igitur hac fabrica in fabricanda tangente. Atq; ita secantes habuimus & tangentes seorsim: sequitur de simul utroq; genere: & quidem proportio effecto ut ante significata.

9. Si è duabus rectis à dato extra puncto prima secat in concavum, reliqua tangit: oblongum è secante & exteriore secantis segmento æquatur quadrato tangentis: etsi oblongum tale æquatur quadrato reliquæ: reliqua ipsa tangit. 36.37.p.3.

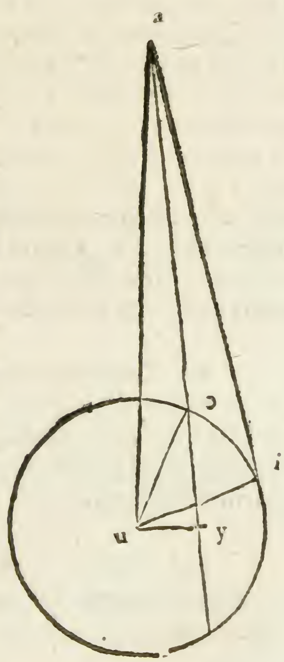
Vogelinus ex effecto causam cõsiderat nempe proportionem in rectis. Verum à posteriori per angulos in sectione alternos secantis & contiguæ. Quare nos Euclidis retinebimus demonstrationes: & effectum ex simili aliquo effecto deducemus.

Secans

Secans igitur tranſit per centrum aut ſecus. Si per centrum tranſit ut hic per u. dico oblongum a e. e o. æquari quadrato tangentis a i. Erit jam radius u i. perpendicularis tangenti per 2. e. Tum quia e o. eſt biſſecta & continuata: per 7.e.13. R. oblongum e a. a o. cum quadrato u o. hoc eſt u i. æquatur quadrato a u.



At quadratum a u per 5.e.12. R. æquatur quadratis a i & i u. jam ab æqualibus oblõgo nempe e a. a o. quadrato i u. & quadratis a i. i u. tollatur æquale quadratum i u. Ergò æquabitur oblongum ductum quadrato tangentis. Si ſecans non tranſeat centrum ut hic in figura: per 12.e.1. invenitur centrum u. tumque per 2.e. u i. tangenti eſt perpendicularis. Ducantur etiam à centro in datum punctum & terminum exterioris ſecantis ſegmenti rectæ u a. u o & perpendicularis u y biſecans o e per 19.e.1.

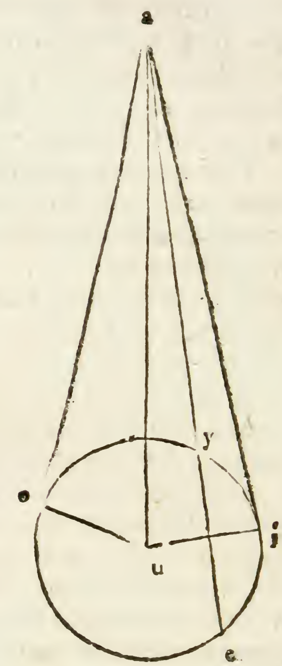


jam per 7.e. 13. R. oblongum a e. a o. cum quadrato o y æquatur quadrato a y. & utrinque addito communi quadrato y u. dictum oblongũ cum quadratis o y. y u. hoc eſt per 5.e.12. R. cum quadrato o u. æquatur quadratis a y. y u. id eſt quadrato a u: id eſt rurſus quadratis a i & i u. hoc eſt u o. cũ i u & u o ſint radii ęquales. itaq; ſi ab æqualibus oblongo a e. a o. quadrato o u. & quadratis a i. atq; o u tollatur idem &

e æquale

æquale quadratum relinquetur oblongum dictum æquale quadrato tangentis. Et sic de antecedente constat.

Conversa similiter demonstratur in adjuncta figura.



Estó enim oblongum e a. a y æquale quadrato a i. dico a i tangere peripheriam. Ducatur per 8.e. tangens a o. item ducantur a u. & radii u o. u i. Erit itaq; a i. a o. æquales. Nam oblongum e a. a y. æquatur quadrato a o. ex jam demonstratis & a i. ex thesi. Quare quadrata & proinde latera a i. a o. inter se æquantur. Quare triangula a i u a o u sunt æquilatera itaque per 1. e. 7. R angulus a i u. æquatur angulo a o u. At hic rectus est. Nam u o est perpendicularis tangenti per 2. e. Quare & alter, & proinde a i est perpendicularis extremæ diametro i u. itaq; per 1. e tangit.

Itaq;

10. *Tangentes ab eodem puncto sunt æquales.*

Nam ipsarum quadrata eidem oblongo æquantur: proinde sunt ipsa ipsorumq; tandem latera æqualia. Nec possunt esse plures duabus tangentes æquales. Est campani ad 36. p. 3. ut & Consectarium sequens.

Et

11. *Oblonga è qualibet ex eodem puncto secante & secantis exteriore segmento æquantur inter se.*

Quia eidem nempe quadrato tangentis æquantur.

Et

Et licet

12. *Datis duabus rectis alteri continuare tertiam: ut oblongum ex continuata & continuatione æquetur quadrato reliquæ. 127.p.1.Vitell.*

Hoc elementum usq; ad parag. Hinc Regio. proxime antecedere debuit para. sequentis paginæ: Fiat itaq; eo diameter &c.

Hæc inventio sive cõtinuationis fabrica facilis est. altera enim datarum continuanda fit diameter peripheriæ: ex cujus diametri extremitate erigitur perpendicularis æqualis reliquæ datarum: tum continuatio diametri in terminum perpendicularis est tertia quæsita.

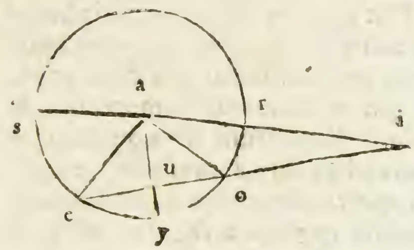
Sint datæ rectæ e o. i a. & sit eo continuanda, ita ut oblongum è tota continuata & continuatione æquetur quadrato a i.

a————————i

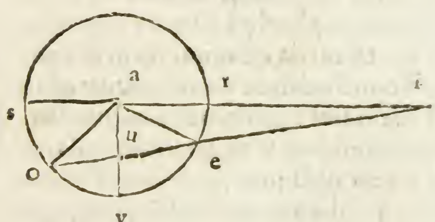
e——————————o

Hinc Regiomontanus 45. p. 1. de triangulis segmenta basis secta perpendiculari investigat. Esto enim triangulum a e i varium. è cujus angulo a descendat perpendicularis in basin.

Erit jam angulus ad basin uterq; acutus aut alteruter obtusus. Nam rectus esse nequit: crus enim ipsum esset perpendicularis per 2. c. 2. e. 8. R.



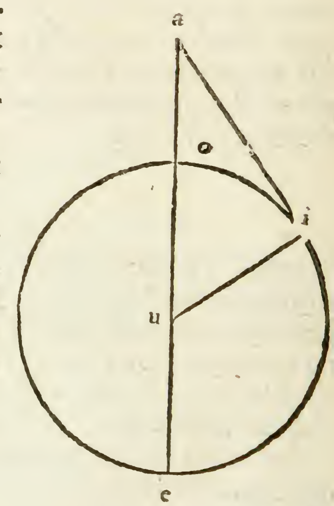
Quod si angulus alteruter obtusus est: perpendicularis cadit extra triangulum per 1. c. 7. e. 8. R. Secus cadit intra per 1. c. 8. e. 8. R.



Sed ut ut cadat segmenta basis perpendiculari secta hinc discemus.

E vertice anguli ex quo est perpendicularis radio cruris minoris describatur circulus:ut hic radio a e circulus e y o r.tum latera trianguli continuentur in concavum peripheriæ ut i a in s & i e in figura secunda in o.Sitq; perpendicularis a u in basin datam aut continuatam. Quæritur segmentum u e. Eo enim dato quæstioni erit satisfactum.Quia itaq; oblongum s i. i r æquatur oblógo e i. o i.erit per 5.e.11.R.ut i e ad i s sic i r ad i o. Hoc est Erit ut basis ad summam crurum sic crurum differentia ad differentiam basis & dupli segmenti basis ab angulo, per quem circulus descriptus erat, ad perpendicularem. ut si detur i e basis 21. & crus i a 20.a e 12.erit ut 21 ad 33.sic 7 ad 11.jam 11. de basi relinquũt 10. duplum segmenti minoris erit ergo segmentum basis datæ u e 5. Quod si jam inventus quartus sit major basi Verbi gratia si i a sit 51.& a e 25.e i 38.erit i o 52. tum perpendicularis cadit extra & differentia ut ante basis & inventi est dupla ad u e.quare u e erit 7.

Fiat itaq; eo diameter peripheriæ i o u.atque à termino diametri i erigatur perpendicularis i a.datæ ęqualis.tum continuetur diameter in a. & quia si diametrum u i continuares non caderet in a.eam cótinuare oportet,quę continuata in a.incidat.itaq; poteris applicare regulam ad a. & centrum & rectam ducere in concavum peripherię a e.cujus segmentũ a o est tertia quæsita. Nam & tum data continuanda continuatur: quia est diameter: & diametri æquantur.



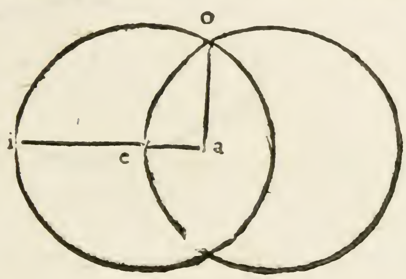
Superest jam Geometria circularis de lineis obliquis peripheriis nempe:iisq; solis aut cum rectis: solis intersectis & contiguis.

13. Si

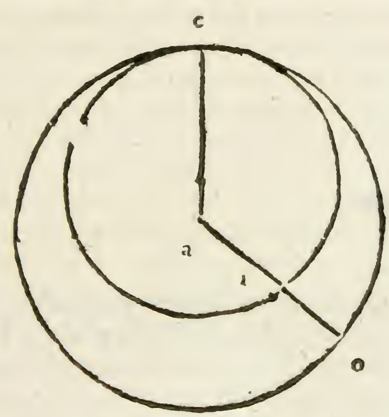
13. Si peripheriæ ſunt interſectæ vel contiguæ: ſunt eccentricæ: illæq; duobus tantū punctis interſecantur, hæ diametros per contactum continuant. 5.6.10.11.12.p.3.

Hæc penè omnia poſtulari poſſent: neq; enim propoſitionis materia potius quam principii eſt. Circulos enim ſe ſecantes aut contingentes vel extra vel intra diverſa habere centra quis eſt qui dubitat? Sed demonſtrationes Euclideæ ex impoſsibili ſic ſe habent. Prima pars de interſectis.

Sint interſectæ duæ peripheriæ i o. o e. Quod ſi ergo commune centrum habent: ſit ſane a. Erunt itaq; a o. a e æquales. itemque a o. a i. per 2. e. 1. & quia e a. a i. eidem æquantur: inter ſe æquantur. nēpe pars toti. Quod eſt abſurdum.



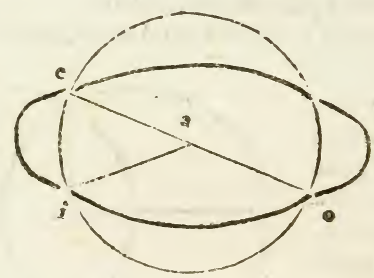
Eadem demonſtratio eſt de contiguis ut in ſequenti figura. ſi ponas commune centrum a. erit a i. æqualis ipſi a o. quia eidem a e radio æquantur. at hoc eſt partem ęquare toti: quod abſurdum eſt. Si peripheriæ ſint foris contiguę res expeditior eſt: neq; demonſtrationē Euclideam meruit.



Quare ſi peripherię ſint interſectæ aut contiguæ ſunt eccentricæ: At (inquis) an non circulo cir-

culum æqualem imponere possem & homocentri cum? an non hic contiguæ peripheriæ homocentricæ? At inquam hic non amplius duo circuli: sed unus est. Nam (ut Vitellio petit) cum plana duo sese contingunt, unum inde efficitur. Et illud est argumentum ἐφαρμόσεως generale, etiã speciale Euclideũ ad 4.8.p.1.

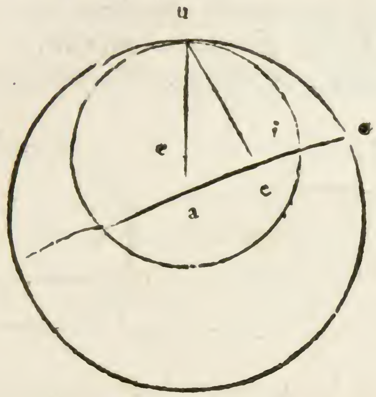
Tertia pars: intersectas duas tantum intersecari duobus punctis duplicem invenit apud Euclidem demonstrationem. At po



sterior è prima parte hujus elementi facilis est ista. Nam si pluribus duob. punctis intersecantur, intersectæ non sunt eccentricę.

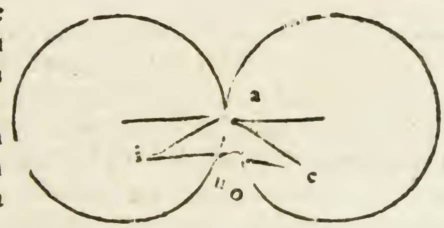
Nam invento centro per 12. e. ductæ rectæ in puncta sectionum tres, quia radii unius peripheriæ, æquantur: sed ducuntur etiam in reliquam peripheriam è puncto intra eam eædem tres æquales. Ergo est ibi reliquæ centrum per 26.e.1. itaque habent intersectæ hæ commune centrum. At hoc fieri non potest ut jam demonstravimus: Ergo neq; pluribus duobus punctis intersectę duæ intersecari possunt.

Quarta pars duabus propositionibus ab Euclide proponitur: primò ubi tactus intus est. ut hic sint centra e & a. dico rectam per a & e cadere in contactum u. Demonstratio per impossibile est. Si enim non cadit in contactum: cadat in punctum o. ducatur igitur radius in contactum e u. Hic in triangulo e u a per 7.e.6.R. u e. e a majora sunt quam u a. hoc est quam a o. jam ablato



ablato communi a e. remanet u e major quam e o. Sed u e æquatur e i. quia sunt radii ejusdem peripheriæ. Ergo & e i major est quam e o pars nempe toto: quod est absurdum. idem erit quocunq; continuatio diametrorum cadere dicatur extra cõtactum.

Si tactus extra sit de quo est Eucl 12. p. 3. idem erit. Nam si recta per centra i & e dicatur alio cadere quam in contactum cadat sane in u & o. jam sint radii i u. i a. & e o. e a utrinq; æquales. Sed i a. e a sunt majores quam i e per 7. e. 6. R.



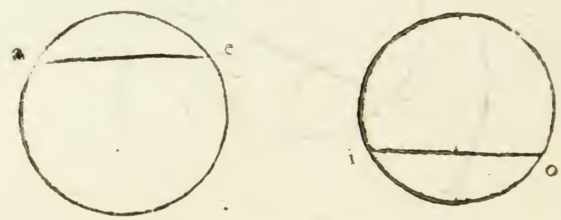
Ergo & i u. e o sunt majores quam i e. partes nempe toto. Vel eadem linea i e major erit i u. o e tanquam suis partibus: & tamen minor iisdem: quia æquantur a i. a e majoribus recta i e. Quod utrumq; absurdum est.

Atq; sic obliquas lineas sive peripherias solas habuimus: De inscriptis autem & peripheriis ratio hæc est:

14. Si

14. Si inscriptæ circulis æqualibus sunt æquales: secant peripherias æquales & contra. 28.29.p.3.

Euclides demonstrat ex posteriori è sectore circuli. At res de-



monstratione non habet opus: ἐφαρμόσει constat. congruant enim circuli: tum & inscriptæ & peripheriæ congruent.

Hinc licet

15. In circulis æqualibus peripheria data unius æqualem abscindere alterius.

Hæc in Messanensi Menelai editione à Maurolyco prima efficitur: & demonstrabilis propositio fit. quæ est ex 14.e. postulatum & fabricæ facilis. Verbi gratia è circulo i o velis abscindere peripheriam æqualem arcui a e. metire inscriptam a e. illa in æquali peripheria i o. subtendet arcum quæsitum.

Ex elemento autem facilis esset error si quis conciperet proportionem inscriptarũ & peripheriarum. ψευδογραφία sanè gravis & periculosa. inscriptæ circulis æqualibus ęquales secant peripherias æquales. Verum est: Ergo si major inscripta, si minor, secat majorem peripheriam, secat minorem. Et id totum verum est. Ergo inscriptæ peripheriis similiter majores, similiter minores, sive proportionales sunt. Falsum id est: ut demonstrat Ptolemæus. Eum igitur de disproportione peripheriarum & subtensarum lib.1. magnę constructionis vide ad caput quod interpreti 9. est. Videbis illic quod ratio majoris peripheriæ ad minorem, major sit: quam majoris inscriptæ ad minorem.

TH. FINKII

TH. FINKII GEOMETRIAE ROTVNDI,

LIBER TERTIVS.

SEgmentum circuli eſt quod comprehenditur extrinſecus à peripheria intus à recta.

Geometria circularis ſimplex poſita nobis fuit in lineis & circuli ſegmentis: lineas hactenus expedivimus: ſequitur circuli ſegmentum, cujus definitio generalis ad ſpecies Euclideas facilius diſtinguendum præmittitur. Rectam intus eſſe dicimus non quod unica illa ſaltem debeat eſſe: aliâs definitio ſectori non competeret. poteſt enim recta eſſe ſimplex aut duplex.

2. *Segmentum circuli eſt ſector aut ſectio.*

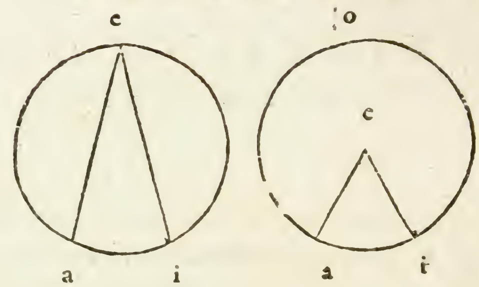
Nomina generis & ſpecierum penè eadem ſunt: at definitionibus tamen, ut Euclides fecit, diſtinguemus.

3. *Sector eſt ſegmentum circuli intus comprehenſum à recta duplici faciente angulum.*

Euclides eum ſaltem vocat ſectorem qui angulum contineat ad centrum: verũ nos generatim ſectorem hic definimus: quem poſtea diſtinguemus ex angulo in centro & in peripheria: quos angulos Euclides retinet: nec ſemper ſectorem vocat ſed periphraſi angulum in centro dicit: qui eſt angulus ſectoris in centro. ideo & angulus in peripheria erit angulus ſectoris in peripheria.

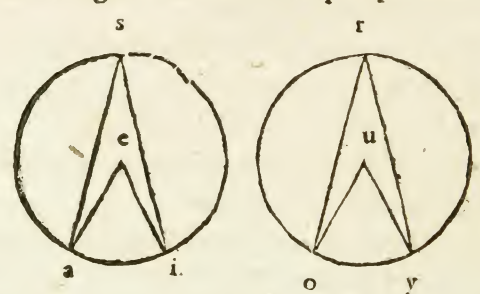
4. *Baſis ſectoris eſt peripheria cui inſiſtit.*

Exempli gratia ſit in circulo recta duplex ad centrum a e. i e. erit a e i. ſector: cujus baſis eſt peripheria a i. Eodem modo ſi concurrat recta illa duplex in peripheria.



5. *Anguli ſectorum circulorum æqualium ſunt ut baſes:& contra. 33.p.6.26.27.p.3.*

Euclidis 33. p. 6. generalis eſt quod anguli ſint ut baſes.inde ſpeciales ſunt 26.27.p.3.de æqualibus angulis & baſibus: & quidem utrinq; de angulis in centro aut peripheria.



Sed de angulis in centro ſatis eſt induci. Sint enim æquales anguli in centro a e i & o u y.per 2.e.7.R.baſes a i.o y erũt æquales:ideo ſecant peripherias a i.o y æquales per 14.e.2. itaq; ſi anguli ſint inæquales peripheriæ etiam tanto erunt inæquales. idem erit de angulis in peripheria.Et converſa ſimiliter patebit. Si enim baſes ſint æquales,iis ſubtenſæ æquabuntur per 14. e.2. & tum per 1. e. 7. R. anguli a e i. o u y æquabuntur. & ſic de inæqualibus baſibus induces. Et ſic revera Euclides inducit. Nam poſita propoſitione è 6.d.5. aſſumptionem de ſimili exceſſu aut defectu aſſumit à nulla præexiſtente demonſtratione.

Itaq;

Itaq;

6. ut sector ad sectorē sic angulus ad angulū. ad 33.p.6.

Sic ut se habet a e i ad o u y. sic angulus a e i. ad o u y.

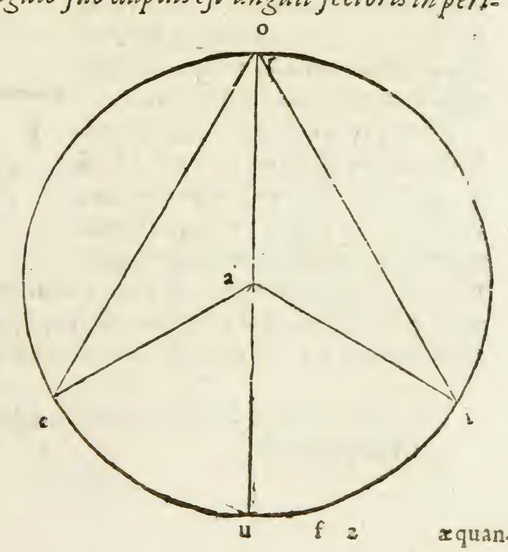
7. Sector est in peripheria aut centro.

Euclides habet angulum in centro & peripheria: illum sectori in centro tribuit: ergo & hic sectori in peripheria tribuetur.

8. Sector in peripheria est cujus recta facit angulum in peripheria: in centro, cujus recta facit angulum in centro. 9.d.3.

ut in præmissis figuris a s i. o r y sunt sectores in peripheria: a e i & o u y in centro. Sed, inquis, nō omnis sector in centro continetur à recta faciente angulū ad centrū ut sector i s e a. segmentum scilicet reliquum circuli de sectore i e a. Fateor quidem Archimedi segmentum hoc dici sectorem majorem: verum nos Euclidis definitioné retinuimus: & pars ea interjecto radio in duos sectores secari potest: ut postea secatur in geodæsia. itaq; minimum sectores in centro tres sunt.

9. Si sector in centro in eandē basin insistat cū sectore in peripheria: angulo suo duplus est anguli sectoris in peripheria. 20.p.3.



Exempli apud Euclidem varietas triplex est. primū cum angulorum diversa sunt latera ut hic

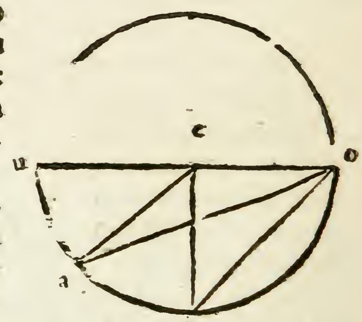
Angulus e o i. & e a i. basi cóveniant: dico angulum e a i. duplum esse anguli e o i. ducatur enim diameter o u. fient utrinq; triangula æquicrura o a e & o a i. itaque per 10. e. 6.R. angulus a o e. & e

æquan-

æquantur item anguli a o i. & i. jam per 2. c. 9. e. 6. R. angulus e a u æquatur interioribus duobus æqualibus quare anguli ad o duplus eſt: eodem modo angulus u a i æqualium duorum alterius etiam anguli a o i duplus eſt. itaq; totus e a i. duplus eſt totus e o i.

Secundum exemplum ubi angulorum duo latera ejuſdem rectæ nempe diametri ſunt. ut in hac figura angulus e duplus eſt anguli o. ob eandem cauſam. Nam o e. e i. æquantur per 2. e. 1. Ergo anguli o. & i æquantur: ſed his æquantur angulus a e i. Ergo alterius duplus eſt.

Tertium exemplum eſt ubi angulus in peripheria non continet angulũ in centro. ut in adjuncto ſchemate.



Sit angulus in centro a e i. communis baſis cum angulo a o i. dico angulum a e i eſſe duplum a o i. Ducatur enim diameter u e o. Hinc ita: Si ſit ut totus ad totum, ſic ablatus ad ablatum: erit ut totus ad totum ſic reliquus ad reliquum.

At ut totus u e i ad totum angulum e o i, ſic ablatus a e u ad ablatũ e o a. Nam totus u e i. æquatur interioribus e o i. & o i e æqualibus: quare duplus eſt alterius verbi gratia anguli e o i. Rurſus ablatus a e u. æquatur angulis e o a. & o a e æqualibus. quare duplus eſt anguli e o a. Ergo reliquus a e i ad reliquum a o i eſt ut totus ad totum duplus.

Itaq;

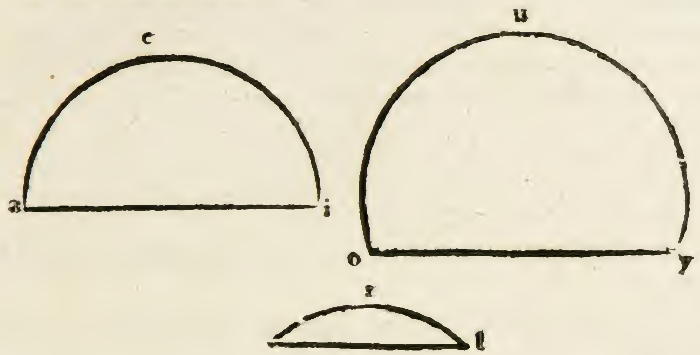
10. *Si angulus in peripheria æquetur angulo in centro: eſt duplus baſi.*

Veritas

Veritas & ratio consectarii manifesta est: & hinc patet æquatio angulorum in centro & peripheria.

11. Sectio est segmentum circuli intus comprehensum ab una recta, quæ basis sectionis dicitur. 19.d.1.

ut hic sectiones sunt.



12. Sectio absolvitur invento centro. 25.p.3.

Absolvere circuli sectionem est datæ sectionis circulum describere. id fit invento centro. centri inventio exposita est 12. 13. 27. e.1.

ut esto sectio a e i. absolvenda. Secetur itaq; basis a i. rectè & bifariam. Nam recta bisecans erit diameter per 12.e.1. prętereà à termino basis a. inscribatur recta a e. quæ etiã bisecetur rectè: & hæc recta erit diameter. itaq; concursus y erit centrum circuli per 13. e.1. quo dato & radio noto sectio absolvitur.

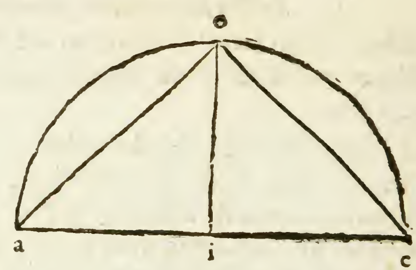
Euclidis fabrica paulo aliter fit. primam diametrum dicta jam ratione ducit. deinde terminum hujus diametri notum, ut exempli gratia hic e connectit cum termino baſis a. Quod ſi jam recta ducta cum dictis terminis claudat angulos æquales, hoc eſt per 10. e. 6. R. ſegmentum diametri à puncto biſectionis nempe y e & biſegmentum baſis a y æquentur: punctum biſectionis eſt centrum abſolvendi circuli per 27.e.1. Nam ab eodem puncto in peripheriam tres rectę, biſegmenta baſis a y. y i. & ſegmentum diametri y e æquantur. Sin recta ducta a e concludat angulos inæquales, hoc eſt per 11.e.6.R. ſi biſegmentum baſis a y & e y ſint inæqualia: angulum qui eſt ad terminum baſis æquat reliquo: atq; tum anguli æquati crus inventum concurrit cum diametro in centro deſcribendi circuli. per 26.e.1.

Sed triplici hac methodo non eſt opus: generalis è 13. e. 1. ſufficit.

13. Peripheria ſectionis biſecatur perpendiculari biſecante baſin. 30.p,3.

ut ſit ſectio a o e biſecanda.

Biſecetur igitur baſis a e: & ducatur perpendicularis per 7. & 9. e. 5. R. dico à perpendiculari biſecari peripheriã in o. Ducantur enim rectæ a o. e o. erunt triangula a i o & o i e æqua angulo æquicruro ad i. nempe recto. Ergo per 2.e.7.R. æqua baſibus a o & o e. quæ ideo per 14.e.2. ſecant peripherias a o & o e æquales.



Angulus in ſectione eſt angulus comprehenſus à duabus rectis è baſis terminis ſibi in peripheria conterminis. 8.d.3.

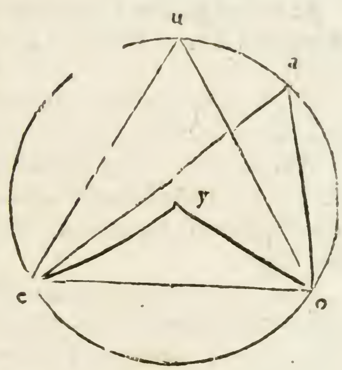
ut in

ut in ſuperiore figura rectæ a o. e o è terminis baſis a & e ſibi in peripheria ad o conterminis comprehendunt angulum in ſectione. Sequitur ratio ſectionis in angulis. Et primò ſectionis unius.

15. Anguli in eadem ſectione ſunt æquales. 21.p.3.

Eſto ſectio a e u o & in ea anguli ad a & u.

Dico angulos a & u æquari. Veritas patet ex 5.e. Nam anguli ſectorum æqualium circulorũ itaq; & ejuſdem circuli (quo ἐφαρμό-σιως genere etiã Euclides 30.p.3. utitur) ſunt ut baſes: quæ hic æquantur: eſt enim tantũ una. Quare anguli a & u æquantur. Et hinc concludes angulũ in ſectione eſſe angulum ſectoris in peripheria: cujus baſis à baſi ſectionis ſubtendatur.



Euclides demonſtratio hæc eſt, ex axiomate arithmetico: proportionalia ad idem æquantur: At anguli a & u ſunt proportionales ad idem nempe dupli ad angulum y in centro per 9.e.

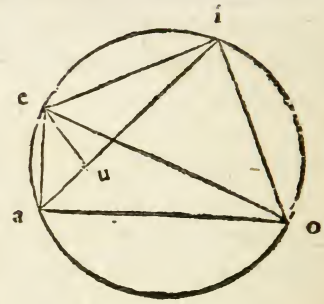
Ergo anguli a & u æquantur. At demonſtratio tantum ſpecialis eſſe angulorum in majore ſectione videtur.

16. Si duorum angulorum in ſectione vertices connectantur: rectangulum crurum ſe ſecantium æquatur rectangulis crurum reliquorum, & baſis atque cnnnectentis. 9.c.1.Ptolem.

Eſto ſint in ſectione a e i o. duo anguli ad e & i, qui connectantur recta e i: & ita erit quadrilaterum a e i o inſcriptum, de quo Ptolemæus loquitur.

Dico rectangulum a i. e o æquari rectangulo a e. o i & rectangulo a o. e i.

Ponatur



Ponatur enim per 5. c.6.e.3. R. aut absoluta sectione per 5.e.angulo o e i æqualis a e u.

Jam Syllogismus demonstrationis hic esto.

Rectangula e o.u i & e o.a u æquantur rectangulis a o.e i.& a e.o i.

At rectangulum a i.e o æquatur rectangulis e o.u i & e o. a u. per 4. e.11.R.

Ergò rectangulum intersectorum crurum a i.e o æquatur rectangulis a o.e i.& a e.o i.

Assumptio probatione alia non eget. Nam rectangula e o.u i & e o. a u sunt è latere uno rectanguli a i. e o & reliqui segmentis. Ergo toti æquantur. Propositio igitur demonstranda erit: Sic ergo:

Bis bina triangula e i u. e o a. & a e u. o e i sunt æquiangula. illic enim anguli a e o. u e i ex additione æqualis sive communis u e o.ad æquales i e o. a e u ex thesi,æquantur: Anguli vero e i u. & e o a per 15. e. æquantur: reliqui ad u & a per c.3.e.7.R. erunt æquales.

Hic vero anguli a e u. o e i ex fabrica,ad a & o ex 15.e.reliqui ad u & i per c.3.e.7.R.æquantur.

Itaque sigillatim bina cruribus sunt proportionalia per 9.e. 7.R. Eritq; illic

ut a o ad u i sic e o ad e i.

Hic vero & alternè

ut a e ad a u sic e o ad o i.

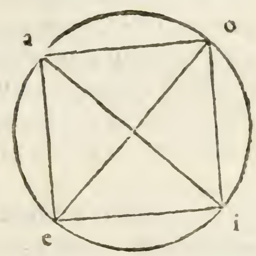
Itaq; per 5.e.11.R.illic rectangulum u i.e o rectangulo a o.e i. hic vero rectangulum a u.e o rectangulo a e. o i æquabitur. Atq; ita syllogismi constat propositio.& elementi veritas.

Habuimus itaq; considerationem solius sectionis: plurium ratio itidem in angulis jam sequitur.

17. *Anguli*

17. Anguli in oppositis sectionibus æquantur duobus rectis. 22.p.3.

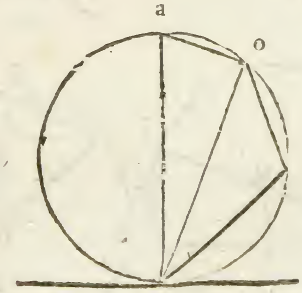
Euclides hic, ut in præmisso elemento Ptolemæus, inscripti quadrilateri meminit. At aptius elementum proponetur ab oppositis sectionibus resectis nempè inscripta: nihil enim adhuc de inscriptione fuit.



Sint ergo oppositæ sectiones o i e & o a e. dico angulos in sectionibus o a e & o i e. æquari duobus rectis. Nam æquantur tribus trianguli e o i. Etenim angulus i primo sibi ipsi æquatur. deinde e a o. æquatur angulis o e i. & e o i suis nempe partibus. quia o a i æquatur angulo o e i. & e a i æquatur angulo e o i per 15.e. At trianguli tres anguli per 9.e.6.R. æquantur duobus rectis.

18. Anguli in oppositis sectionibus æquantur alternis angulis secantis & contiguæ. 32.p.3.

Euclides & cum eo alii nonnulli elementum hoc doctrinæ cujusdam sectionis nempe semicirculi postponunt. At ego memor quos mihi in syntaxi hac cancellos circumdederim, quod sectionis erat in genere, præmittere volui.



Sint ergo oppositæ sectiones e i o. & e a o tangens vero u e y.

Dico angulum o a e æquari angulo o e y. & angulum o i e æquari angulo o e u. Quod si e a non sit diameter, fiat sane diameter ob demōstrationem. manebit enim angulorum in sectione æqualitas per 15.e. jam quod ad primam æquationem attinet tres anguli

a e u. a e o. o e y æquantur duobus rectis per 1. c. 8. e. 5. R. At duobus rectis æquantur etiam anguli e a o. a o e. a e o. per 9. e. 6. R. jam ab æqualibus his auferantur primo communis a e o deinde recti (qui etiam æquales sunt) a e u. ex thesi, quia u e y tangens est, & a o e rectus per 15. e. 1. & 8. e. 3. R. relinquentur ergo æquales e a o. o e y. Deinde quod ad alteram æquationem: anguli ad a & i æquantur duobus rectis per 17. e.

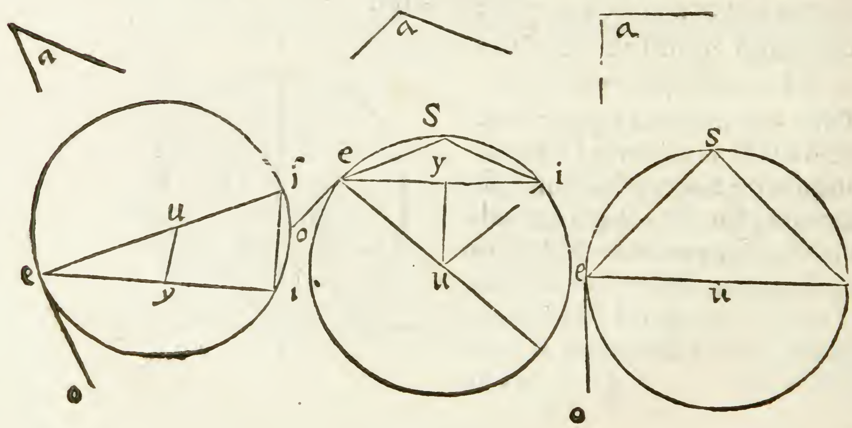
At duobus rectis etiam æquantur anguli o e u. o e y per 1. c. 8. e. 5. R. ablatis ergo æqualibus angulo a & o e y relinquuntur æquales ad i. & angulus o e u.

Hinc Euclidi sequuntur duæ propositiones mechanicæ sive problematicæ: quæ nobis hinc sunt consectaria specialia.

Itaq;

19. Si ad terminum datæ rectæ æquetur angulus rectilineus dato, & ab æquati vertice perpendicularis reliquo lateri concurrat cum perpendiculari à medio datæ: concursus erit centrum circuli per æquatum angulum descripti, in cujus opposita sectione super datam angulus æquabitur dato. è 33. p. 3.

Esto recta e i & datus angulus a. jam sit describendus circulus per angulum dato æqualem ut super rectam constituatur angulus dato æqualis in sectione: quod Euclides proposuit.



Fiat

Fiat itaq; ad terminum e angulus o e y æqualis dato : & à vertice e item medio datæ rectæ y concurrant duç perpendiculares e u. y u in u. Dico ibi esse centrum circuli per verticem æquati anguli describendi : atque angulum in opposita sectione e u i. nempe e s i æquari dato. itaq; si angulus datus rectus sit data, recta est diameter circuli optati: & proinde centrum datur. in acuto angulo & obtuso demonstrabitur simul.

Concursus diametrorum habet centrum.

At concursus dictarum perpendicularium est concursus diametrorum. Nam perpendicularis à vertice anguli est diameter per 2.e.2. item perpendicularis à medio basis per 12.e.1. Ergo concursus perpendicularium erit centrum.

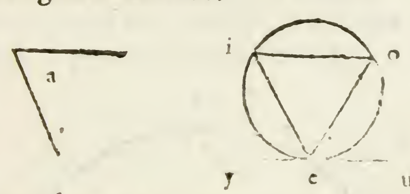
De angulo in sectione simili argumento in triplici angulorum genere concludes. Nam angulus in sectione opposita æquatur angulo o e y per 18.e. At angulus o e y æquatur dato. Quare hinc licet super datam rectam sectionem describere, quæ capiat angulum rectilineum æqualem dato.

Et

20. *Si angulus secantis & contiguæ æquetur dato angulo rectilineo: angulus in opposita sectione eidem pariter æquabitur. 34.p.3.*

Euclides proponit: à dato circulo secare sectionem quę angulum contineat æqualem dato angulo rectilineo.

Fabrica itaq; hujus in dicto elemento exposita est. Sit datus angulus a. & circulus e i o. Secanda jam sit optata sectio. Ducat̃ itaq; ad punctũ e tangens y e u.

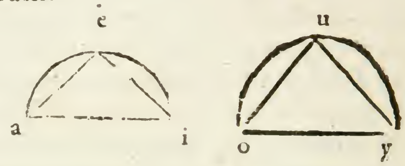


jam è puncto e recta faciens angulum o e u æqualem dato secat sectionem e i o quæ contineat angulũ o i e æqualem o e u: per 18 e. hoc est angulo dato. Ratio sectionis ita est: similitudo sequitur

21. *Si sectiones capiunt angulos æquales: sunt similes. è.10.d.3.*

Euclides definitionem hinc 10. fecit vel 11. ut Cōmandini fert exemplar. Sed non tam definitio eſt quam è definitione generali ſimilium figurarum conſectarium.

ut hic a e i. o u y.



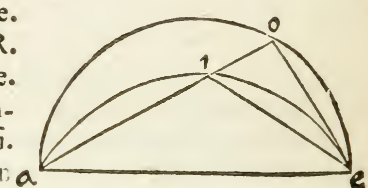
Hinc peripheriarum & baſiū conſequitur proportio, ut Pappus ait 13. the. 5. lib. Et triangula inſcripta quia ſunt æquiangula ex theſi, erunt etiam ſimilia. per 9. e. 7. R.

22. *Si ſectiones ſimiles ſunt in æquali baſi: ſunt æquales.* 23. 24. p. 3.

Euclides hinc fecit duas propoſitiones: diligentius tamen examinatæ in idem recidunt. Nam prior negans in affirmatam mutata idem enunciat quod ſequens affirmata. Demonſtratio eſt per abſurdū. Nam ſi nō ſunt æquales in æquali ſive eadē baſi:

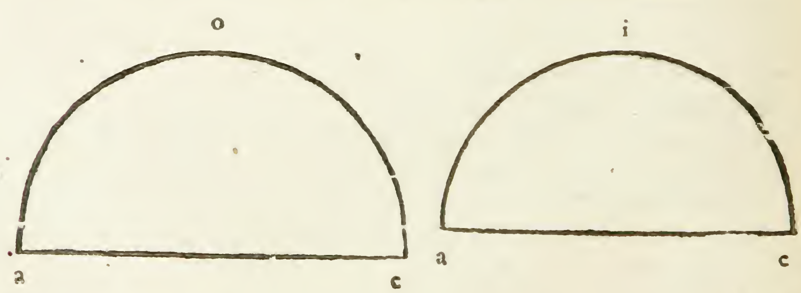
Sint inæquales ut ſectio a i e & a o e.

Itaque anguli a i e. a o e per 21. e. ſunt æquales. At per 4. c. 9. e. 6. R. angulus a i e eſt major angulo a o e. Quare nō poſſunt eſſe ſectiones inæquales ſi ſimiles ſint in eadē baſi.



Rurſus ſint ſectiones ſimiles in æqualibus baſibus a o e & a i e. Dico eſſe æquales.

Id Euclidi demonſtratur per ἐφάρμοσιν: & abſurdum.



Nam

Nam a e ſibi congruunt. Sectio autem ſectioni etiam cõgruet. Secus intus cadet aut extra, aut partim extra partim intra.

At hoc eſt cõtra primam partem. quia ſectiones ſimiles in eadem baſi (qualis hic per ἐφαρμόσιν eſt) non eſſent æquales. præterea circulus circulnm ſecaret in pluribus punctis quam duobus: nempe in terminis baſis & loco aliquo in peripheria ubi cadit extra: at hoc fieri nequit per 13.e.2.

23. Angulus ſectionis eſt qui comprehenditur à terminis ſectionis. 7.d.3.

Sic in præmiſsis ſchematis angulus o a e. & o e a ſectionis angulus dicitur: comprehenſus ſcilicet à peripheria & baſi.

TH. FINKII GEOMETRIAE ROTVNDI, LIBER QVARTVS.

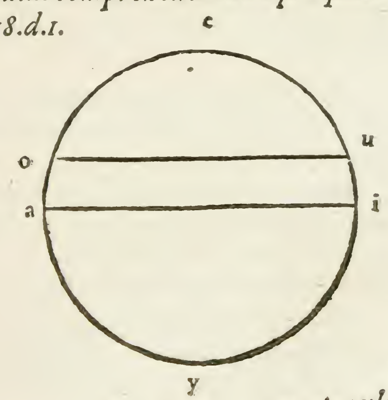
De cruribus anguli in ſemicirculo.

SEctio eſt ſemicirculus aut inæqualis ſemicirculo.

2. Semicirculus eſt ſectio dimidia circuli.

Itaq;

3. Semicirculus comprehenditur à peripheria & diametro. 18.d.1.



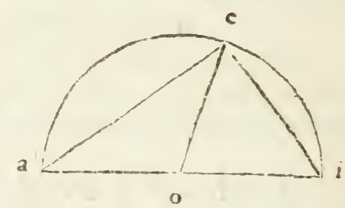
Sic a y i vel a e i eſt ſemicirculus. nempe dimidia ſectio circuli. comprehenditur autem à diametro & peripheria: quia circulus non niſi diametro biſecatur.

o y u eſt major ſectio.

o e u eſt minor ſectio.

4. Angulus in ſemicirculo rectus eſt, ſemicirculi minor recto rectilineo, major quovis acuto: in maiore ſectione eſt minor recto, majoris major: in minore major, minoris minor. è 31. 16. p. 3.

Elementi partes ſunt ſeptem. prima angulum in ſemicirculo eſſe rectum, ut hic angulum a e i: patet per 15. e. 1. Nam a e, i e è diametri terminis ſibi conterminantur in peripheria. Ergo per 15. e. 1. ſunt inter ſe rectæ: & proinde per 8. e. 3. R. angulus a e i eſt rectus.



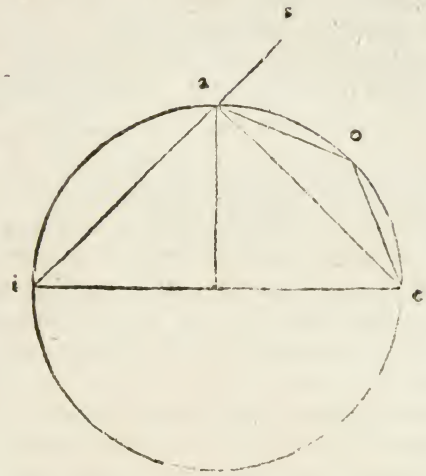
Poterit & hoc modo demonſtrari dimiſſo radio e o. quia a o. & o e æquantur: anguli a o e. & o e a æquantur per 10. e. 6. R. eodemque modo o e i & o i e. Quare totus a e i æquatur duobus e a i. & e i a. At ſi angulus trianguli ęquatur reliquis rectus eſt per 1. c. 3. e. 8. R.

Ariſtoteles hanc Geometriam in organo logico & philoſophia comprehendens angulum in ſemicirculo rectum dicit: propterea quod ſit dimidius duorum rectorum: nempe angulus a e i eſt rectus, quia eſt dimidius duorũ ad o qui æquátur duobus rectis. vel ſi a e cõtinuetur: ſiunt recti duo: ergo unus eorum dimidius.

Secunda pars, quod angulus ſemicirculi ſit minor recto rectilineo: patet ex eo quod ſit pars recti rectilinei: qui ſit è tangente & diametro.

Tertia: quod ſit major quovis acuto patet per 4. e. 2. ſecus ſi non eſt major: tangens non erit ſingularis eadẽ parte: ſed cadet quædam inter circumferentiam & tangentem quæ faciat angulum acutum majorem. At hoc fieri nequit.

Quarta angulum in majore ſectione eſſe minorem recto patet. ut hic angulum a i e. Nam in triangulo ſaltem poteſt eſſe re-



ctus unus. Quare cũ e a i. tanquam in ſemicirculo ſit rectus. a i e erit recto minor. Et ſi crus neutrum ſit per centrum: conſtitui tamen poteſt angulus, cujus crus ſit diameter: angulus tamẽ ęqualis, nempe in eadem ſectione.

Quinta, angulum majoris ſectionis i a e eſſe majorẽ recto patet, quia rectum rectilineum continet qui eſt ad a in triangulo i a e.

Sexta ſic patet: angulus in minore ſectione e o a eſt major recto: quia cum angulo in oppoſita majori ſectione a i e æquatur duobus rectis per 17. e. At angulus a i e eſt minor uno recto. Quare angulus e o a eſt major recto.

Septima: angulus minoris ſectionis o a e eſt minor recto: quia eſt recti pars. quemadmodum patet ſi protrahatur latus a i. fiet angulus exterior e a s rectus per 15.e.1.& 8.e.5.R. & hic angulum minoris ſectionis continet.

Atq; hæc Geometria eſt angulorum ſectionum circularium, ubi ſingularis & mirifica patet peripheriæ natura. Datur in circulo major recto: datur minor recto illic in majori hic in minori ſectione. Ecquid igitur negabis (inquiunt qui ſemicirculi angulum rectilineo recto æqualem volunt) in æquali ſectione quæ eſt inter majorem & minorem etiam angulum inter majorem & minorem nempe rectum dari. Cur ni, inquam, negarem? an tu contentum continenti æquale facies, partem ſcilicet toti? an non rectum rectilineum eſſe didiciſti diametri & tangentis? rurſus an non inter diametrum & tangentem peripheria ſecat angulos

gulos duos? horum ne alterum ut toti parem facias lumen naturæ insitum docuit?at(inquis)inter tangentem & peripheriam nihil intercipitur:nulla interjacet magnitudo. Cur id non probas?an non supra lib. 2. te docui interjacere magnitudinem & quidem infinitè dividuam?an non ἐφάρμοσιν documentum mathematicum hac in re tibi proposui? Sed vide quid tibi largiar: petis angulum semicirculi esse rectum: quod si nihil aliud petis:dabo tibi. Nam rectus est in suo genere:& sic recti in sphæra dicuntur, & medius inter majorem & minorem sui generis est: sic perpendiculum sic angulus rectus generatim definiuntur. At ideo rectum rectilineum esse contendis? aut recto saltem rectilineo æqualem? id tibi non datur: cur non detur intellexisti. At nec id tibi concedi potest angulos semicirculorum esse sibi æquales: sed Geometria nobis imperat ut inæquales statuamus. Si enim de æqualibus exempli gratia rectis recticruris tollantur inæquales anguli tangentium & peripheriarum inæqualium, relinquuntur anguli semicirculorum inæquales. At(inquis)quis tibi dedit angulos illos cōtactuum in peripheriis inæqualibus esse inæquales: id, inquam, ἐφάρμοσις Geometrica 6.e.2. me docuit.

Videatur itaq; sane res mira: dari majus, dari minus, & dari id inter quod æquale: æquale tamen non dari: vera tamen interim habeatur: verum, inquam, statuamus rectum rectilineum angulo semicirculi majorem. Sic mira est res in ratione inscriptarum dari rationem æqualitatis, rationem inæqualitatis utriusq;: non tamen dari proportionem. interim veram tamen Ptolemæus convincit.

Sed ad opes anguli nostri in semicirculo revertamur: quæque inde oriantur videamus.

Itaq;

5. *Si duæ rectæ diametro circuli contermine contermi-nentur in peripheria: comprehendunt angulum rectum.*

Fiet enim angulus in semicirculo: & angulus in semicirculo rectus est. Hinc Vogelinus 2. p. 2. sui elementalis quærit centrum

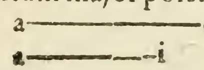
trum circuli. Nam inscribit in circulum datum duas inscriptas inter se rectas: quæ crura recti sunt: ea connectit basi recti: quę est diameter: in cujus medio centrum est. Atq; sic quartam habemus centri inventionem.

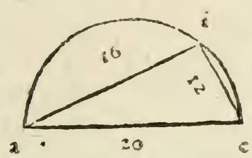
Et

6. Si duarum rectarum major fiat diameter circuli, minorq; majori contermina & inscripta connectatur: major plus poterit quam minor quadrato cõnectentis. ad 13.p.10.

Lemma duplex assumitur ad 14.p.10. in trãslatione Commandini: datis duabus rectis invenire quantum major sit potentior minore: & quæ possit utramq;. At intellecto primo secundum ex 5. e. non erit obscurum.

Esto sint duæ rectæ a e & a i. quæritur quantum major possit plus quam minor. Respõdetur hoc elemento. Fiat itaq; major a e diameter circuli: & à termino diametri inscribatur a i. cõnectaturq; cũ reliquo diametri termino e. dico a e plus posse quam a i potentiã e i. Nam triangulum a i e fit rectangulum per 4.e. Ergo per 5.e.12.R. basis a e æque potest cruribus: proindeq; plus potest quam a i quadrato e i. Atq; hinc licet trium ejusmodi rectarum datis duabus tertiam invenire. Et sic Ptolemæus 9.c.1. data diametro & inscripta, reliqui arcus ad semiperipheriam chordam investigat. Exempli gratia si a e major esset 20. a i vero 16. potentia a e est 400. potentia vero a i est 256. Ergo potentia e i est 144. ejusq; latus nempe recta e i erit 12.

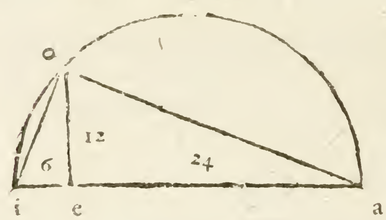




Et

7. Si recta continuata è duabus rectis fiat diameter circuli: perpendicularis à puncto continuationis in peripheriam erit proportionalis inter datas. 13.p.6.

Ex iis quæſtionibus quæ Mathematicos veteres valde exercuerunt: non poſtrema fuit ea quæ de cubi eſt duplicatione. Eam ad rem tandem ab Hippocrate Chio è mercatore Mathematico Meſolabium inventum tota poſteritas approbavit. At Meſolabium illud inventio erat duarum mediarum continuè proportionalium: cujus uſus ad quælibet corpora optata ſimilia fabricanda poſtea traductus eſt. Quare & ratio inventa eſt deſcribendi & fabricandi ſuperficies ſimiles: & id quidem per unam mediam: cujus fabrica etiam Meſolabium dici poſſet aut Meſographus. Fabrica autem è theſauris anguli in ſemicirculo depromitur. Sint enim duæ rectæ a e & e i. quibus invenienda ſit media proportionalis.



a————————e
e——i

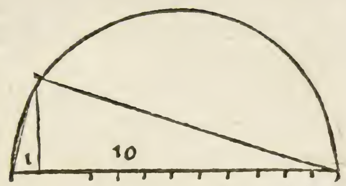
Continuentur itaq; in unam rectam: è cõtinuata a i fiat diameter circuli i o a. è puncto continuationis e erigatur perpendicularis e o. in peripheriam: dico ut a e eſt ad e o. ſic eſſe e o ad e i. Nam perpendicularis in triangulo ab angulo recto in baſin eſt proportionalis inter ſegmenta baſis 1.c.4.e.8.R. At e o eſt perpendicularis in triangulo ab angulo recto, quia ſemicirculi, in baſin.

Hinc trium harum rectarum datis duabus in numeris tertia invenietur ex doctrina continuæ proportionis & inventionis lateris quadrati. ut ſi verbi gratia a e eſſet 24. & e i eſſet 6. erit e o 12. Nam 24 per 6 facit 144. cujus latus eſt 12. Sic ſi detur i e 6. & e o 12. jam multiplica 12 per ſe, factum 144 diuide in 6. quotus 24 eſt longitudo a e. Itaq; hinc licet

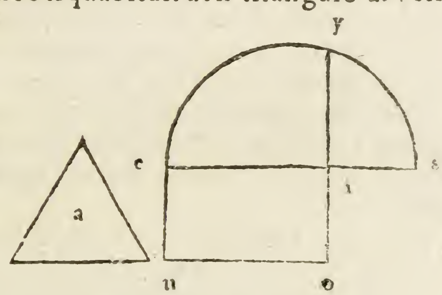
8. *Rectilineo conſtituere æquale quadratum.* 14.p.2.

Nam datum rectilineum aut oblongum eſt aut triangulum & alias triangulatum. Si oblongi latera continuata fiant diameter: è puncto item erigatur perpendicularis: ea erit proportionalis inter

inter latera oblongi.itaq; per 4. e. 12. R. latus quadrati æqualis dato oblongo. ut ſi ſit oblõgum decempedale: cujus latera ſint pedum 10. & 1. tum latus pedum 10 continuetur cum latere pedis unius: proportionalis media erit latus. ut in adjuncta figura.



Si rectilineum præter oblongum detur aliud: oblongo æquari neceſſe eſt per 1. & 2. c. 10. e. 10. R. Tum oblongo æquato æquale quadratum, dato rectilineo æquabitur. ut ſi triangulo a. velis conſtituere quadratũ æquale. per 1. c. 10. e. 10. R. fiat oblongum e i o u æquale dato triangulo. jam latera e i. i o fiant diameter, erit i y latus quadrati quæſiti.

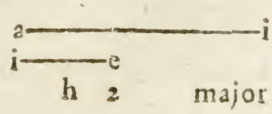


Campanus addit rationem ſecandi datarum majoré ut inter ſegmenta minor ſit proportionalis: at fabrica eſt ſpecialis. nam quantitas minoris determinatur ne ſcilicet major ſit quam dimidia majoris. Fabrica ſic eſt.

Et

9. Si duarum rectarum major minimum dupla minoris fiat diameter circuli, minorq́, extremæ diametro perpendicularis connectatur cum peripheria per rectam diametro parallelam: recta à connexione in diametrum perpendicularis ſecat ſegmenta quibus proportionaliter minor interjacet.

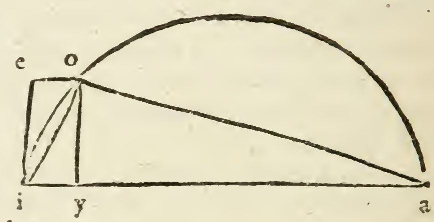
Eſto recta major a i minor i e quæ ſummum dimidia ſit majoris. Nam ſi



major

major ſit verbi gratia ſubſeſquialtera ab ejus termino diametro parallela nunquam cum peripheria concurret: cum radio peripheriæ major ſit.

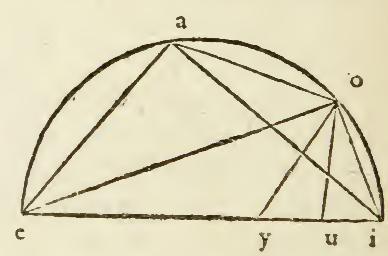
jam ſit ita major ſecanda ut inter ſegmenta minor ſit ꝓportionalis media. fiat itaq; major diameter, aq; termino ejus i. minor erigatur perpendicularis, atq; terminus



e connectatur cum peripheria per rectam e o diametro parallelam. tum à termino connexionis o. ſit perpendicularis. jam per 7. e. o y hoc eſt per 5.c.12.e. 5. R. data e i eſt proportionalis inter a y. y i. ſegmenta majoris. Hinc licet rectangulum, cujus latera conditiones elementi admittant, ita ſecare ut rectangulum è ſegmentis certis majoris, æquetur quadrato minoris. Quæ res neceſſaria ſæpe eſt in dividundo agro rectangulo aut ſimili re inter hæredes.

10. Perpendicularis à puncto biſectionis peripheriæ cruris anguli in ſemicirculo in diametrum, ſecat ſegmentum dimidium differentiæ reliqui cruris & diametri. 9.c.1. Ptol.

Eſto angulus in ſemicirculo e a i. atq; à puncto biſectionis peripheriæ a i deſcendat perpendicularis o u. duo ſegmentum diametri u i eſſe dimidium differentiæ reliqui cruris e a hoc e y & diametri, hoc eſt eſſe dimidium recta y i.



Ducantur enim a o, o i, o y, o e. jam quia in triangulis a e o, o e y. anguli ad e æquales ſunt ex theſi & 5. e. 3. & æquicruri: æquabitur baſis a o, baſi o y per 2. e. 7. R. ſed a o ex theſi & 14. e. 2. ęquatur recta o i. ergo triangulum y o i æquicrurum eſt in baſi: & proinde per

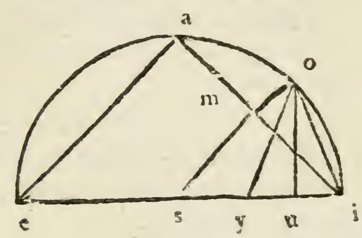
2. e. 7. R.

2.e.7.R.perpendicularis o u biſecat baſin y i. Quod ſi jam e i ſit 20.a i vero 16.erit a e 12.& proinde u i 4.

Itaq;

11. Si radius biſecet crus anguli in ſemicirculo: ſegmentum radii inter centrum & crus dictum eſt dimidium reliqui cruris. 4.th.1. Copern.

Eſto rurſus in ſemicirculo angulus e a i. atq; crus a i biſecetur à radio s o in m. dico ſegmentum s m eſſe dimidium ipſius a e. ponatur enim e y æqualis rectæ e a. jam hæc ſit demonſtratio. Recta u s eſt dimidia rectæ a e.



Nam ea æquatur rectis e s, s y ex theſi, hoc eſt i s, s y. atqui y u, i u æquantur. quare addito cõmuni s y. erit s u item s y cum i u ſemiſsis i s & s y hoc eſt a e.

At s m æquatur ipſi s u. Nam tringula o s u, i s m cum rectos habeant ad u & m & angulum s communem erunt æquiangula: & per 9.e.7.R.cũ s i æquetur ipſi s o & s u æquabitur ipſi s m. Ergo s m eſt dimidia rectæ e a.

Copernicus facilius ſic demonſtrat. Quia duo triangula a e i, s m i ſunt æquiangula. anguli enim a & m recti ſunt illic quia in ſemicirculo hic ex theſi. & angulus i cõmunis. Ergo ut a i dupla eſt ipſius m i ſic & a e dupla eſt rectæ s m.

Ad 6.e propoſita fuit ratio inveniendæ inſcriptæ peripheriæ reliquæ ad ſemipheriam data diametro & inſcripta: idem hinc poſſumus præſtare. Nam datur s i radius & m i dimidia inſcripta. Ergo per 5.e.12.R. Dabitur s m. atq; hujus duplum latus a e. atq; hic calculus eſt facilior: quanto facilius pauciores numeros quam plures numeramus. ut ſi exemplum ejus loci repetatur ſit diameter 20 erit radius 10. inſcripta a i 16. erit m i 8. jam potentia radii minus potẽtia m i erit 36 cujus latus eſt 6. nempe s m. ergo erit e a. 12.

 Et

Et licet in numeris

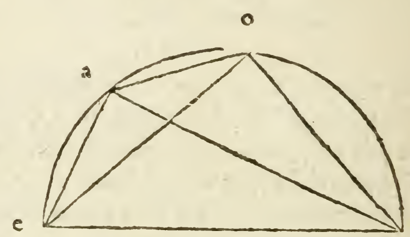
12. *Inter punctum bisectionis & terminum diametri inscriptam invenire.*

Hoc Ptolemæo est propositum: data inscripta alicujus arcus & diametro nota: inscriptam arcus dimidii investigare. Hinc Theoni theorema κατὰ διχοτομίαν dicitur.

Inventio autem est è proportione continua, docente proportionem 2.c.4.e.8.R. Quia enim in præmisso schemate angulus e o i est rectus, & o u perpendicularis: erit ut e i ad o i. sic o i ad u i. jam detur e i 20. & u i 4. erit inde factus 80. cujus latus erit $8\frac{16}{17}$ vel 9 pro o i.

13. *Planus crurum, in semicirculo angulorum, se secantium plano reliquorum crurum minutus, est planus diametri inscriptæq; inter angulorum vertices.*

Esto sint in semicirculo e a i, e o i. anguli. Erit per 16. e.3. Rectangulum e o, a i æquale rectangulis o i, a e & e i, a o. Ergo ex proprietate numerationis primæ planus e a, o i subtractus de plano a i, o e relinquit planum e i, a o.



Itaq; hinc licet

14. *Datis ab eodem diametri notæ termino inscriptis inæqualium arcuum, inscriptam inter terminos datarum reperire.*

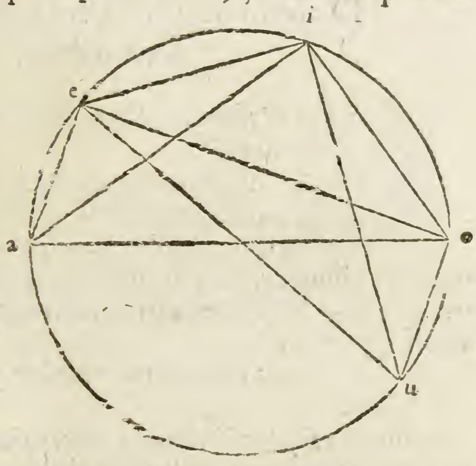
Hoc Theoni est theorema καθ' ὑπεροχήν. Esto sit diameter e i 20. inscripta o i 12. & a i 16. quæritur inscripta a o. per 6.e. aut 11.e. invenietur e o 16. a e 12. Hinc planus e o, a i est 256. planus e a, o i est 144. jam minore de majore ablato relinquitur planus e i, a o 112. Eo igitur diviso in e i 20. quotus $5\frac{3}{5}$ est pro inscripta a o.

Et

Et

15. Datis duarum peripheriarum inſcriptis contermi-nis, ſummæ peripheriarum inſcriptam invenire.

Hoc Theoni eſt theorema κατὰ σύνθεσιν. Sint arcuum contermi-norum a e, e i inſcriptæ a e $5\frac{3}{5}$ e i 12. quæritur inſcripta a i. Hic quia datur a e $5\frac{3}{5}$ dabitur per ſuperiora e o $19\frac{1}{5}$. Rurſus quia datur e i 12. dabitur etiã i u 16. jam ſi de plano e o, i u tollas planum e i & o u. hoc eſt a e. qa utraq; eſt rectæ e o cõtinuatio in diametrũ: hoc eſt de $307\frac{1}{5}$ tollas $67\frac{1}{5}$ relinquitur planus diametri e u, i o. quare datur præ præmiſſum i o 12. jam nota i o cum diametro a o 20. per 6. e. aut 11. e. dabitur a i 16. inſcripta quæſita.



Atque ſic Ptolemæo, quæ debere ipſi volvi, exolvi: referens ipſius theoremata de ſubtenſis: & ad ſuum locum ut puto referens. Hipparchus de ſubtenſis ſcripſiſſe refertur libros 12. Menelaus, quem vulgo Arabum interpretes Mileum vocãt, de iiſdem libros ſex confecerat. At Ptolemæus Hipparchi 12. & 6. Menelai quinq; theorematis contraxit. Tantum in eo, hoc quidem in loco, brevitatis fuiſſe ſtudium videmus. Brevitas hęc tam grata accidit: ut à poſteris Ptolemaica theoremata retenta fuerint. Retinuit Purbachius: retinuit Copernicus in opere ſuo mirando: retinuere alii. Et nos retinere voluimus ob pleniorem textus Ptolemaici intellectum: qui miſerè hodie è ſcholis, ſuperioribus tamen annis avide receptus, excluſus videtur.

TH. FINKII

TH. FINKII GEOMETRIAE ROTVNDI,

LIBER QVINTUS.

De semicirculi sinibus, tangentibus, secantibus.

IN semicirculo considerantur sinus & his connexa recta.

Paulatim jam accedimus propius ad subtilissimam mensuram, quæ vulgo per sinus fieri dicitur aut subtensas. Eam nos ex semicirculo deducere voluimus: quia in eo primò residet non tam doctrina sinuum quam his connexarum rectarum: quæ mensuram illam tandem perficient.

2. *Sinus est recta à termino peripheriæ perpendicularis.*

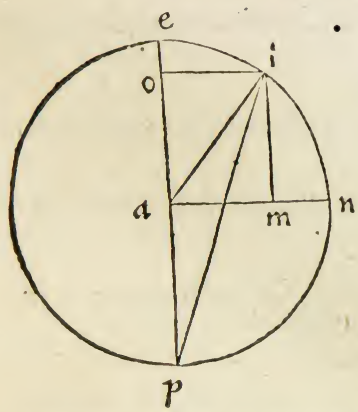
Sinus vocabulum in hac significatione est Arabicum: & ideo barbarum: attamen multorum seculorum consensu in arte retentum fuit. Est enim gratissima in Mathematicis concinna orationis brevitas. Et Aristoteles topico & capite 2. res nominibus tritis & usitatis appellandas censet. Neque commodum magis aut magis Latinum vocabulum habemus. Nam quod quidam Ptolemæum imitati semissem subtensæ dicunt: id, præterquam quod periphrasticum ac longum sit, parum accuratè dici videtur. non enim omni sinui hæc tribui potest appellatio,

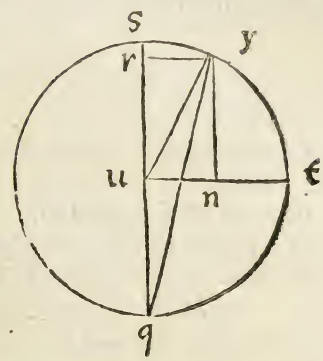
3. *Similium peripheriarum sinus radiis suorum circulorum sunt directè proportionales.*

Sint enim similes arcus e i. s y eorumq; sinus i o, y r, & o e, r s.

Erit ut u y ad a i hoc est u s ad a e radius nempe ad radium sic y r ad i o item r s ad o e.

Prima





Prima pars patet ex 9.e.7.R. Sunt enim triangula y r u & i o a æquiangula: cum anguli ad r & o per 2.e. recti, ſint æquales per c.8.e.3.R.& anguli ad a & u ſunt æquales. Nam ductis i p & y q. anguli ad p & q. æquantur ex theſi & 21.e.3.ſed horum ęqualium dupli ſunt anguli ad a & u. Ergò æquales. Et per c.3.e.7.R. anguli ad i & y. æquantur. Quare dicta triangula cruribus ſunt proportionalia.

Secunda pars ſimiliter conſtat ex axiomate illo arithmetico: ſi ſit ut totus ad totum ſic ablatus ad ablatum: erit ut totus ad totum ſic reliquus ad reliquum.

Hic autem eſt ut totus s u ad totum e a ſic ablatus u r ad ablatum a o. Ergo ut u s ad a e ſic reliquus r s ad reliquum o e.

Itaq;

4. *Peripheriarum ſimilium ſinus ſunt directè proportionales.*

Sint in præmiſsis figuris arcus i n, y t item e i, s y ſimiles eorumq; ſinus i m. hoc eſt per 6.e.10.R. o a & y n hoc eſt r u item i o & y r. Erit ut o a ad r u. ſic o i ad y r. Nam
ut o a ad r u ſic i a ad y u.
ut i a ad y u ſic i o ad r y. Ergo ex æquatione rationum, ut o a ad r u ſic i o ad r y.

i Eodem

Eodem modo in finibus dictorum arcuum o e, s r, & m n, ut veritas patet simili syllogismo.

Et

5. ut sinus ad sinum, sic peripheria ad similem peripheriam.

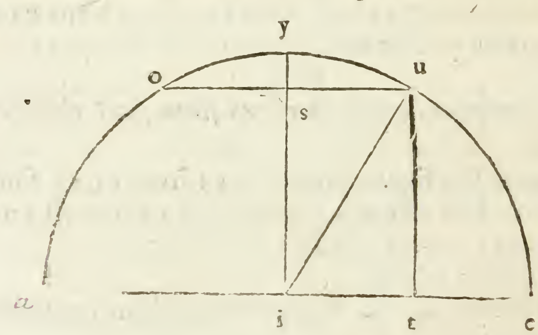
Exempli gratia in schemate præmisso est ut y r ad i o sic y s ad i e. Nam
ut y r ad o i sic u y ad a i.
ut u y ad a i sic tota peripheria t s q ad totam n e p per 8.e i.
ut tota ad totam sic y s ad i e ex thesi.
Ergò ex æquatione rationum
ut y r ad o i sic y s ad i e.

6. Sinus primus est aut secundus.

Sinum recentiores penè omnes secant in rectum ac versum: rectum demum in primum aut secundum. At distributio illa parum videtur logica. Nam & sinus versus est perpendicularis & rectus: jam vero philosopho in topicis distributio ea vitiosa est, ubi partes conveniant.

7. Sinus primus est sinus in diametrum per reliquum arcus terminum ductam.

Sinus generatim fuit definitus perpendicularis à termino pe-



ripheriæ:

ripheriæ:in primo autem dimittitur eousq; perpendicularis, ut cadat in diametrum ductam per alterum terminum peripheriæ. Sic arcus y u sinus primus est perpendicularis à termino uno verbi gratia u in diametrum i y ductam per alterum terminum datæ peripheriæ. Sic arcus u e sinus primus est recta u t. ita arcus o y sinus primus est recta o s. Atque hic sinus semper intelligitur quando sinus simpliciter vocatur: Et nos hanc synecdochen ob brevitatem deinceps retinebimus.

Itaq;

8. Sinus peripheriæ, est sinus reliquæ ad semipheriam. Theon.11.c.1.Ptol.

Sic in præmissa figura sinus arcus u e nempe u t est etiam sinus arcus u y a reliqui ad semiperipheriam. patet ex definitione sinus. Nam ut est perpendicularis ab uno termino in diametrum per reliquum terminum ductam.

Atq; hinc est quod vulgo dicunt eundem sinum esse duorum arcuum: unius quidem quadrantis peripheria minoris, alterius vero majoris. Quæ res in calculo sæpè imperitis imponit: ut angulum inveniant acutum ubi obtusus est.

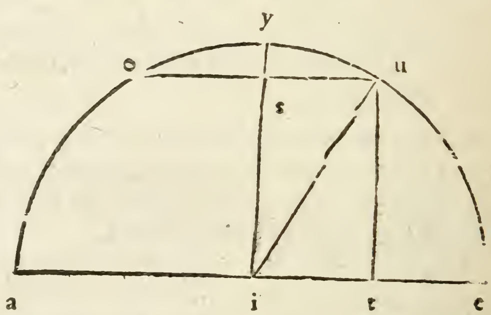
9. Radius æquè potest sinibus peripheriæ & complementi. 1.p.Regio de sinib.

Sequitur deinceps comparatio sinuum cum radio & inscriptis: & primo ratio æqualitatis in potentiæ sinuum & radii comparatione. Complementum dati arcus vocatur ipsius & peripheriæ quadrantis differentia. Exempli gratia, arcus y u est complementum arcus u e, item arcus u a. quia est differentia y e & dictorum arcuum. Sit jam sinus arcus u e recta u t. & complementi u y sinus sit recta u s. Dico radium i u æquè posse sinibus u s, u t. per 5. e. 12. R. Nam triangulum u s i est rectangulum. Ergo i u æquè. potest cruribus u s & s i hoc est, per 2.c.6.e.10. R. u t. Hinc jam dato radio cum sinu peripheriæ datur sinus

complementi.ut si detur u t 8. & radius sit 10.erit potentia radii 100.potentia vero u t 64. Ergo hac ab illa subducta potentia u s relinquetur 36.cujus latus 6.est ipse sinus u s.

Sinus complementi vulgò dicitur sinus rectus secundus arcus cujus est complementum. verum & intelligenter magis dicitur complementi sinus:& in bonis authoribus sinus complementi dicitur. Sequitur ratio inæqualitatis cum inscriptis.

10. *Sinus est dimidius inscriptæ arcus dupli.*



ut in hac figura u s est dimidius o u subtendentis peripheriā u y o. duplam peripheriæ u y cujus sinus est recta u s.Nam radius i y rectè secat inscriptā o u per 7.e. Ergò per 19.e.1. bisecat.est itaq; u s dimidius ipsius o u. sed & arcus o y u est duplus arcus y u. quia bisecatur in y per 13.e.3.

Ex hoc elemento vulgo definiunt sinum chordam (sic appellant inscriptam) dimidiatam: & elegantioris literaturæ studiosi semissem subtendentis duplum arcum appellant. Verum cum hæc appellatio cuidam tantum sinui nempe primo competat:& verò cum de rebus convenit,ut Cicero monet, in verborum usu faciles nos esse oporteat: quæ causa est recedendi à nomine sinus hactenus retento.

Atq; hinc patet quomodo Ptolemaica de inscriptis ad sinus, & contra sint accommodanda. Nam sinus dimidiati arcus dati duplicatus est chorda seu subtensa dati arcus : & inscripta dupli arcus dimidiata est sinus dati arcus.

Itaq;

11. *Inscripta peripheriæ potest quadruplum sinus dimidiæ.*

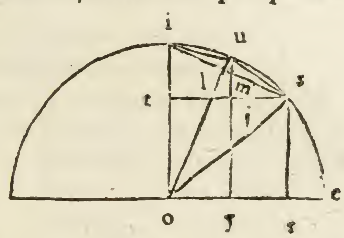
Hoc

Hoc consectario utitur Regiomontanus 2.p.de sinibus,ubi sinum quærit vigesimæquartæ partis peripheriæ graduum nempe 15.Veritas autem consectarii patet per 7.e.12.R.

Esto enim inscripta o u.dico eam posse quadruplum u s.Nam est bisecta ad s per 10.e.& y u est dimidia peripheria totius o y u. itaq; o u potest quadruplum dimidii per 7.e.12.R. Sequitur ratio inæqualitatis segmentorum diametri.

12. *Sinus à terminis æqualium arcuum secant segmentum diametri majus propius centro.* 6.p.Reg.de Sinib.

Extat propositio hæc etiam in commentariis Theonis l.2.& apud Iordanum 2.p.de ponderibus:ubi ejus usus cóspici potest.



Sint æquales peripheriæ i u & u s.à quorum terminis sint sinus i o,u y & s r. Dico segmentũ diametri o y esse majus segméto y r. Ducantur enim radii in u & s.anguli i o u & u o s per 5. e. 3. æquabuntur: & ducta i s per 2. e. 7.R. rectæ i l & l s æquantur. Quare i m erit major quam m s. Sed sit quoq; sinus s t. jam quia m j est parallela ipsi i t per 12.e.5.R. Erit per 13.e.5.R.

ut i m ad m s sic t j ad j s. hoc est o y ad y r.

At i m est maior quam m s.Quare & o y major quam y r.

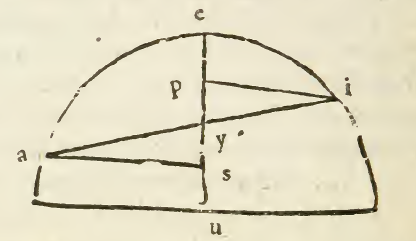
Sequitur jam comparatio sinuum cum inscriptis in qualitate nempe proportio.

13. *Si radius secet inscriptam in semicirculo: segmenta sinibus conterminorum arcuum inter radium & inscriptam sunt directè proportionalia.* 11.c.1.Ptolem.

Dicimus inscriptam in semicirculo sive basis sit hoc est diameter,sive alia quæcunq;:Ptolemæus tamen basin excludit: postulat enim arcus inter terminos inscriptæ,& diametri conjunctim minores semiperipheria:verum hoc fecit propter finem & usum quem hinc haurire voluit nempe discretionem peripheriæ in duas optatas.

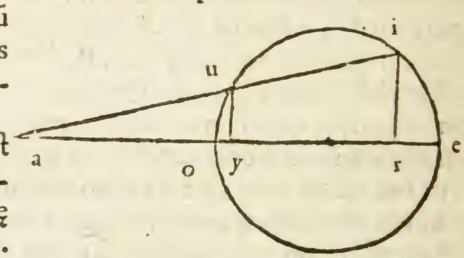
Sit itaq; inſcripta a i eamq; ſecet radius u y e. Sintq; arcuum i e & a e ſinus i p & a s.

Dico a y eſſe ad a s ut eſt y i ad i p. Quia enim i p & s a ſunt parallelæ per 12. e. 5. R. angulus enim ad p & s rectus eſt: erit per 14. e. 5. R. ut i p ad i y ſic s a ad a y. quare alternè & inverſè.



14. Si à termino diametri continuatæ recta ſecet peripheriam: ſinus à punctis ſectionum rectæ ductæ & exteriore ejus ſegmento directè ſunt proportionales. 11. c. 1. Ptol.

Eſto enim diameter o e continuata in punctum a. atq; à puncto a recta ſecet peripheriam in u & i. & à punctis ſectionum ſint ſinus u y, i r. Dico eſſe ſinũ i r ad ductã i a ut eſt ſinus u y ad exterius ſegmentum u a.



Demonſtratio eadẽ eſt cum præcedente. Nam rectæ a i & a r ſunt interſectę parallelis u y, i r per 12. e. 5. R. Ergo ſegmenta conterminis parallelis ſunt proportionalia per 14. e. 5. R.

Atq; ita comparatio fuit ſinuum inter ſe & cum inſcriptis: ſupereſt ſinuum cum peripheriis ratio una.

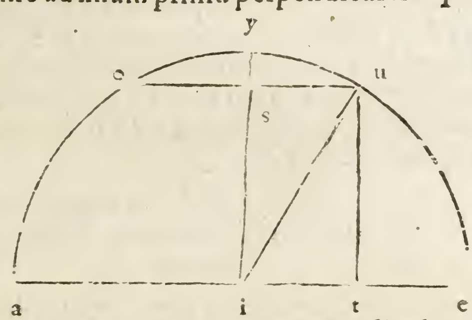
15. Si ſinus peripheriarum quadrantis peripheria minorum ſunt æquales: peripheriæ ſunt æquales: & contra.

Elementi veritas è 10. e. & 14. e. 2. eſt manifeſta. ut enim illic inſcriptæ æquales ſecabãt peripherias æquales: ſic hic ſemiſſes inſcriptarum habent ſemiſſes peripheriarum æquales. itaq; ut illic erat ratio ęqualitatis etiam inde inæqualitatis cõſequebatur: ita & hic.

& hic. Sed id tamen notari oportet peripherias quadrantis peripheria minores hic postulari. Secus ψευδογραφία gravis fuerit: de qua etiam ad 8.e dixi. cum idem sinus peripheriarum duarum sit. Est itaq; speciale elementum sinuum primorum. Nam sinus secundi arcuum habent arcus æquales. nec requiritur hic ut arcus sit quadrantis peripheria minor.

16. *Sinus secundus est sinus ab arcus termino in sinum primum.*

Sinus secundus vulgo versus sinus dicitur: à nonnullis authoribus nescio qua similitudine sagitta: definiri potest segmentum diametri ab arcus termino ad sinum primũ perpendiculare: quo tamen verborũ cumulo nõ est opus. Nã si sinus est: constat qđ sit perpendicularis: et si in sinum primũ cadit non põt fieri quin sit segmẽtum diametri. Sic peripheriæ y u sinus secundus est recta y s à termino y in sinũ primum u s perpendicularis. Sic sinus secundus arcus u e est recta e t. arcus u y a est recta a i t.



Itaq;

17. *Radius æquatur sinui complementi peripheriæ quadrantis peripheria minoris & sinui secundo.*

ut in præmissa figura radius i e ęquatur sinui i t cõplementi peripheriæ u e. & sinui secundo t e nempe dictæ peripheriæ u e. Hinc facile dati arcus datur sinus secundus. Nam si sit arcus u y sinus nempe i t 6. & radius 10. erit t e. 4. sinus secundus arcus u e.

Et

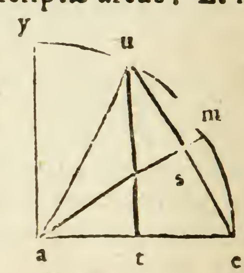
18. *Sinus secundus peripheriæ quadrantis peripheria majoris æquatur radio & sui complementi sinui.*

Esto arcus u y a ejusq; complementum u y. erit a t sinus secundus

dus dati arcus æqualis a i radio,& i t ſinui complementi. Quare ſumma è radio & ſinu cōplementi,eſt dictæ peripheriæ ſinus ſecundus. ut radius ſit 10 ſinus i t 6. erit a t 16. pro ſinu ſecundo optato. Sequitur comparatio alia ſinuum inter ſe cum inſcripta.

19. Sinus primus & ſecundus poſſunt inſcriptam ſuæ peripheriæ.

Supra dixi inſcriptam poſſe quadruplum ſinus peripheriæ dimidiæ. jam docetur inventio potentiæ inſcriptæ arcus: Et ſic Regiomontanus quærit. ut in ſubjecta figura ſi ducatur inſcripta u e. per 5.e.12.R. u t & t e poterunt inſcriptam u e. Verbi gratia ſi ſit u t 8. & t e 4. potentiæ 64. & 16. hoc eſt 80 erunt quadratum inſcriptæ: cujus quarta pars 20 per 11.e. eſt potentia ſinus ſemiſsis peripheriæ u e ergo ſinus ſemiſsis dictæ erit $4\frac{4}{9}$.



20. Sinus peripheriæ in quadrante eſt medius proportionalis inter ſemiradium & ſinum ſecundum peripheriæ duplæ. 3.p.Reg.de Sinib.

Eſto data peripheria e m ad quam dupla ſit e u. ſitq; ſinus arcus u e recta u t, peripheriæ vero e m ſit recta e s. Dico ſemiſſem a e hoc eſt radii eſſe ad e s. ut e s eſt ad e t ſinum ſecundum peripheriæ duplæ.

Sunt enim triangula a s e & u t e. æquiangula ob rectos ad s & t. communem ad e. erit ergo per 9.e.7.R. ut a e radius ad inſcriptam u e. hoc eſt ſemiradius ad ſemiſſem inſcriptæ nempe ad e s. ut e s ad e t.

Hinc Regiomontanus inveſtigat ſinus dimidiarum peripheriarum. ut verbi gratia ſi quæreretur e s. daretur autem u t 8.& a t 4 erit t e 6. radius a e 10. jam ut 5 ad s e. ſic s e ad 6. quare factus à 5 per 6. nempe 30 erit etiam factus ab e s per ſe. Ergo ejus latus erit $5\frac{5}{11}$. pro ſinu e s arcus dimidii ad datum e u.

Sed

Sed cognita proportione elementum ad plures poteſt uſus transferri.

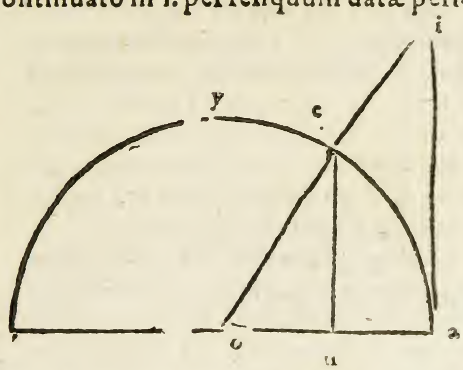
Atque hactenus ſinus expedivimus: ſupereſt ut ad rectas his connexas deſcendamus.

21. Recta ſinibus connexa eſt tangens peripheriæ aut eam ſecans.

Propoſuimus ut in circulo inſcriptas, ſic in ſemicirculo ſinus perpendendos ſed & rectas ſinibus connexas. Eas plenioris intellectus cauſa in tangentes & ſecantes dislocamus. Verbis ſi, hac in re non nova, novis: tamen ut ſperamus accommodatis.

22. Tangens eſt à termino peripheriæ altero perpendicularis in radium extra per reliquum terminum continuatum.

Eſto peripheria a e. ſitq; ab ejus termino a perpendicularis a i terminata in radio o e continuato in i. per reliquum datæ peripheriæ terminum. erit a i tangens datæ peripheriæ. Sic vocare placuit quia ſit perpendicularis extremæ diametro. hoc dato per 1.e.2 erit tangens: itaq; Geometria ipſa commodum ſuppeditavit nomen: nec aliunde adferri cõmodius poterit. Nam quod quidam numerum fœcundum rectam a i vocant. id ii videant quomodo defendant: mihi non probabunt. Damus aliquid peritiſſimo illi artifici Regiomontano homini Germano: qui primus hujus vocabuli author dicitur: damus etiam aliquid receptæ conſuetudini. Verum id non facile damus ut verba ea in uſu retineamus quibus elegantiora, breviora, ſignificantiora, veriora habeamus.

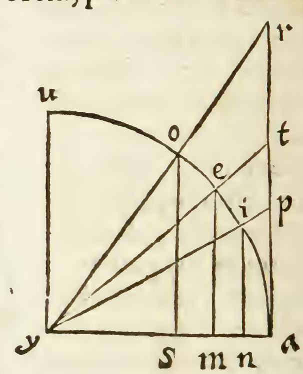


Rheticus in tabula à ſe edita, & triangulorum canone non tantum rectam a i ſed & ſinum modo baſin trianguli modo cathetum & perpendiculum oppoſiti anguli ratione vocat. Quæ ſi ratio commutationis verborum ejuſdem lineæ cui placeat per me certe licet. interim tamē ſi baſin ille vocet ego legitimè perpendiculum vocare poſſum. quia ſi baſi recta eſt perpendicularis & baſis erit eidem rectæ perpendicularis. Sed de nominibus litigare nolumus. Quæ nobis bona Rheticus communicavit, grato animo agnoſcimus, agnoſcent alii. Sequitur tangentium comparatio cum ſinibus: primò ratio.

23. *Tangens ſemiſsis peripheriæ quadrātis, radio æquatur: majoris radio major eſt, minoris minor.*

Sit enim arcus a e ſemiſsis peripheriæ quadrantis: a o ſit major. a i minor. ſintq́; ſingulorum tangentes a t, a r, a p.

Dico t a eſſe æqualē radio: & r a majorem, p a minorem. Nam quia triangula t y a, e y m ſunt æquiangula ob rectos duos & communē ad y erit per 9.e.7.R. y m ad m e ſic y a radius ad a t. ſed y m æquatur rectę m e. Nam ex theſi arcus a e eſt ſemiſsis totis a u. Ergo ſinus u e recta y m æquatur ſinui e a qui eſt e m per 15 e.



Reliquę partes hinc ſequūtur. Nam r a tangens arcus majoris ſemiſſe peripheriæ quadrantis, eſt major quam t a. at t a eſt æqualis radio. ita p a eſt minor quam t a hoc eſt radius. unde proportio ſequitur.

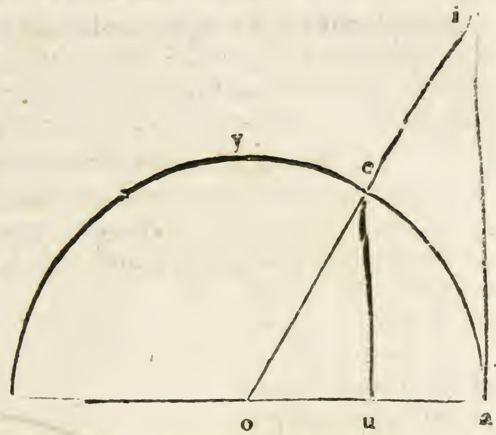
Itaq;

24. *Sinus peripheriæ eſt ad ſinum complementi, ut radius ad tangentem dicti complementi.*

Elemento hoc Eraſmus Rheinholdus in tabulis ſuis directionum canonem ſecundū è canone ſinuum cōponit præcepto 8.

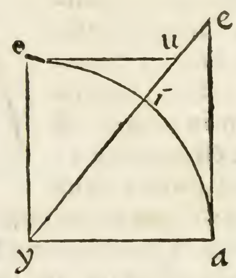
Sit

Sit in ſubjecta figura peripheriæ quidē y e ſinus o u. ſitq; cōplemētum e a cujus ſinus e u. ſitque tangens cōplementi i a. Quia triāgula æquiangula erit ut o u ad u e ſic o a radius ad a i. Et hinc intelligi poteſt tangentē peripheriæ competere etiam reliquæ ad ſemiperipheriam.



25. Radius eſt medius proportionalis inter tangentes peripheriæ & complementi.

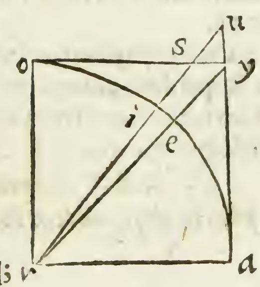
Sit peripheria a i ejuſq́ue complementum i o. ſitq; tangens illic a e hic vero o u. jam quia duo triangula y o u, e a y ſunt rectangula & angulus y e a angulo u y o æquetur per 12.e.5.R. erunt ęquiangula. itaq; per 9.e.7.R. ut a e ad a y radium ſic y o radius ad o u.



26. Tangentes peripheriarum tangentibus complementorum ſunt reciprocè proportionales.

Eſto ſint peripheriæ a e & a i earumq́ue complementa o e, o i. ſintq́ue earum tangentes a y, a u, o s, o y. Dico jam a u eſſe ad a y ut eſt o y ad o s. Nam per præmiſſum elementum eſt

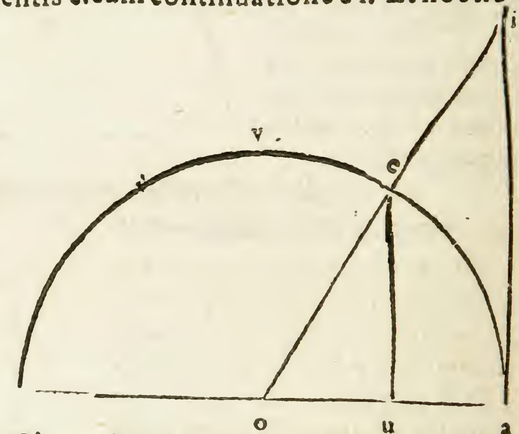
ut a u ad radium ſic rad. ad o s.
ut a y ad radium ſic rad. ad o y.



Itaq; planus a u, o s æquatur plano radii itemque planus a y, o y plano radii per 4.e.11.R. Ergo planus a u,o s æquatur plano a y,o y. Quare per 14.e.10 vel 5.e.11.R. ut a u ad a y ſic o y ad o s. Atq; ſic doctrinam tangentis peripheriæ expedivimus.

27. Secans eſt radius per terminum peripheriæ in tangentem continuatus cum continuatione.

Sic peripheriæ a e ſecans eſt o e i.nempe radius o e continuatus in terminum tangentis e.cum continuatione e i. Et hoc nomē huic rectę accommodatum putamus.



Joachimus Rheticus hypotenuſam trianguli rectanguli vocat reſpectu anguli recti ad a cui ſubtenditur. Verũ cum referatur non ad angulum rectum, ſed angulum in centro ad o hoc eſt arcum a e: an non potius arcus ſecans dicatur quam recti anguli hypotenuſa judicent alii.

Maurolycus canonem Rhetici paulò mutatum in Meſſanenſi Menelai editione, nomine etiam mutato edidit: & beneficum vocavit: tanquam recta o i ſeu numerus hanc definiens beneficus diceretur. At recta o i non magis eſt benefica, quam recta a i fœcunda.

Sequitur comparatio ſecantium cum ſinibus, & cum tangentibus. & quidem comparatio cum ſinibus priori loco: & proportio. Nam ratio ſecantium ad radium, ad ſinus eſt inæqualitatis, & quidem majoris: quam determinare non fuerit neceſſum.

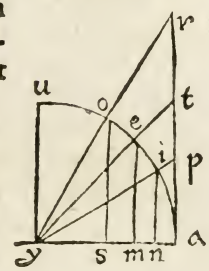
28. Radius eſt medius proportionalis inter ſinum peripheriæ & ſecantem complementi.

Hoc

Hoc etiam elementum Er.Rheinholdi eſt in tabulis directionum præcepto 8. Eſto in præmiſſo ſchemate arcus dati a e ſinus e u, ſecans o i. dico eſſe ut o u ſinum complementi datę peripheriæ ad o a radium ſic o e radium ad o i. Sunt enim triangula i o a, e o u æquiangula. ideoq; per 9.e.7.R. cruribus proportionalia.

29 *Secantes peripheriarum ſinibus complementorum ſunt reciprocè proportionales.*

Sint enim in ſubjecta figura datæ peripheriæ a e, a o: quarum complementa ſint e u, o u. jam ſint datarum peripheriarum ſecantes y t, y r. complementorum vero ſinus y m, y s. Dico eſſe y r ad y t ut eſt y m ad y s. Nam per 28.e.

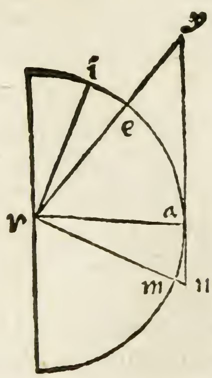


ut y r ad radium ſic rad. ad y s.
ut y t ad radium ſic rad. ad y m.

Quare planus y r, y s item y t, y m ſigillatim æquatur plano radii per 4.e.11. R. Ergo & inter ſe: eritque per 14.e.10.R. ut y r ad y t ſic y m ad y s. Sequitur deinceps comparatio ſecantis cum tangente. & primo ratio æqualitatis.

30. *Secans peripheriæ æquatur tangenti & datæ & ſemiſſis complementi.*

Sit peripheria data a e. ſemiſsisq; complementi ejus a m. jam ſit ſecans datæ r y. tangens ejuſdem y a. ſemiſsis cōplementi a n. hoc eſt tangens datæ & ſemiſsis cōplementi ſit y n. Dico y r & y n æquari. Elementi veritas patet è 10.e. 6.R. Nam angulus r n y & y r n æquantur. Etenim.



Reliqua æqualium de æqualibus æquantur.

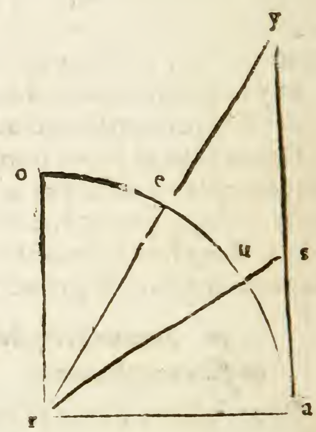
At anguli r n y & y r n ſunt reliqui æqualium i r e & a r n ex theſi (uterq; enim ſubtenditur à ſemiſſe complementi datæ

 peripheriæ,

peripheriæ.itaq; per 5.e.3.sunt æquales)sunt reliqui,inquam,æqualium de æqualibus i r n & r a y quia hi recti sunt.Quod autem angulus a r n de angulo r a y relinquat angulum r n a patet per 2.c.9.e.6.R. Rectus enim æquatur duobus oppositis interioribus.Ergo minutus altero interiorum,est reliquus.

Ergo dicti anguli sunt æquales: proinde à lateribus æqualibus subtensi.

31. Secans peripheriæ, quadrantis peripheriæ semisse minoris,cum tangente ejusdem æquatur tangenti datæ semisse sui complementi auctæ.



Sit data peripheria a u minor semisse peripheriæ quadrantis: ejus complementum sit u o. cujus semisis sit e u. jam sit tangens datæ a s secans r s. at arcus a e tangens sit a y. Dico r s, s a æquari rectæ a y. Vis veritatis in eo consistit, ut monstretur æqualitas r s & s y. Nam a s per se sibi æquatur. per 12. e. 5. R. itaq; o r e hoc est ex thesi y r s æquatur angulo r y s. Quare per 10. e. 6. R. r s & s y æquantur. Sequitur ratio inæqualitatis segmentorum tangentis & secantis.

32. Si secantes per terminos æqualium peripheriarum secent tangentem summæ æqualium,segmentum tangentis & secantium extra peripheriam propius extremæ diametro minus est remotiore.

Sint in præmissa figura æquales peripheriæ a u,u e. Sitq; tangens summæ æqualium nempe a u e recta a y & per terminos æqualium secent tangentem secantes r s,r y. Erit segmentum s a minus segmento s y.& u s minus quam e y. De segmentis tangentis constat. Quia enim e a peripheria bisecatur in u per r s & angu-

&c angulus y r a erit bisectus per 5.e.3.itaq; per 12.e.6. R.ut r a ad r y sic a s ad s y.

Verum r a est minor quam r y per 11.e.6 R.

De secantibus facilis res est: r s minor est quam r y per 11.e. 6.R.auferatur æqualis radius: u s minor quam e y relinquetur.

Itaq;

33 *Si secantes secent e tangente segmenta æqualia: segmentum peripheriæ propius extremæ diametro remotiore majus est.*

Hoc elemento Euclides in opticis utitur 4 & 7 theor.ad demonstrationem inæqualitatis rerum visarum quo ad visum: quę æquales tamen existunt.Est autem manifesta veritas è pręcedente. Ponatur enim in præmisso figura segmentum a s æquale segmento s y.Dico peripheriam a u esse majorem peripheria u e.

Nam si peripheriæ a u,u e non sunt inęquales:secantes per earum terminos secabunt segmenta a s,s y inæqualia.per 32.e.

At hic ex thesi non secant inæqualia. Quare sunt inæquales. Et a u major est quam u e.Si enim esset æqualis & proinde multo magis si minor:a s esset minor quam s y.

At non est minor ex thesi. posset institui alia demonstratio si centro a.radio r s describeretur peripheria:fieret sector major ad extremam diametrum:minor qui esset remotior: unde angulus s r a major concluderetur augulo y r s per 6.e.3.& hinc per 5. e. 3. peripheria a u major quam u e. Alio adhuc modo scholiastes Euclidis opticus demonstrat. Ducta enim recta ex s quæ parallela sit ad r a,facit angulum cum r s ęqualem angulo s r a per 12.e.5.R. Et cum ille major sit angulo y r s per 11.e.6.R. angulus s r a etiam eodem major erit.itaq; & a u major quam u e. Quemadmodum illic videri poterit.

Expedivimus itaq; doctrinam circularem eam quæ in lineis fuit atq; segmentis.

TH. FINKII

TH. FINKII GEOMETRIAE ROTVNDI,

LIBER SEXTVS.

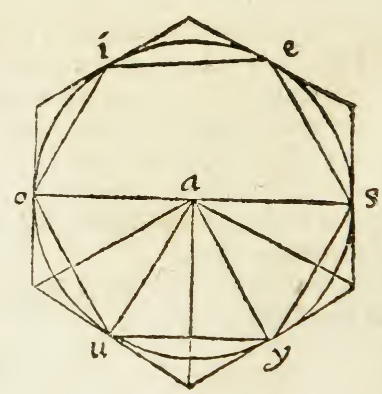
De adscriptione circuli & trianguli.

Geometria circularis simplex expedita est: sequitur cōjuncta in adscriptione circuli & rectilinei.

Geometriam circularem nobis duplicē proposuimus: simplicem quidem solius circuli cum suis lineis & segmentis: conjunctam vero in adscriptione circuli & rectilinei ponimus. Cognitionem rectilineorum è Geometria vel Euclidis vel Rami huc adferri necesse est: nos ea saltem docebimus quæ circulum attingunt.

2. *Si rectilineum adscriptum circulo est æquilaterum: est æquiangulum.*

Campanus hoc ad 15. p. 4. proponit. Veritas autem docenda erit & in inscripto & circumscripto. Tot enim species adscriptionis sunt 10. e. 1. R. primo itaq; de inscripto. id si triangulum est per 2. c. 10. e. 6. R. est æquiangulū. De triangulato itaque res demonstranda: ut hic si inscriptæ o u & s y sint æquales subtendent peripherias æquales per 14. e. 2. jam si æqualibus o u & s y addas æquale nempe peripheriā o i e s. summæ u o i e s & y s e i o sunt æquales. Ergo per 5. e. 3. angulus u y s an-

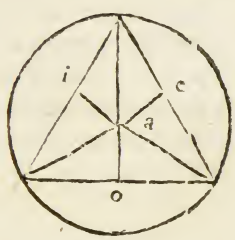


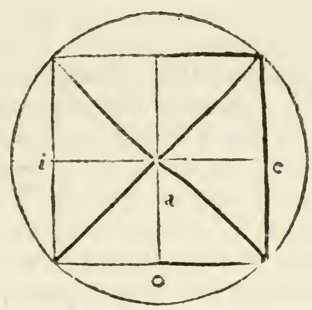
gulo

gulo y u o eſt æqualis. Eodemq; modo reliqui anguli æquales probabuntur. De circumſcripto etiam patet ſi intelligas circumſcriptum circulum. perpendiculares enim à centro a. in latera circumſcripti cum radiis facient utrinq; triangula æquilatera per 5.e.12.R. & per 1.e.7.R. æquiangula. Quod ſi ergo anguli à radiis & circumſcripti lateribus æquantur toti à lateribus comprehenſi æquabuntur.

3. *Si rectilineum adſcriptum circulo eſt æquilaterum: æquatur triangulo æqualis baſis quidem perimetro, altitudinis autem perpendiculari à centro in latus.*

Sit inſcriptum triangulum itemq; quadratum.





Dico triangulo huic æquari triangulum, cujus baſis ſit continuata è tribus lateribus, altitudo ſit perpendicularis è centro communi rectilinei & adſcripti circuli. Veritas eſt manifeſta ex 1.c.6.e.7.R. Nam ſi à centro ducantur radii tres in angulos dati trianguli fient tria triangula quæ ſunt æquealta. Altitudo enim eſt perpendicularis à vertice in baſin hoc eſt à centro in latus: ea autem hic eſt æqualis per 23.e.1.in tribus triangulis. Et in æquali ſunt baſi nempe latere trianguli æquilateri. jam quia triangulum è baſi perimetro, altitudine perpendiculari à centro in latus eſt æquealtum triangulis tribus, & baſin habet baſibus horum

æqualem nempe ex iis compoſitam: æquatur univerſis: æqualibus dato triangulo.

Eodem modo in quadrato res patet. Nam ductis diagoniis fiunt quatuor triangula æquealta æqualia dato. at his quatuor, æquealtum & æquale baſibus triangulum è perimetro quadrati & perpendiculari dicta æquatur. Et ſic de quolibet rectilineo ordinato induces.

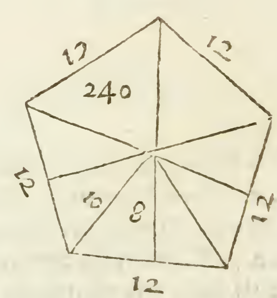
Itaq;

4. *Planus è perpendicularis è centro in latus & perimetri altero per reliqui dimidium eſt area rectilinei ordinati.*

Hæc geodæſia rectilinei ordinati communis eſt. Ac in triangulo expeditior eſt 4.c.6.e.10.R. aut 9.e.12. Quadrati quoq; compendioſior 6.e.11.R. Quare ſupereſt uſus hujus elementi in multangulis ordinatis reliquis. Veritas autem in promptu eſt. Nam triangulum æqualis, altitudinis cum perpendiculari à centro in latus, baſis perimetro æqualis, æquatur dato multangulo.

At area trianguli eſt planus baſis & ſemiſsis altitudinis aut altitudinis & ſemiſsis baſis per 4.c.6.e.10.R. & 6.11.R.

Eſto quinquangulum ordinatum, cujus perimeter ſit 60. erit latus 12. Sitq; perpendicularis à centro in latus 8. Quæritur area quinquanguli. planus è 30 & 8. eſt 240. pro quæſita area. Vel planus è 60 per 4 eſt 240. rurſus pro area optata.



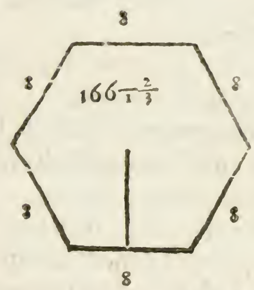
Eſto rurſus ſexangulum: cujus latus ſit pedum 8, & perpendicularis à centro in latus ſit $6\frac{12}{13}$.

Planus ergo à $6\frac{12}{13}$ per 24 eſt $166\frac{2}{13}$ per area.

At dices: unde ſcis perpendicularem eſſe in quinquangulo dato pedum 8 in ſexangulo pedum $6\frac{12}{13}$.

Id, inquam, hic dari oportet: aliàs patebit inventio ejus è radio

dio circumſcripti circuli & ſemiſſe lateris multanguli, in triangulo rectangulo. Nam quadratum ſemiſsis lateris de quadrato radii relinquit quadratum perpendicularis per 5.e.12.R. ut melius intelligetur ad doctrinam circũſcriptionis circuli. Nos hic cum matre filiam conjungere maluimus: quam interpoſitis aliis ſejũgere.



Sunt qui geodęſiã proponunt ſpecialem: Verbi gratia Cardanus in practica ſua arithmetica dicit, eſſe quadratum lateris quinquanguli ad aream ejus, ut 2,939 ad 5,056. Quod verum non eſt: Nam ſi ſit ut 2,939 ad 5,056 ſic quadratum lateris ad aream: erit ut 2,939 ad 5,056 ſic 144 ad 248 fere. aream nempe quinquanguli. Quæ certè magna ψευδογραφία eſt. pro 240 aſſumere pedes quadratos 248. Quod ſi ejuſmodi ſpeciales ſunt conſtituendæ geodæſiæ: ex inventa area, & latere menſuræ quanto majoris tanto ad hanc rem melioris: termini rationis minimi potentiæ lateris ad aream conſtitui debent: & tamen nullius eſſe momenti deprehendes. in noſtro exemplo area eſt 240. quadratum lateris 144. Horum maxima menſura communis eſt 48. Ergo 240. & 144, in terminis minimis proportionalibus ſunt 5. ad 3. Quare ut 3 ad 5 ſic quadratum lateris quinquanguli ad aream. Verum eſt in hoc exemplo. At ſit latus quinquanguli 20. erit perpendicularis pedum $13\frac{38191}{50000}$.

jam planus è 50 ſemiſsis perimetri & perpendiculari eſt 688. & parum ultra pro area quinquanguli. Sed ſi terminos primi noſtri exempli teneas: erit ut 3 ad 5 ſic 400, quadratum lateris ad 666 pro area. En novam ψευδογραφίαν in primo exemplo ter-

 mini

mini hi erant rationis quadrati lateris ad aream: at in hoc vides quantum aberrent. Hinc ergo judicium facerę poteris: quid de ſpecialibus ejuſmodi non tam Cardani quam aliorum geodæſiis ſit ſentiendum.

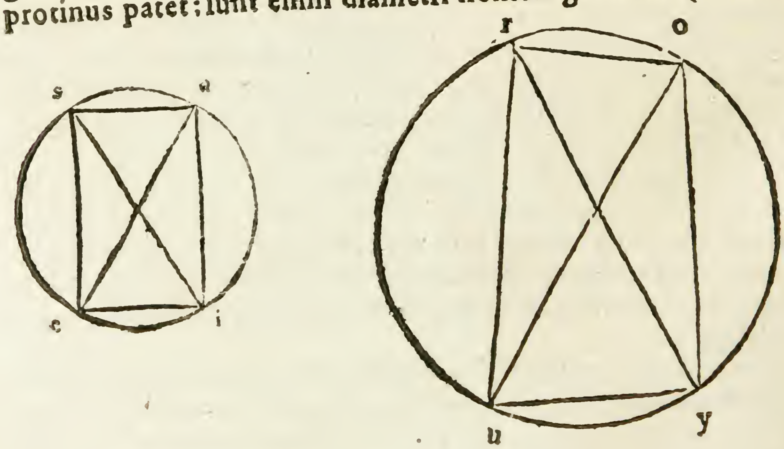
Hic merito ſibi optaſſet tantum quotannis agri quantum ψευδογραφία illa periret.

Geodæſia generalis vera, & facilis eſt. Ea igitur teneatur.

5. *Rectilinea ſimilia circulis inſcripta ſunt ut à diametris quadrata.* 1.p.12.

Euclides de polygonis ſeu multangulis proponit, de ſpecie ſcilicet quod erat generis. Elementum vero hinc patet quod plana ſimilia habeant duplicatam rationem homologorum laterum per 1.e.6.R. Eſt autem duplicare rationem homologorum laterum quadrare diametros.

Jam vero in inſcriptis rectilineis diametri ſunt homologa latera aut homologis lateribus proportionales. ut patet primò in triangulis. Ea ſint rectangula aut obliquangula. Sint ergo rectangula a e i, o u y. quia itaque a e & o u ſunt diametri per 4.e.4. res protinus patet; ſunt enim diametri homologa latera ipſorum



triangu-

triangulorum. Sed ſint jam obliquangula i s e, y r u. diametri erunt homologis lateribus proportionales nempe lateribus e i, u y ut jam patuit: & ideo ut diametri a e, o u. Si triangulata ſint, in ſua reſolventur triangula per 2.e.10.R.& res patebit ſimiliter. Itaq;

6. Si ſit ut diameter circuli ad latus rectilinei inſcripti, ſic diameter ſecundi circuli ad latus ſecundi rectilinei inſcripti, triangulaq; inſcriptorum ſingularia ſimilia ſimiliterq; ſita: rectilinea inſcripta erunt ſimilia ſimiliterq; ſita.

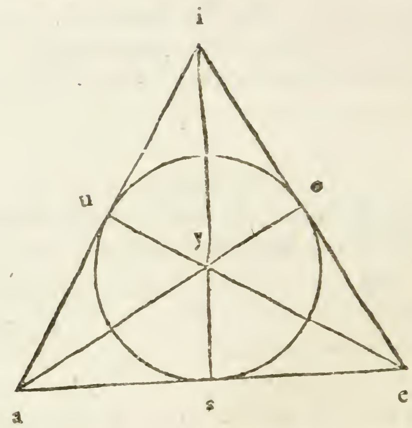
Conſectarium hoc Euclides 2. p. 12. ſic ſumpſit: & ideo nos hinc concludimus. ut ſi ſit in præmiſſa figura ut, o u ad u y ſic a e ad e i & triangula o u y, a e i ſimilia ſimiliterq; ſita itemq; triangula o u r, a e s: erũt inſcripta quadrangula ſimilia ſimiliterq; ſita

7. Rectilineum inſcribitur circulo quando peripheria tangit angulos: Circumſcribitur cum à ſingulis lateribus peripheria tangitur. 3.4.d.4.

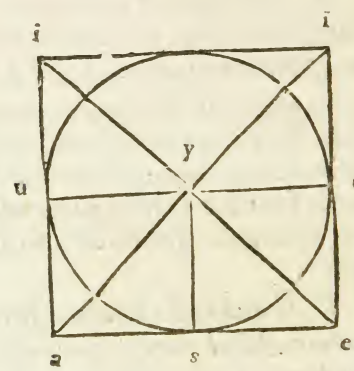
Hæc ex generali adſcriptionis doctrina in haſce ſpecies deducuntur. Eſt enim circuli terminus peripheria: Si ergo peripheria rectilinei terminis terminetur: circulus & rectilineum adſcripta inter ſe ſunt &c. Et hinc patet adſcriptionem quidem circuli eſſe cum triangulo quolibet: cum triangulato autem duntaxat ordinato.

8. Si duæ rectæ biſecent duos angulos dati rectilinei: circulus radii ab earum concurſu in latus perpendicularis inſcribetur dato rectilineo. 4. 8. 13. p. 4.

Eſto verbi gratia triangulum a e i. & rectæ e u, a o biſecent angulos i a e, i e a: & ab earum concurſu ſint perpendiculares in latus y s, y o, y u. dico radio y u vel y o vel y s centro y deſcribi circulum. per 27. e. 1. Sunt enim tres ab eodem puncto æquales. Nam biſecantes cum perpendicularibus faciunt per 2.e.7. R. triangula æquilatera. Nam triangula s e y, y e o ſunt æqua angulis ad e ex theſi & s, o, æqualis baſis y e quia communis. Ergo s y, y o



æquãtur. rursus s a y, y a u eodem modo sunt æquilatera. unde & u y & y s ęquantur. proindeque tres perpẽdiculares sunt æquales ab eodem puncto & quidẽ centro. idem argumentum in triangulato erit. ut exempli gratia hic in quadrato videbis.



Nam & hic perpendiculares tres y o. y s. y u simili ratiocinatione ęquãtur pro radiis circuli cui quadratũ est circumscriptũ. Est autem tum rectilineum circumscriptum quiá peripheria à lateribus tangitur.

9. Si duæ rectæ rectè bisecent duo latera dati rectilinei: circulus radii ab earum concursu in angulum circumscribetur dato rectilineo. 5. 9. 14. p. 4.

ut in superioribus figuris rectæ y o, y u rectè bisecent latera e i, a i dico circulum à centro y radio y e, vel y i vel y a circumscribi dato rectilineo, sunt enim tres dictæ æquales. Nam ducta recta y i triangulum y o i, triangulo y o e est angulo æquicruro ęquum ex thesi

ex thesi. Sic & triangulum y u i triangulo y u a. Ergo per 2.e.7. R. e y, y i & y i, y a æquantur. & proinde per 27 e i. centrum est in y. ex quo radio y e circumscribitur circulus

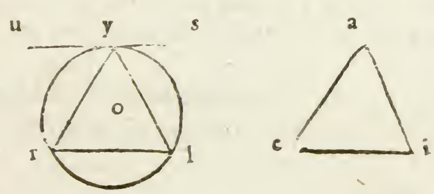
Itaq;

10. *Segmenta ex concursu bisecantium angulos rectilinei ordinati, sunt radii circumscripti circulis.*

ut in superioribus figuris u e, & o a bisecant angulos. harum concursus est in y centro rectilinei, circuli & inscripti & circumscripti. quare y e, y a sunt radii circumscripti. Atq; hinc patet inventio perpendicularis à centro in latus cujus cognitio ad geodæsiam rectilinei necessaria est. Quia enim y e, y a sunt radii: a e erit inscripta & in eam y s est perpendicularis: a s & s e ęquabuntur. jam si de quadrato y e tollas quadratum s e semissis lateris relinquetur quadratum perpendicularis quæsitæ. Atq; hactenus adscriptio circuli fuit: sequitur adscriptio rectilinei, & primo quidem trianguli.

11. *Si duæ inscriptæ à contactu rectæ & peripheriæ æquent duos utrinq; angulos duobus angulis dati trianguli: connexæ inscribent triangulum dato circulo æquiangulum dato triangulo.* 2.p.4.

Detur itaq; triangulum a e i: offeratur etiam circulus o. Huic inscribendum sit triangulum æquiangulum dato. Ducatur itaq; tangens u y s quæ tangat, inquam, peripheriam y r l. atq; a contactu inscriptæ y r & y l æquent angulos u y r, s y l angulis dati trianguli a e i, a i e & cō-nectantur inscriptæ recta r l. Dico triangulum y r l esse æquiangulum dato. Nam si bini anguli duorū triangulorum æquātur, reliqui æquantur per c.3.e.7.R.

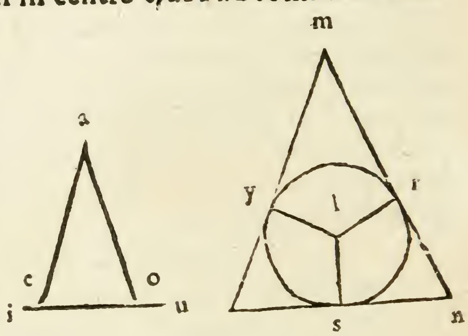


At hic anguli l & i anguli e & i æquantur. Quia angulus y l r æquatur

æquatur angulo u y r per 18. e. 3. hoc est angulo a e i. item angulus y r l. æquatur angulo s y l hoc est angulo a i e. Ergo angulus a æquatur angulo y.

12. *Si duo anguli in centro dati circuli æquētur ad commune latus exterioribus angulis dati trianguli: rectæ tangentes peripheriam in cruribus angulorum circumscribent triangulum dato circulo æquiangulum dato.* 3.p.4.

Esto triangulum a e o. ejusq; exteriores anguli a e i, a o u. circulus autem datus y s r. tum in centro ejus l ad commune latus



l r æquentur anguli exterioribus dati trianguli, y l r quidē angulo a e i, at r l s angulo a o u. jam sint tangentes ad r, y, s quæ circumscribant triangulum. Dico id esse dato æquiangulum. Nam per 4. e. 6. R. anguli in quadrangulo y m r l æquantur quatuor rectis. Sed anguli ad y & r sunt recti ex fabrica & 1. e. 2. Ergo reliqui ad l & m æquantur duobus rectis. quibus etiam æquantur anguli a e i, a e o per 1. c. 8. e. 5. R. Ergo ablatis æqualibus a e i & y l r ex fabrica de æqualibus relinquitur y m r æqualis angulo a e o.

Eodem argumento angulus trianguli circumscripti oppositus angulo s l r nempe r n s æquatur angulo a o e. itaq; cum bini æquentur reliquus reliquo etiam erit æqualis per c. 3. e. 7. R.

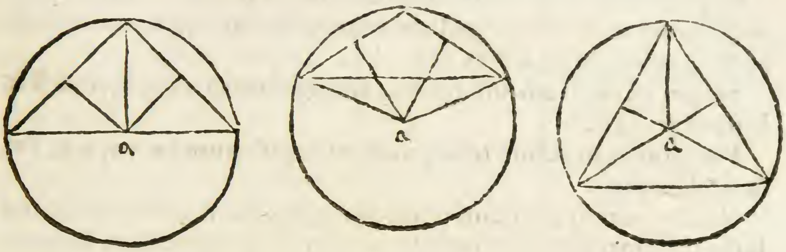
Itaq;

13. *Si triangulum est rectangulum, obtusangulū, acutangulum: centrum circumscripti circuli est in latere, extra latera, intra latera: & contra.*

Elementi veritas è premisso auxilio 4. e. 4. manifesta est. Nam rectangulum tantum inscribitur semicirculo: ergo in latere sive basi, hoc est diametro, est centrum.

Obtusan-

Obtuſangulum tantum inſcribi poteſt minori ſectioni. Ergo centrum extra baſin. Sic acutangulum inſcribitur majori ſectioni ergo intra latera centrum conſiſtat neceſſe eſt, ut hic ſubjectæ figuræ centrum a exhibent.



TH. FINKII GEOMETRIAE ROTVNDI,

LIBER SEPTIMVS.

De adſcriptione triangulati.

SI rectæ tangant peripheriam in angulis inſcripti triangulati ordinati: circumſcribent triangulatũ circulo homogeneum inſcripto. 7.12.p.4.

Expedita eſt adſcriptio circuli, adſcriptio item trianguli: ſupereſt adſcriptio triangulati & quidem ordinati: cujus circumſcriptio generalis hic proponitur ex antecedente tamen inſcriptione quę ſpecialia requirit elementa. Exempla autẽ per ſpecies deinceps cum inſcriptis proponẽtur. Et hoc generale ſemel & primo dicere ſatius eſt quam in quadrãgulo quinquangulo ſpeciatim repetere. Sequitur ergo inſcriptio & quidem per unicum latus, quod repetitum quoties opus eſt peripheriam compleat. Fecit hoc Euclides in quindecangulo: nec malè tamen in reliquis eadem ratio inſtituitur.

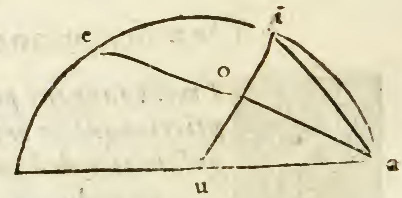
2. Si radius biſecet latus inſcripti ordinati: recta inter terminum radii atq; lateris eſt latus inſcripti ordinati ad prædictum lateribus dupli.

Ita per triangulum æquilaterum inſcribi poteſt triangulatum angulorum 6. per ſexangulum triangulatum angulorum 12. & hinc angulorum 24,48,96 &c.

Sic per quadratum inſcribetur triangulatum angulorum 8 & hinc 16,32,64 &c.

Per quinquangulum triangulatum angulorum 10, 20, 40, 80, & ſic deinceps.

Veritas autem elementi patet per 13.e.3. Nam radius biſecans latus inſcripti per 19.e.1.rectè ſecat.ergo & peripheriam biſecat. Quare quam peripheriam ſubtendebat recta una: eam jam duæ æquales rectæ ſubtendunt. itaq; inſcribitur hoc modo ordinatum duplum.Quod ſi latus inſcripti dati detur in numeris: nota diametro ex ſuperioribus præceptis latus inſcripti dupli manifeſtum erit per 12.e.4.aut per 19.e.5. Exempli gratia,eſto inſcriptum latus trianguli æquilateri a e 173,205. radius biſecãs a e in o, biſecat etiam arcum a e in i. erit ergo a i latus ſexanguli ordinati. id inveniet hoc modo: datur a o tanquam ſinus peripheriæ a i. ergo per 9. e.5.dabitur o u ſinus complementi 50,000.jam ablato o u de radio 100,000.relinquitur o i ſinus ſecundus arcus a i. jam per 19 e.5.a i dabitur 100,000 latus inſcripti ſexanguli. Et ſic in reliquis. Ptolemaica via è 12. e. 4. eodem modo inſcriptam a i inveniet.



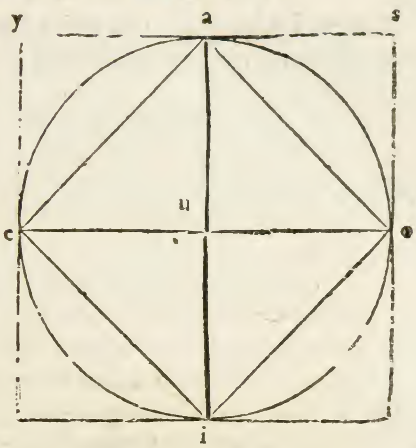
3. Si diametri rectè interſecantur: ſubtenſa recto erit latus quadrati. 6.p.4.

Eſto circulus a e i o. cujus diametri e o, a i rectè interſecentur ad u.rectoq; ſubtenſa ſit a o,dico a o eſſe latus quadrati inſcripti.

Crura

Crura enim recti u a, u o item u a, u e & u e, u i & u i, u o ſunt radii: & rectis connexi triangula æqua angulo ęquicruro ſunt: ergo per 2.e.7.R.baſes a o, a e, e i, i o æquantur: quare quælibet ſubtēdit quartam peripheriæ partem per 14.e.2.

jam cum habeatur inſcriptum quadratū facile comparabitur circumſcriptum per i e.



Itaq;

4. *Latus inſcripti quadrati poteſt duplum circularis radii.*

ut in præmiſſa figura a o æquè poteſt per 5.e.12.R.cruribus a u, u o. At horum quadrata ſunt æqualia: cum latera ſint æqualia. Ergo ad alterutrum quadratum, quadratum a o erit duplum. Et hinc Ptolemæus 9.c.1.inveſtigat latus inſcripti quadrati. Nam ſi quadratum radii duplicetur, latus duplicati erit inſcripti quadrati latus. ut ſi radius ſit 10,000,000. a o erit 14,142,136 ferè.

Et

5. *Quadratum inſcriptum eſt dimidium circumſcripti. in 2.p.12.*

In promptu cauſa eſt quia e o diameter hoc eſt latus circumſcripti y s æque poteſt cruribus e a, a o lateribus nempe inſcripti. At latera hæc ſunt æqualia. Quare quadratum y s eſt duplum ad quadratum lateris inſcripti.

Et

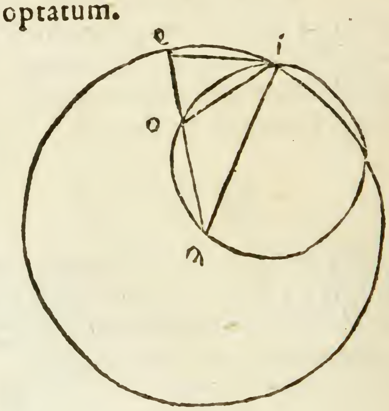
6. *Quadratum inſcriptum eſt majus dimidio circumſcripti circuli.*

Nam per ἐφάρμοσιν quadratum circumſcriptum eſt majus circulo: ideo etiam ſemiquadratum hoc eſt inſcriptum quadratum majus ſemicirculo.

Sequitur jam inſcriptio quinquanguli: ad cujus inſcriptionem opus eſt triangulo cujus uterq; angulus ad baſin duplus ſit anguli reliqui. Et ſic in imparilateris reliquis opus eſſet triangulo cujus uterque angulus ad baſin ſit multiplus reliqui: ſi haberi poſſet.

7. *Si recta ſecetur proportionaliter: trianguli crurum ſectæ æqualium, baſis majori ſegmento æqualis, uterq; angulus ad baſin erit duplus reliqui: & baſis erit latus quinquanguli in circulum cum triangulo inſcripti.* 10.11.p.4.

Hic primo fabrica trianguli optati erit. Sit recta a e pro radio circuli: ea ſecetur proportionaliter per 3.e.14.R. in puncto o. deſcribatur circulus radio a e. tumq; per 11.e.1. à termino e inſcribatur e i æqualis majori ſegmento a o. & terminus i connectatur cum centro a. Fiet triangulum æquicrurum cujus anguli e & i æquantur: ut quod de uno probatum ſit de reliquo quoq; verum habeatur. Et hoc eſt triangulum optatum.



Ducatur enim recta i o & per 9.e.6. triangulo i o a circulus circumſcribatur.

Hunc circulum tanget recta e i. Nam ex theſi ut a e ad a o hoc eſt e i ſic e i ad e o. Ergo per 4.e.12.R. oblongum a e, e o æquatur quadrato reliquę e i. Ergo per 9.e.2. reliqua ipſa e i tangit datam peripheriam.

Itaque hinc ſequitur quod angulus e i a duplus ſit anguli ad a.

Nam æqualis duobus ęqualibus eſt duplus alterius: At angulus

lus e i a eſt æqualis angulis a i o, o a i æqualibus inter ſe.

Aſſumptionis partes duæ ſunt: quod e i a angulus æquetur di ctis duobus: quod duo iſti æquentur. prima hinc patet. quia angulus e i o æquatur angulo o a i in alterno ſegmento per 18.e.3. & reliquus a i o æquatur ſibi ipſi.

Secundo, æqualitas horum angulorum o i a, o a i inter ſe hinc manifeſta eſt.

Angulus e o i æquatur angulis o i a, o a i per 2.c.9.e.6.R.

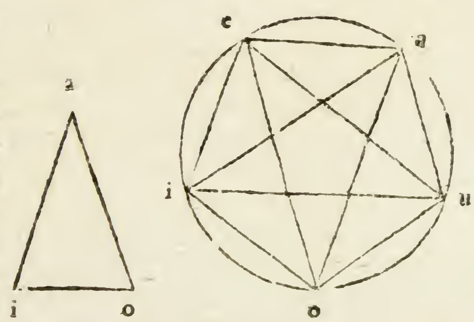
Anguli o i a, o a i æquantur angulo e i a ut jam patuit.

At angulus e i a æquatur angulo o e i ex theſi.

Quare angulus e o i æquatur angulo o e i: itaq; per 10.e.6.R. o i æquabitur lateri e i. At e i ex theſi æquatur lateri a o. ergo o i æquatur lateri o a. & proinde anguli o i a & o a i per 10.e.6.R. æquantur. Et hinc jam generalis complexio eſt angulum e i a eſſe duplum anguli e a i.

Et ſic fabrica trianguli optati fuit: Secundo jam erit inſcriptio quinquanguli. Dico itaq; baſin e i eſſe latus quinquanguli cum triangulo inſcripti.

Inſcribatur enim triangulum a i o æquiangulum invento per 11. e.6. erit baſis i o latus quinquanguli inſcripti. Etenim ſint duæ rectę biſecantes angulum utrunque reliqui duplum: & cõnectantur cum angulis & inter ſe fient latera i o, i e, e a, a u, u o, æqualia. Nam cum anguli quinq;, biſecti duo & reliquus ſint æquales: per 5.e.3. peripheriæ quinque æquabuntur: & per 14.e.2. inſcriptæ, hoc eſt ipſa latera ſunt æqualia. jam inſcripto quinquangulo facile erit per 1.e quinquangulum ordinatum circumſcribere.

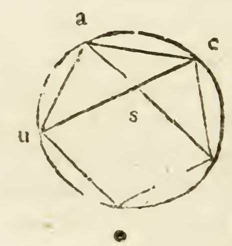


Et ſic recta proportionaliter ſecta quinquanguli adſcriptionem machinatur: ſed & inde viciſsim redditur linea proportionaliter ſecta.

8. Si duæ rectæ ſubtendunt duos deinceps angulos inſcripti quinquanguli ſecantur proportionaliter, & majora ſegmenta ſunt latera inſcripti. 8.p.13.

Elementi partes duæ ſunt: rectas ſecari proportionaliter: majora ſegmenta eſſe æqualia lateribus inſcripti quinquanguli.

Eſto igitur inſcriptum quinquangulum: & rectæ a i, u e ſubtendẽtes angulos deinceps a e i, u a e. Dico eas proportionaliter ſecari in s. ſecundo majora ſegmenta s i, s u æquari lateribus inſcripti. Quia itaque triangula a e i, u a e ſunt æqua angulo per 2.e.6. æquicruro ex theſi: ſunt æquilatera per 2.e.7.R. & per 1.e.7.R. æquiangula. & anguli a i e, a e u æquales. Sed in triangulis



s e a, e a i angulus a communis eſt. Quare per c.3.e.7.R. anguli a s e & a e i æquantur. itaque per 9.e.7.R. ut i a ad a e hoc eſt (ut mox patebit) i s ſic a e ad a s. Quare recta i a ſecta eſt proportionaliter in s. per 1.e.14.R. Eodem argumento u e ſecta eſt proportionaliter in s. jam ſecundo quod i s æquetur lateri inſcripti quinquanguli nempe a e inde patet.

Eidem æqualia ſibi ſunt æqualia.

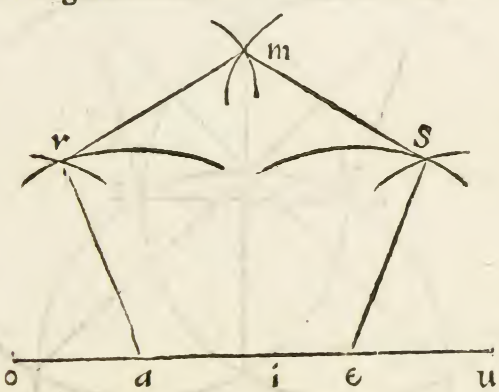
At i s & a e ſunt eidem æqualia nempe e i. & quidẽ a e ex theſi i s vero per 10.e.6.R. Anguli enim i e s, e s i æquantur quia eidem ſunt dupli. Nam i s e æquatur duobus interioribus s a e, a e s per 2.c.9.e.6.R. æqualibus per 5. e. 3. itaq; alterius duplus eſt nempe anguli a e s. Sed & hujus duplus eſt angulus u e i per 5.e.3. inſiſtens nempe in dupla peripheria. Et hinc ſequitur complexio quæſita. Ergo e i, i s ſunt æquales. Atq; hinc patet fabrica quinquanguli ordinati ſuper datam rectam: quam ſequenti conſectario exponemus.

Itaq;

Itaq;

9. Si data recta secta proportionaliter continuetur utrinq; majori segmento, sexq; peripheriæ radio datæ concurrant, binæ utrinq; à terminis datæ & continuatæ, duæ reliquæ ab earum concursu: rectæ per concursus & terminos datæ constituent super datam quinquangulum ordinatum.

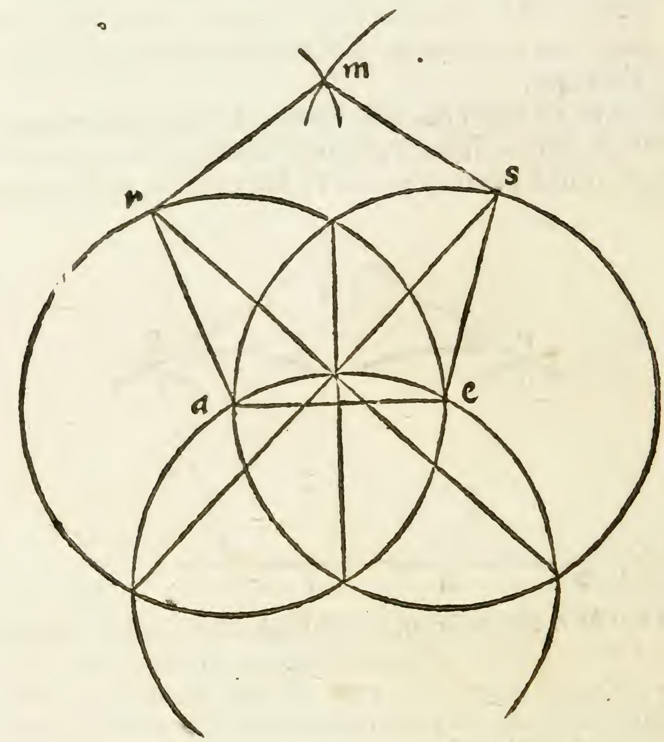
Esto recta a e super quam construendum sit quinquangulum ordinatum. Ea itaq; secetur proportionaliter in i. continueturq; utrinq; majori segmento a i illic e u hic a o: tum radio datæ ter-



minis e,u & o,a concurrant peripheriæ illic in s hic in r. jam terminis s & r eodem radio concurrant peripheriæ in m. & à terminis a & e ducantur rectæ a r, e s & ab r & s ducantur r m, s m. Erit quinquangulum a e s m r optatum. Nam cum recta a e secetur proportionaliter, & continuetur majori segmento: cótinuata secabitur proportionaliter per c. 3. e. 14. R. & data a e erit segmentum majus. & per 8. e. latus quinquanguli. Mechanici hodie compendium fabricæ habent tale: *Si quinq;* peripheriæ radio datæ concurrant, duæ à terminis datæ, tertia ab earum concursu, reliquæ à terminis rectarum è concursu tertiæ cum primis per sectionem bisecantis datam infinitè & tertiæ in primas: re-

ctæ

ctæ à concursu reliquarum per sua centra segmentum majus in terminos datæ constituent super datam quinquangulum ordinatum. ut hic data recta a e experiri potes.



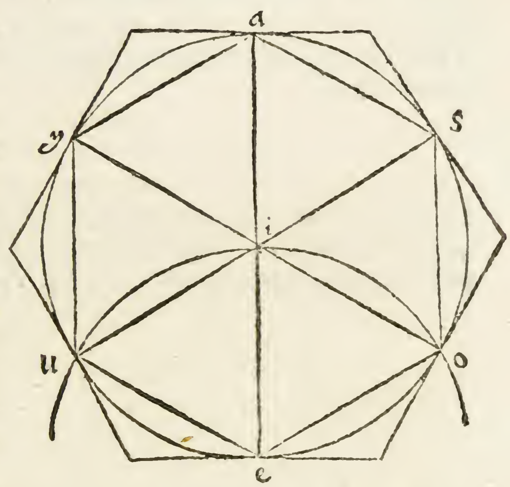
10. Si diameter circuli quinquangulo circumscripti est rationalis: est irrationalis ad latus inscripti quinquanguli. è 11. p. 13.

Euclides hæc ita proponit: sed addit latus quinquanguli esse minorem irrationalem: quod ad Geometriam non multum attinet.

net. pendet autem elementum inde quod ſegmenta rectæ proportionaliter ſectæ ſunt irrationalia 2. e. 14. R. Sequitur inſcriptio ſexanguli.

11. *Radius circuli eſt latus inſcripti ſexanguli.* 15.p.4.

Sexangulũ per 2.e.inſcribi poſſet inſcripto triangulo ęquilatero biſectis nẽpe trib.angulis: ſed brevius hoc modo p radiũ inſcribit, ſexies repetitũ.



Eſto in circulo diameter a e. jam centro e radio e i. Deſcribatur peripheria u i o. & diametri à punctis u & o ſint u s, o y. Hæ connexæ & inter ſe & cum diametro a e inſcribent ſexangulum ęquilaterum dato circulo lateris radio æqualis: & etiam æquiangulum proindeq; ordinatum. utrumque demonſtrabitur hoc modo: primum

Triangula æqua angulo æquicruro æquantur baſibus. At hic triangula ſex æquicrura ſunt: Nam crura ſunt radii ejuſdem peripheriæ ergo æquales per 2. e. 1. ut i u, i e, & i e, i o item i o, i s & ſic deinceps: & æqua angulo æquicruro. Nam cum u i & u e item e i ſint radii æqualium peripheriarum ex fabrica. triangulum u i e eſt æquilaterum: quare u i e eſt $\frac{2}{3}$ recti per 3.c.10.e.6.R. Sed eodem argumento e i o eſt etiam $\frac{2}{3}$ recti proinde priori æqualis: ſed his duobus æquantur illic a i s hic y i a. per 2.c.8.e.5.R. ſed & a i s, e i o hoc eſt 1 $\frac{1}{3}$ recti de duobus rectis per 1.c.8.e.5 R. relinquent $\frac{2}{3}$ recti pro angulo s i o hoc eſt pro angulo y i u. Sex anguli itaq; æquantur.

Ergo baſes u e, e o, o s, s a, a y, y u ſunt ſex æqualia latera inſcri-

pta:æqualia radio ut in demonſtratione patuit:ſunt enim triangula æquilatera.

Quod ad ſecundum attinet nempe eſſe ſexangulum æquiangulum,id ex 2.e.6.ſequitur. Nam eſt jam demonſtratum æquilaterum.

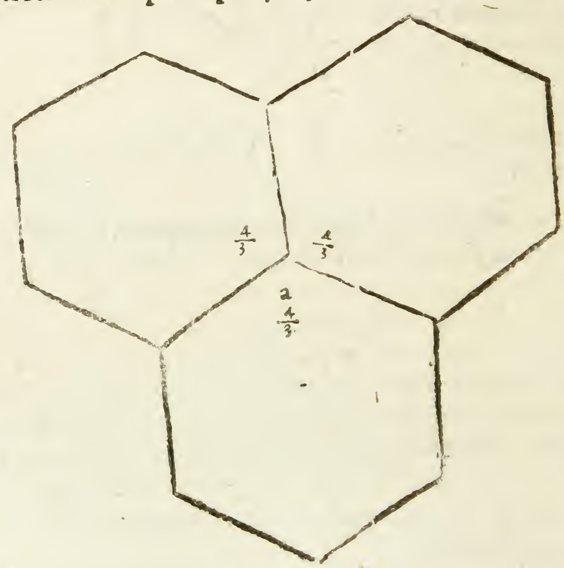
Et eſt elementi hujus vulgatus uſus apud doliarios.ii enim volentes aptare dolio operculum congruum: inſcribunt ſexangulum dolio:& latus ſexanguli pro radio operculi retinent.

Circumſcriptio autem ſexanguli,è generali circumſcriptione ad 1 e dicta, eſt.

Itaq;

12. *Sexangula tria ordinata complent locum.*

Quatuor recti anguli complent locum planum 16. e. 4. R. At tres anguli circa idem punctum trium ſexangulorum æquantur quatuor rectis.Nam unus ſexanguli valet $\frac{4}{3}$ hoc eſt $1\frac{1}{3}$ recti, ut in præcedenti ſchemate patet per 5.e.3. Vel quia duplus eſt exem-



pli

pli gratia angulus u e o anguli u e i qui valet $\frac{2}{3}$ recti. erit ipse $\frac{4}{3}$ recti. aut quia 6 valent 8 rectos per 4.e.6.R. valebit unus $1\frac{1}{3}$. Ergo triplex erit $\frac{1\,2}{3}$ recti hoc est 4 recti.

Ergo sexangula tria complent locum. ut hic anguli ad a complent locum.

Aristoteles in doctrina cœli 8.c. 3. ordinatas figuras in planis tres locum complere ait: triangulum, quadratum, sexangulum. De triangulo constat è 4.c.10.e.6.R.

Quod ad quadratum attinet: speciatim nos τετράγωνον Aristotelis intelligere non est opus nisi synecdochicè pro rectangulo quolibet accipiatur. Nam quatuor rectangula qualiacunq; tandem complent locum 2.c.2.e.11.R. De sexangulo ex hoc elemento patuit.

Nec præter has est in planis figura ulla quæ locum compleat. Nam quinquanguli angulus est $1\frac{1}{5}$ recti per 4.e.6.R. Quare triplex erit $3\frac{3}{5}$ rect: quadruplex $4\frac{4}{5}$. Atq; ut illic quantitas necessaria deficit: sic hic redundat multoque magis si quinque aut sex quinquangula assumantur.

Eodem modo septanguli angulus est $1\frac{3}{7}$ recti. Nam 7 æquales æquantur 10 rectis. Ergo æqualium unus est $1\frac{3}{7}$ recti. Est itaq; duplex $2\frac{6}{7}$ recti triplex $4\frac{2}{7}$ illic minor hic major. ita angulus octanguli est $1\frac{1}{2}$ recti. Ergo duplex est 3 rectorum: sed triplex $4\frac{1}{2}$.

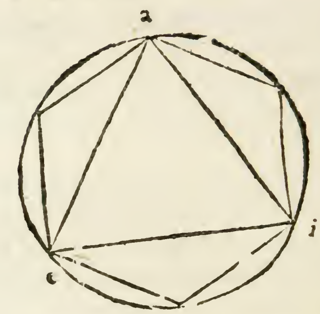
Et sic deinceps inducenti constare poterit tribus tantum planis compleri locum ordinatis triangulo & sexangulo ordinato: rectangulo quolibet.

Et

13. *Si rectæ ab uno inscripti sexanguli angulo in tertium utrinq; angulum connectantur: inscribent triangulum æquilaterum dato circulo.*

Hoc Theon è sua demonstratione deducit. Sit enim inscriptum sexangulum, jamque ab uno angulo sexanguli ad a in tertium utrinq; angulum e & i inscribantur rectæ a e, a i & connectantur: inscribent triangulum a e i æquilaterum. Nam latera

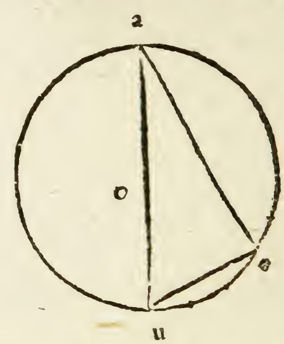
 subten-



ſubtenduntur peripheriis æqualibus a e, a i, e i. ergo per 14.e.2. latera ſunt æqualia. Et eſt hoc tantum hic ſpeciale de connexione duarum inſcriptarum. At latus ab uno inſcripti angulo in tertium eſſe latus inſcripti ordinati dimidii numero laterum 2.e. docebit.

14. Latus inſcripti trianguli poteſt triplum circularis radii. 12.p.13.

Eſto latus inſcripti trianguli æquilateri a e. dico id poſſe triplum radii o u. Nam a e ſubtendit duas tertias ſemiperipheriæ: & reliquam tertiam hoc eſt peripheriæ totius ſextã ſecat recta e u. Quare e u per 11. e. eſt latus ſexanguli vel radius. Hinc ita colligitur. latus trianguli & ſexanguli a e, e u æque poſſunt baſi, hoc eſt diametro a u per 5.e. 12.R. cum angulus ad e rectus ſit per 4.e.4.



At diameter poteſt quadruplum dimidii nempe radii o u, hoc eſt e u, per 7.e.12.R.

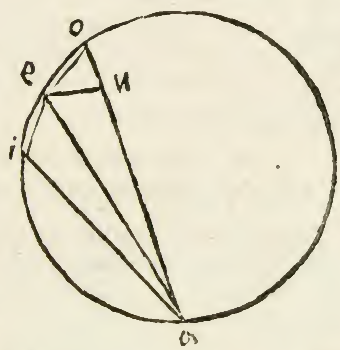
Ergo latus trianguli & ſexanguli poſſunt quadruplum lateris ſexanguli: ablato itaq; e u latere ſexanguli remanebit potentia a e ad e u potentiam tripla.

Hinc infinita ferè ſpecialia theoremata pro inventione lateris trianguli extiterunt. Et ſane Ptolemæus hinc latus trianguli inſcripti inveſtigat: ſi potentia radii triplicetur: triplicatę latus eſt latus inſcripti trianguli. ut ſi ſit radius 10,000,000. ejus potentia triplicata eſt 300,000,000,000,000. cuius latus

tus est 17, 320, 508 ferè. pro latere trianguli inscripti. Alii adhibent methodum generalem 6. aut 11. e. 4. alij aliter speciale theorema hoc fecerunt. Exempli gratia: trium quartarum quadrati diametri latus est trianguli æquilateri inscripti latus. Quod eodem recidit. Sed quia in thesauris sexanguli atq; trianguli versamur nec hoc μυροθήκιον occlusum relinquemus.

15. Si à termino lateris trianguli æquilateri inscripti duæ rectæ in puncta peripheriæ æqualiter à reliquo dicti lateris termino remota inscribantur: differentia inscriptarum æquatur inscriptæ inter reliquum terminum & alterutram inscriptam.

Esto in adjuncta figura latus trianguli æquilateri inscripti: sintq; à termino a inscriptæ duæ quæ cadant in puncta i & o æqualiter remota ab altero termino e. Sitq; a i æqualis a u erit ergo u o differentia inscriptarum.



Dico eam esse æqualem inscriptæ e o vel e i. Nam completè inscripto triangulo: angulus in sectione e o a erit $\frac{2}{3}$ recti. Sed e i supra eadem basi e a ęquatur angulus e o a per 15. e. 3. Sed & angulo e o u æqualis est angulus e u o per 10. e. 6. R. Nam anguli i a e, e a u æquantur & æquicruri sunt ex fabrica. Ergo per 2. e. 7. R. e u æquatur ipsi i e: hoc est ex thesi & 14. e. 2. e o. Quare e o & e u subtendent angulos æquales jam $\frac{2}{3}$ & $\frac{2}{3}$ recti de duobus rectis relinquunt $\frac{2}{3}$ pro angulo o e u. Ergo & angulus o e u, æquatur angulo o u e. proinde per 10. e. 6. R. o e & o u ęquantur. Atq; ita patet veritas. poterit vero transferri elementum etiam ad sinus per 10. e. 5. & ita quidem enunciari: *Differentia* sinuum peripheriarũ à sextante totius peripheriæ æquali differentia majoris & minoris æquatur sinui differen-

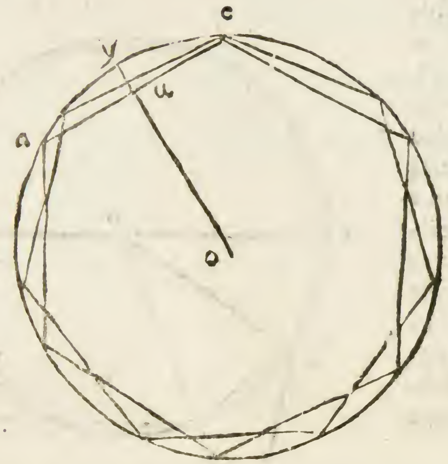
 tiæ.

tiæ. Conſectarium certe pro conſtructione canonis ſinuum notiæ. Conſectarium certe pro conſtructione canonis ſinuum nobiliſsimum. Atque ita ſexanguli & adjuncti trianguli inſcriptio fuit. Figura verò ſexangula vulgo Geometria apum vocatur. Apis enim, ait alicubi Varro, ſexangulam cellam ſibi architectatur, quot habet ipſa pedes: quod Geometræ ἑξάγωνον fieri in orbe rotundo oſtendunt: ut plurimum loci includatur. Hoc idem Pappo in Proœmio libri quinti copioſius demonſtratur. Et ita adſcriptionem habemus trianguli, quadrati, quinquanguli, ſexanguli, octanguli. jam ad ſeptangulum & nonangulum opus eſſet triangulo, cujus uterq; angulus ad baſin eſſet illic triplus hic quadruplus reliqui. in hujus inventione multum poſuiſſe operæ atq; ſtudii Geometras veteres accepimus. Quidam, ait Proclus, ab Archimedis helicibus incitati in datam rationem datum angulum rectilineum ſecuerunt. Conatus illos Geometricos P. Ramus ſcholis ſuis Mathematicis inſeruit lib. 12. in 4. Euclidis. qui illic perlegi poſſunt: artificium eſt difficile, multiplex & Varium: nec temporum proportio in Archimedeo theoremate ſatis Geometrica videtur: Et refutata à Nonio hæc eſt adſcriptio. Mechanici tamen ſeptangulum inſcribunt opera ſexanguli hoc modo.

16. *Perpendicularis à centro in latus inſcripti ſexanguli, eſt latus inſcripti ſeptanguli.*

Eſto enim inſcriptum ſexangulum: & à centro o ſit perpendicularis o u. Dico perpendicularem o u eſſe latus inſcripti ſeptanguli. Detur radius partium 10 totidem erit latus a e. jam quia a e biſecatur in u per 19. e. 1. erit e u partium 5. & proinde u o erit $8\frac{11}{17}$. Quare u y erit $1\frac{6}{17}$. jam ſex perpendiculares ſunt æquales pro ſex lateribus: pro ſeptimo ſunt ſegmenta radiorum ſex inter latera ſexanguli & peripheriam: quæ æquantur uni è perpendicularibus dictis. Nam ſegmentum unum eſt $1\frac{6}{17}$. ergo ſex reſtituunt ferè unam perpendicularem.

Atq;



Atq; hæc Mechanicorum ſeptanguli eſt inſcriptio: per quam etiam circumſcriptio eſt ex 1 e. Sequitur de decangulo.

17. Si latus ſexanguli ſecetur proportionaliter: majus ſegmentum erit latus decanguli.

Eſt Pappi lib.5.theor.24.& habet Campanus ad 3.p.14.Commandinus ad 9.p.13.

Secetur enim radius a o ſeu latus ſexanguli proportionaliter per 3.e.14.R.& ſit a e æqualis majori ſegmento. Dico a e eſſe latus decanguli. Eſt enim peripheria a e ſubdecupla totius peripheriæ. Ergo & inſcripta ejus eſt latus decanguli.

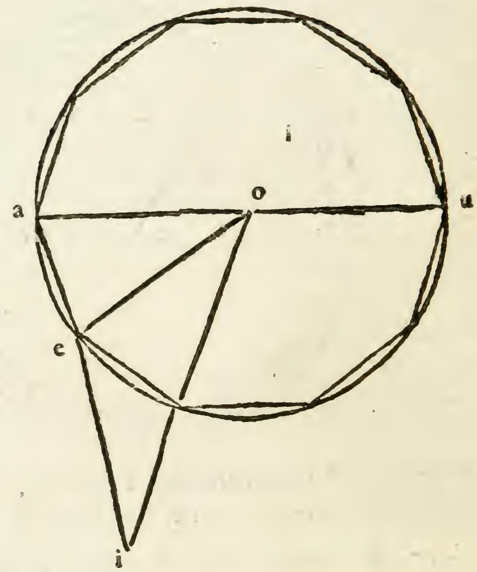
Hujus ἀπόδειξις hæc eſt. A' centro o in terminum e ducatur radius: fiet triangulum æquicrurum a o e cujus anguli ad a & e per 10. e. 6. æquantur: & uterque duplus eſt anguli a o e per 7. e. Nam a o ex theſi ſecatur proportionaliter & a e & majus ſegmentum.

Ergo angulus a & e ſimul eſt quadruplus anguli o.

At angulis o a e,a e o æquatur angulus e o u per 2. c.9. e.6.R.

Quare

Quare & angulus e o u est quadruplus anguli e o a. & per 5.e.3. peripheria u e quadrupla est peripherię a e. jam addita a e ad e u semiperipheria circuli erit quintupla ipsius a e, & proinde tota peripheria decupla. Quare subtensa a e est latus decanguli.



Itaq;

18. *Si decangulum & sexangulum eidem circulo inscribantur: recta è latere utriusq; continuata secabitur proportionaliter, & majus segmentum erit latus sexanguli: & si majus segmentum rectæ proportionaliter sectæ est latus sexanguli: reliquum erit latus decanguli.* 9.p 13.

ut in præmissa figura a e sit latus decanguli inscripti, e i verò radius hoc est latus sexanguli. dico rectam a i secari proportionaliter in e. Nam per 17.e. a e est majus segmentum rectæ e i sectæ proportionaliter. est enim ex thesi latus decanguli: & continuatur recta e i majori segmento. Ergo per c.3.e.14. tota continuata secabitur proportionaliter in e. & majus segmentum erit data e i latus nēpe sexanguli. unde etiá consequitur fore ut a e sit

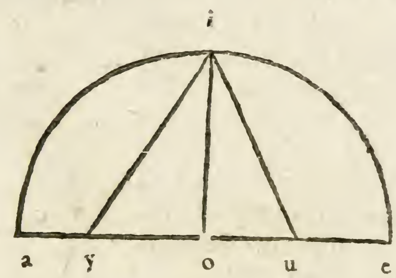
latus

latus decanguli ſi e i ſit latus ſexanguli & tota a i ſecta proportionaliter. Et hinc ſequitur inventio lateris decanguli Ptolemaica.

Et

19. *Latus ſimul utriuſq; quadrati radii & ſemiradii minus ſemiradio eſt latus inſcripti decanguli.*

Sint in dato circulo diametri inter ſe rectæ a e, & i o. biſecetur radius o e in u connectaturq; i u. atq; rectæ i u æqualis ponatur u y. Dico jam o y eſſe latus inſcripti decanguli. Nam recta e y eſt ſecta proportionaliter in o & ſegmẽtum majus e o eſt latus ſexanguli, quia ex theſi eſt radius. ideo per 18. e. y o erit latus decanguli.



Quod autem ſectio proportionalis ſit ad o hinc patet. Quia enim o e eſt biſecta & continuata per 7. e. 13. R. oblongum e y, y o cum quadrato o u æquatur quadrato y u hoc eſt ex fabrica i u.

At quadratum i u æquatur quadratis i o, o u. Quare oblongum e y, y o cum quadrato o u æquatur quadratis i o, o u & ablato quadrato communi o u. oblongum e y, y o æquabitur quadrato i o hoc eſt ex theſi o e. itaq; per 4. e. 12. R. ut e y ad o e ſic o e ad y o. & proinde per 1. e. 14. R. recta e y eſt ſecta proportionaliter in o. Quare patet y o eſſe latus decanguli. jam quia datur radius i o & ſemiradius o u dabitur per 5. e. 12. R. recta i u nempe baſis trianguli rectanguli. Sed i u æqualis ex theſi eſt y u. Ergo inde ſublato u o ſemiradio relinquetur o y latus decanguli.

ut ſi detur radius 10,000,000. erit ejus potentia

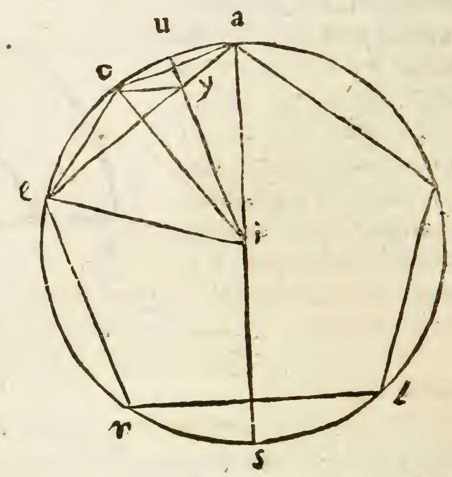
100,000,000,000,000 & potentia ſemiradii
25,000,000,000,000

125,000,000,000,000 Hujus latus tetragonicum eſt
o 11,180,

11,180,340. inde ablatus ſemiradius relinquit 6,180,340. pro latere inſcripti decanguli.

20. Si decangulum, ſexangulum & quinquangulum eidem circulo inſcribantur: latus quinquanguli poteſt latera reliquorum: & ſi recta poteſt latera ſexanguli & decanguli: eſt latus quinquanguli. 10.p.13.

Eſto latus inſcripti quinquanguli a e, ſexanguli e i, decanguli a o. Dico jam a e poſſe reliqua. Ducantur enim perpendiculares i o biſecans latus quinquanguli, i u biſecans latus decanguli cõcurrens cũ latere quinquãguli in y.



Syllogiſmus demonſtrationis ſic eſt:

Oblõga è latere quinquanguli & ejus ſegmẽtis a e, e y & e a, a y æquantur quadratis reliquorum laterum illic e i hic a o.

At quadratum lateris quinquanguli æquatur oblongis è toto & ſegmentis per 3.e.13.R.

Ergo quadratum lateris quinquanguli æquatur quadratis lateris ſexanguli & decanguli.

Aſſumptio dubia non eſt: propoſitio eſt. Ejus igitur proſyllogiſmus attendatur.

Bis bina triangula a e i, y e i & a o e, a o y ſunt æquiangula.

Ergo per 9. e. 7. R. ſigillatim bina cruribus ſunt proportionalia. eritq; illic

ut a e ad e i ſic e i ad e y

hic verò

ut e a ad a o ſic a o ad a y.

Et

Et per 4.e.12.R.illic oblongum a e,e y quadrato e i hic oblongum e a,a y quadrato ao æquale exiſtit.Et ita ſyllogiſmi conſtat propoſitio. Sed & pro ſyllogiſmi antecedens hoc modo demonſtratur. & primo quod triangula a e i,y e i ſint æquiangula.Nam angulus ad e communis eſt:& angulus e i y,angulo e a i ęquatur.

Nam ejuſdem dimidia æquantur ex Arithmetico axiomate.

Anguli e i y,e a i ſunt anguli e i s dimidii.

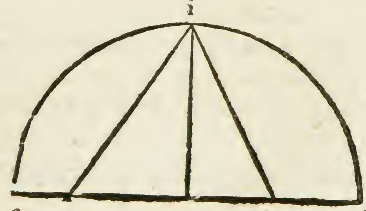
Nam primo angulus e i s æquatur angulis i e a,i a e per 2.c.9.e. 6.R.æqualibus per 10.e.6.R.Ergo alterius e a i eſt duplus.

Secundo peripheria s r e eſt peripheriæ e o u dupla. Nam ſemiperipheria a l s æquatur ſemiperipheriæ a r s. item a l ipſi a r. Ergo l s æquatur reliquæ r s. ſed his etiam æquatur e o.per 1 e. Quare r l hoc eſt e r dupla eſt ipſius r s hoc eſt eo.& r s dupla ipſius o u.nam ex theſi o a hoc eſt e o id eſt r s biſecatur in u. Quare tota s r e dupla eſt peripheriæ e o u.

Itaq; per 5 e 3.angulus e i s duplus eſt anguli e i y. jam cum in triangulis a e i & y e i bini æquentur anguli per c.3.e.7.R. erunt æquiangula.

Secundo triangula quoque a o e, a o y eſſe æquiangula patet. communis enim angulus ad a eſt. & angulus a o y æqualis eſt angulo a e o. Nam eidem æquantur nempe angulo o a y illic per 2.e.7.R.hic per 10.e.6.R.illic enim perpendicularis i u biſecans a o facit triangula duo o u y, y u a æqua angulo ad u recto æquicruro.quare æquilatera & æquiangula: hic vero e o,o a per 10.e.6.R.ſubtendunt angulos æquales. Cum itaque bini anguli ſint æquales triangula a o e, a o y erunt æquiangula per c.3.e. 7.R. Atque ita conſtat & demonſtrationis propoſitio,propoſitionisq; proſyllogiſmus.

Converſâ Euclides adhibet in demonſtratione inſcriptionis Icoſaëdri in ſphæra 16.p.13. Et Prol.9.c.1. inde latus quinquāguli inveſtigat hoc modo.



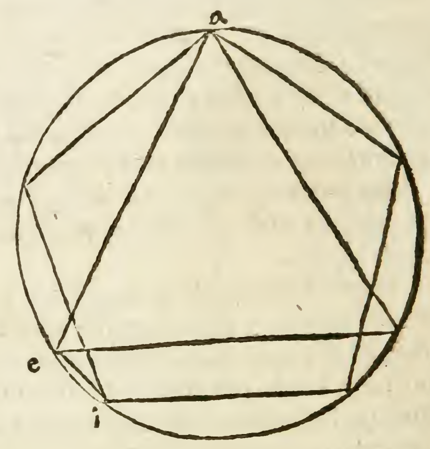
Repetita figura 19. e. patuit o z yo

y o esse latus decanguli: & i o est latus sexanguli. Quia jam hæc, inquit, potest per 5. e. 12. R. recta i y: ea erit latus quinquanguli. ideoque notis y o & i o facile dabitur i y. Sit enim y o partium 6,180,340. & i o partium 10,000,000. erit i y latus quinquanguli partium 11,755,704 ferè.

21. Si triangulum & quinquangulum eidem circulo inscribantur ad idem punctum: recta inscripta inter utriusq; basin dicto puncto oppositam erit latus inscripti quindecanguli. 16.p.4.

Inscribantur enim à puncto a. eidem circulo triangulũ & quinquangulum: & inter utriusq; basin puncto dicto oppositam sit inscripta ei. dico eam esse latus quindecanguli. Est em̃ peripheria ei $\frac{1}{15}$ totius.

Nam peripheria a e est $\frac{1}{3}$ totius & a i sunt $\frac{2}{5}$ totius. jam si tollas $\frac{1}{3}$ de $\frac{2}{5}$ relinquetur $\frac{1}{15}$ totius peripheriæ. Et hinc patet inventio lateris quindecanguli ista.



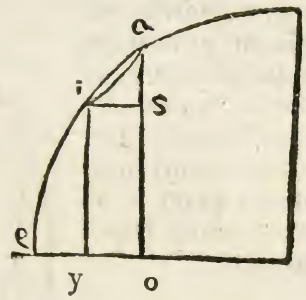
Itaq;

22. Differentiæ sinuum sextæ ac decimæ peripheriæ partis primorum & secundorum possunt inscriptam quindecanguli.

Esto sit a e $\frac{1}{6}$ peripheriæ e i vero $\frac{1}{10}$: sintque horum sinus a o, i y primi quidem, secundi o e, y e sitq; differentia primorum a s. secundorum y o hoc est per 2.c.6.e.10.R. i s. jam per 5.e.12.R. i s & s a possunt inscriptam a i. Ea autem est latus quindecanguli. Nam si compleatur circulus, dupla a e erit $\frac{1}{3}$ dupla i e $\frac{1}{5}$ periphe-

riæ

riæ & i y, a o dupla per 19. e. 1. fient latera trianguli & quinquanguli & quidē bases inscriptorum ab eodem puncto. Ergo per 21. a i erit latus quindecanguli. Hinc jam patet inventio ejus. Nam latus trianguli inventum est ergo & semissis nempe a o dabitur pro sinu sextæ partis 8,660,254.



At lateris quinquanguli semissis 5,877,852.
Ergo a s erit 2,782,402.

Atque cum dentur arcuum a e, i e sinus primi: dabuntur etiam sinus complementorum per 9. e. 5 illic 5,000,000 hic 8,090,170.

Et per 17 e 5 sinus secundus	o e	5,000,000
Et	e y	1,909,830
Itaq; y o vel i s		3,090,170.

Quare per 5. e. 12. R. inscripta a i hoc est latus quindecanguli erit partium 4,158,234 ferè.

Et

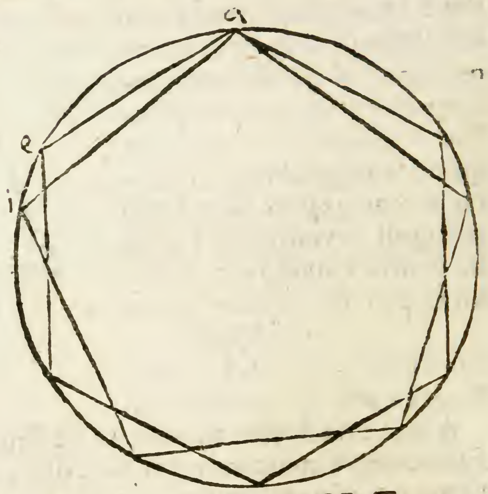
23. *Si quinquangulum & sexangulum eidem circulo inscribantur ad idem punctum: peripheria inter utriusq; latera erit pars tricesima totius peripheriæ.*

Hæc est inscriptio quindecanguli duplicati, hoc est triangulati ordinati cujus anguli sunt 30. itaque & sexangulo utitur, duplicato scilicet triangulo. inscribatur ad idem punctum a sexangulum & quinquangulum. Sitque peripheria e i inter ipsorum latera. dico eam esse $\frac{1}{30}$ peripheriæ totius. Nam a e $\frac{1}{6}$ de a i $\frac{1}{5}$ relinquit e i $\frac{1}{30}$.

Quare hinc patet inscriptio trigintanguli. Nam inscripta e i est latus.

Generalis tamen duplicati ordinati multanguli inscriptio

per 2. e. expeditior est. Elemẽtum proximum latus quindecãguli sup peditavit. Ergo 2.e. latus trigintanguli docebit. & ejus etiam longitudinem : quam si inquiras invenies fere 2,090,570.



TH. FINKII GEOMETRIAE ROTVNDI, LIBER OCTAVVS.

De tetragonismo circuli.

Geometria circularis conjuncta: & adscriptio rectilinei & circuli geodæsiã nobis subministravit multanguli ordinati 4.e.6. Eadem geodæsiam ac dimensionem circuli concedet. Dimensio isthæc tetragonismus & vulgo quadratura circuli dicitur. Argumentum certè mathematicum nobilibus omnium ætatum animis tanquam perpetuum exercitium propositum. Brisonis quadratura ab Aristotele citatur primo poster. & Elench. ubi legi poterit: additis Alexandri commentariis, quibus summa demonstrationis Brisonicæ repetitur. Verum ut hic parum profecit: ita & Aristoteli parum effecisse Antipho visus est: imo 2. c.1. physicorum gravis ejus paralogismus notatur: & veluti responsione

Geometrica

Geometrica indignus à philoſopho ablegatur. Quare in arenam hanc cum deſcenderint & veteres plurimi è recentioribus etiam non pauciores: maximè negotium hoc Archimedes promoviſſe videtur. Nodus quæſtionis eſt in ratione diametri & peripheriæ. Eam Archimedes talem demonſtravit: qualis etiamnum hodie in geodæſia retinetur. Rationem certè aſcibilem nō arbitror: cum ratio curvi ad rectum ſæpe dati poſsit: & peripheria in planū motu ſuo extendi poſsit: ut in rotis currum obſervare licet. Etſi vero in peripheria non ſit principium nec finis: hypotheticum tamen principium ejus dari, & hypothetico fine claudi non eſt dubium. Sed rationem Archimedeam doceamus.

1. *Peripheria eſt tripla diametri & ferè ſeſquiſeptima.*

Demonſtrat Archimedes habere peripheriam ad diametrum rationem minorem tripla ſeſquiſeptima, majorem tamen tripla ſuperdecupartiente ſeptuageſimas primas. Sed quia exceſſus vicinior erat ſeſquiſeptimæ quam ſeſquioctavæ, ideo ſeſquiſeptima retenta fuit: pro ipſo vero, quod vero erat propinquum.

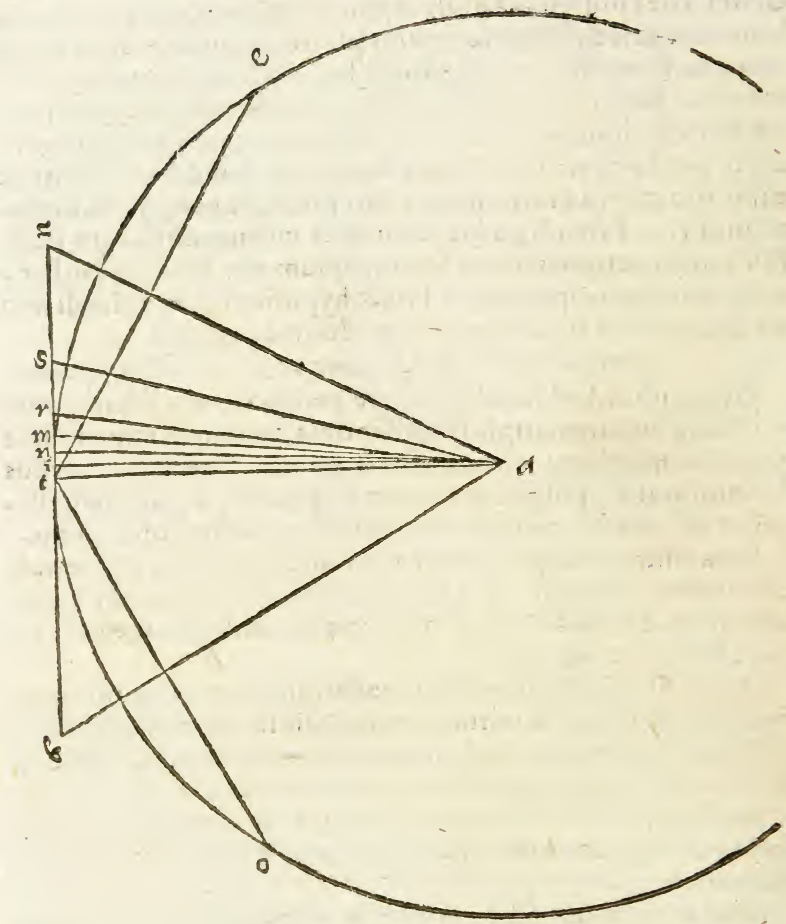
Demonſtratio ergo duarum partium eſt: prima peripheriam eſſe triplam diametri, & paulo ſeſquiſeptima diametri minorem: Secunda eſſe triplam & majorem decem ſeptuageſimis primis ideoq; una octava.

Demonſtratio autem eſt facta collatione illic majorum hic minorum: collatione nimirum multanguli laterum 96 æqualium adſcripti cum circulo. Sed primam partem primò expediamus. ac latus multanguli optati circumſcripti inveniamus.

Sit circulus e i o ex centro a deſcriptus: & ſit tangens u y. ſitq; radius utrinq; à termino i inſcriptus nempe i o, i e pro ſexanguli lateribus.

jam i o arcus hoc eſt $\frac{1}{6}$ peripheriæ biſecetur per 13. e. 3. recta a y. erit ergo per 5. e. 3. angulus i a y $\frac{1}{12}$ totius peripheriæ hoc eſt $\frac{1}{3}$ anguli recti. Nam $\frac{1}{12}$ quatuor rectorum eſt $\frac{1}{3}$ unius recti. jam ponatur i u æqualis rectæ i y & connectatur a u. erunt duo triangula u a i. i a y æqua angulo ad i æquicruro. Ergo & æqua baſibus a y, a u. itaq; & æquiangula per 1. e. 7. R.

Quare



Quare angulus u a y erit $\frac{2}{5}$ recti. Nam uterque particulatim æquatur sibi: & unus ex thesi est $\frac{1}{5}$.

Sed & anguli a u i, a y i æquantur. Est autem alterutrer $\frac{4}{5}$ recti nempe sublato angulo i a y $\frac{1}{5}$ de recto uno. Est itaq; triangulum u a y

u a y æquilaterum:& proinde a u erit duplum ad u i. Quare ſi u i ſit 153, biſecans a u valebit 306. jam ſi quadratum perpendicularis u i 23,409 de quadrato ſecantis a u 93, 636 ſubduxeris: reſtabit quadratum radii a i per 5.e.12.R.70,227. Cujus latus erit 265 ac paulo plus, id ſic deinceps notetur 265+- .Quare ex axiomate arithmetico quod eſt 8 p 5 Euclid. ratio radii a i ad tangentem u i ſit major quam 265 ad 153. Hoc ergo triangulum a i u lateribus ſuis ita dimenſum fundamentum eſt multiplicis ſectionis anguli u a i.& laterum in factis hinc triangulis inventionis. primum itaq; angulus ſubtertius recti u a i biſecetur per 6.e.5.R.recta a s:& fiet angulus s a i $\frac{1}{6}$ recti.itaq; per 12.e.6 R.

ut a u ad a i ſic u s ad s i.

& per additionem proportionum

ut a u & u s
a i ad a i ſic s i ad s i

Et alterne

ut a u & u s &
a i ad s i ſic a i ad s i.

Quare ratio ſimul utriuſque cruris a u & a i 306 & 265+- id eſt 571+- ad 153 erit etiam ratio a i ad conterminum ſegmentum s i: major nempe ratione 571 ad 153.ut ſi ſegmentum s i valeat 153 radius valebit plus quam 571.

Et ſic in triangulo a i s crura recti ſunt menſa. ubi numerus ſegmenti i s eſt qui fuit totius i u.quod etiam deinceps ſervabitur. jam vero quadratum i s ſit 23, 409. at quadratum a i 326, 041. totum 349,450 +- erit per 5. e. 12. R. pro ſecunda biſecante nempe a s.latus vero erit 591$\frac{1}{8}$+-.

Et hæc fuit prima diviſio anguli tertiarii, ſequitur ſecunda. Biſecetur itaq; angulus s a i:erit proinde r a i $\frac{1}{12}$ recti, nempe dimidius dimidii unius tertiæ unius recti $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{3}$ hoc eſt $\frac{1}{12}$. Erit itaq; rurſus

ut a s ad a i ſic s r ad r i. Et
ut a s & a i ad s r & r i ſic a i ad r i.

Quare ratio ſimul utriuſque cruris a s, a i 591$\frac{1}{8}$+- & 571+- hoc

eſt 1,162$\frac{1}{8}$+- ad 153 erit etiam ratio a i ad conterminum ſegmentum r i: major nempe ratione 1,162$\frac{1}{8}$ ad 153. Fiant jam quadrata radii 1,162$\frac{1}{8}$ & contermini ſegmenti r i erunt 1,350,534$\frac{33}{64}$ & 23,409 totũ aũt per 5. e 12. R. pro biſecante a r & ipſa a r 1,172$\frac{1}{8}$+-.

Atque hæc ſecunda anguli fuit biſectio: triangulumq́; tertium lateribus dimenſum. Sequitur tertia biſectio anguli ſcilicet r a i. Fiat itaq; angulus m à i $\frac{1}{24}$ unius recti. eſt eñ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{3}$ hoc eſt $\frac{1}{24}$. erit ergo per præmiſſas ratiocinationes ratio ſimul utriuſq; cruris a r, a i 1,172$\frac{1}{8}$+- & 1,162$\frac{1}{8}$+- hoc eſt 2,334$\frac{1}{4}$+- ad 153. Eademq́ue a i ad i m major nempe ratione 2,334$\frac{1}{4}$ ad 153. jam adde quadrata crurum 5,448,723$\frac{1}{16}$ & 23,409: totum erit pro quadrato a m. & latus 2,339$\frac{1}{4}$+- pro latere a m.

Atq; hæc fuit tertia biſectio, quartiq́; trianguli laterum dimenſio. & hic habemus $\frac{1}{96}$ totius peripheriæ. Nam u a i fuit $\frac{1}{12}$. Ergo $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{12}$ erit $\frac{1}{96}$.

Sed ratio totius diametri ad ſegmentum tangentis non ſuccederet. itaq; angulus æqualis jam inventus aliter inveniendus eſt, addendo æqualem ad inventi dimidium.

Quare ſequatur jam quarta biſectio anguli nempe m a i per rectam a n. Erit ergo n a i $\frac{1}{48}$ unius recti nempe $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{3}$ hoc eſt $\frac{1}{48}$.

Hic jam ratio per præcedentes ratiocinationes a i ad i n, eſt ratio utriuſque cruris m a & a i hoc eſt 2,339$\frac{1}{4}$+- & 2,334$\frac{1}{4}$+- id eſt 4,673$\frac{1}{2}$+- ad 153. Atq; hactenus anguli ſectio quintuplex fuit.

jam ponatur angulo n a i æqualis angulus i a t: Erit ergo angulus n a t æqualis angulo m a i hoc eſt $\frac{1}{96}$ totius peripheriæ, ut paulo ante patuit. n t itaq; erit latus multanguli laterum 96 circumſcribendi: quod nonagies ſexies deinceps paribus intervallis circa peripheriam circumſcriptum totam figurã complebit.

Hiſce fabricis biſectionis angulorum & triangulorum dimenſionibus abſolutis ratio propoſita ſic colligitur.

Radius a i ad ſegmentum i n majorem rationem habet quam 4,673$\frac{1}{2}$ ad 153. ut jam innotuit.

At ut radius ad i n ſic diameter ad n t. Eſt enim diameter dupla

pla radii & n t dupla i n: & partea multiplis sunt proportionales.

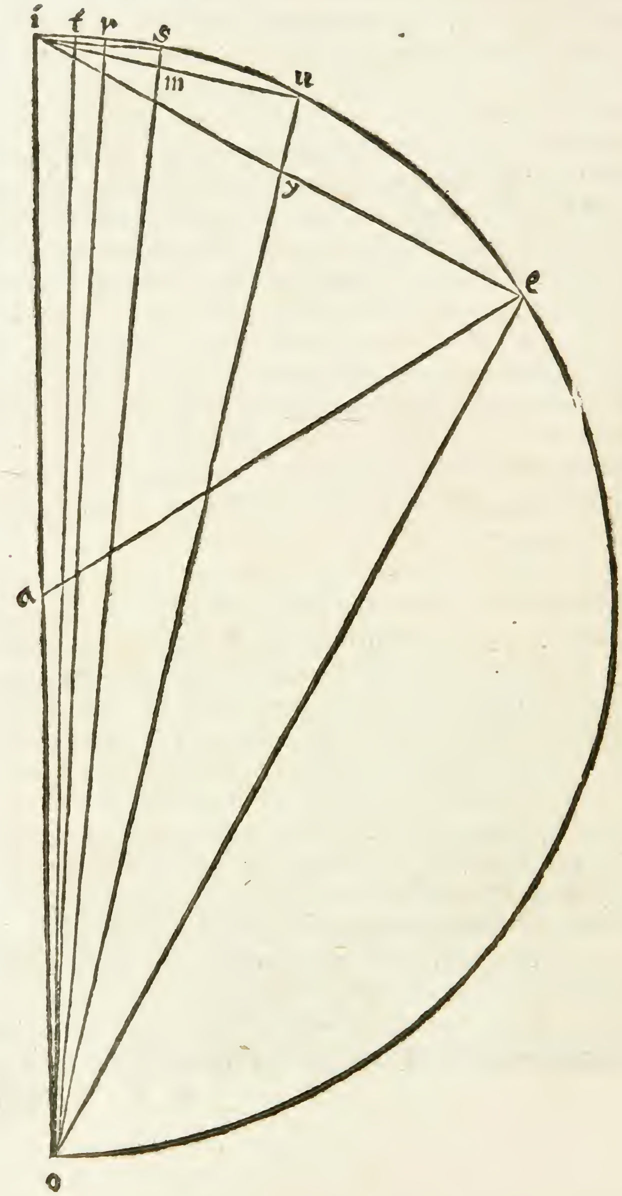
Tota igitur diameter ad n t majorem habet rationem quam $4,673\frac{1}{2}$ ad 153. At si 153 per 96 multiplices facies perimetrum circumscripti multanguli 14,688. jam ratio perimetri multanguli ad diametrum est tripla major $667\frac{1}{2}$ ipsius diametri: quemadmodum ex arithmetica rationum investigatione patet. Vel sic: à perimetro ter tollas 4,673 restabunt 669. inde rursus $\frac{1}{2}$ ter ablata relinquit $667\frac{1}{2}$ ipsius diametri. At hæc minor est quam septima pars diametri. Nam septies $667\frac{1}{2}$ sunt $4,672\frac{1}{2}$. numerus unitate scilicet minor quam diameter. Ergo perimeter multanguli laterum 96 circumscripti tripla est diametri circularis, sed paulo minor sesquiseptima, unitate nimirum unius septimæ. Quare & peripheria tripla quidem diametri est, sed multo minor sesquiseptima. Est enim contenta peripheria inscripti circuli. Ergo continente perimetro nonasexanguli circumscripti minor.

Atq; hæc demonstrationis pars prima fuit: altera superest:

Peripheriam ad diametrum triplam esse & multo sesquioctava majorem.

Fabrica est hic primo ut ante inscriptionis nonasexanguli per quintuplicem anguli sectionem & triangulum quintuplex. inde ratio inscripti docetur ad diametrum tripla & major sesquioctava. Sed fabrica doceatur. Sit circulus o e i. cum inscripto sexangu li latere i e. connectaturq; e cum termino diametri o: erit angulus i o e $\frac{1}{3}$ recti. Nam ducto radio e a triangulum i a e est æquilaterũ, & per 3.c.10.e.6.R. angulus i a e est $\frac{2}{3}$ recti. Quare per 9. e.3. angulus i o e erit $\frac{1}{3}$ recti: nempe dimidius anguli in centro.

Atq; hæc ut prima anguli sectio est: sic triangulum e i o est primum in fabrica hac. Ejus latus o i est duplum lateris e i: diameter nempe sui radii. itaq; si sumamus mensuram radii 780 (Orontius ut ante tangentis e i mensuram posuit 11 sic hic radium facit 15) erit diameter o i 1,560. jam tolle quadratum radii 608,400 de quadrato diametri 2,433,600. relinquetur per 5.e.12.R. 1,825,200 pro quadrato e o: cujus quadrati latus est paulo minus quam 1,351. id sic deinceps notari poterit 1,351—. Quare secans e o ha-



bebit

bebit minorem rationem ad i e quam 1,351 ad 780. Atq; sic primi trianguli latera dimensa sunt.

Sequitur quadruplex subbisectio anguli i o e. Bisecetur itaque rectâ o u. & connectatur i & u. fient triangula o e y, o u i æquiangula. Nam anguli ad u & e sunt ęquales per 15.e.3: angulus etiam i o u æquatur angulo u o e ex fabrica. Ergo per c.3.e.7.R. reliquus reliquo erit æqualis. Quare per 9.e.7.R.

ut o e ad e y sic u o ad u i.

At ut i o ad i y sic o e ad e y per 12.e.6.R.

Ergo ut i o ad i y sic u o ad u i.

Et quia est ut e o ad o i sic e y ad y i: erit per compositionem proportionum.

ut e o & e y &

o i ad o i sic y i ad y i.

itaq; alterne

ut o i ad y i sic e o & o i ad e y, & y i hoc est e i.

Sed ut o i ad i y sic u o ad u i.

Ergo ratio u o ad u i est ratio e o & o i ad e i.

Hic enim numerus perpendicularis manet, qui postea si non ratione: terminis tamen rationis variatur.

Addantur itaq; e o, & o i hoc est 1,351— & 1,560. totū 2,911— erit o u. Quare minor ratio est u o ad u i quam 2,911 ad 780. jam addantur quadratum u o 8,473,921— & quadratū u i 608, 400: totum per 5.e.12.R. erit quadratum diametri i o. totiusque latus erit 3,013$\frac{4}{6}$, $\frac{1\ 5\ 2}{0\ 2\ 7}$ hoc est 3,013$\frac{3}{4}$— Quare diameter o i ad u i minorem habet rationem quam 3,013$\frac{3}{4}$ ad 780.

Atq; sic prima fuit bisectio anguli & secūdi trianguli dimēsio.

Secundo itaq; bisecetur angulus i o u rectâ i s. erit ergo angulus i o s $\frac{1}{12}$ recti unius nempe $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{3}$ hoc est $\frac{1}{12}$. jam connectatur recta o s secans, cum termino i.

Quia igitur per præmissum ratiocinium triangula m u o, i s o sunt æquiangula: erit

ut o u ad u m sic o s ad s i.

At ut i o ad i m sic o u ad u m p 12.e.6.R. & alternatis terminis.

Ergo ut i o ad i m sic o s ad s i.

Et hinc ut ante concludetur per compoſitionem proportionum eſſe s o ad s i ut u o & o i hoc eſt 2,911— & 3,013$\frac{3}{4}$ id eſt 5,924$\frac{3}{4}$ ad s i 780. Reducatur jam uterque terminus ad integra proportionalia, multiplicatione nempe per 4: habebuntur integri 23,699— & 3,120: rurſusq; integri reducantur per maximam menſurā communē 13 ad minores proportionales 1,823— & 240 hoc eſt ratio s o & s i minor eſt ratione 1,823 ad 240.

jam addantur quadrata reductorum 3,323,329— & 57,600. totū erit pro diametro o i 3,380,929. totiusq; latus erit 1,838$\frac{2685}{3677}$. Quare diameter o i ad s i minoris eſt rationis quam 1,838$\frac{9}{11}$ ad 240. Et hæc ſecunda fuit biſectio & tertii trianguli dimenſio.

Tertio biſecetur angulus s o i per rectam o r. Triangula erunt ut ante æquiangula. ratio o r ad r i connectentē, erit ratio o i diametri & s o hoc eſt 1,823— & 1,838$\frac{9}{11}$— id eſt 3,661$\frac{9}{11}$— ad 240. Reducito hic partes ad integra proportionalia multiplicatione per 11 facies 40,280 & 2,640 hoc eſt facta diviſione per communem menſuram maximam 1,007— ad 66. Horum jam quadratis 1,014,049— & 4,356 additis, totum erit quadratum diametri, diameterque ipſa 1,009$\frac{1}{6}$— Quare i o ad r i eſt rationis minoris quam 1,009$\frac{1}{6}$ ad 66.

Hæc tertia erat biſectio ubi angulus r o i eſt $\frac{1}{24}$ recti: nempe $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{3}$ hoc eſt $\frac{1}{24}$: Et dimenſio fuit trianguli quarti.

Quarto jam biſecetur angulus r o i per rectam o t. Triangula erunt ut prius æquiangula: & ratio i o diametri & r o hoc eſt 1,007— & 1,009$\frac{1}{6}$— id eſt 2,016$\frac{1}{6}$— ad 66 erit ratio o t ad t i. jam adde quadrata o t, t i 4,064,828$\frac{1}{36}$— & 4,356. totum erit pro quadrato diametri. & tum diameter erit 2,017$\frac{1}{4}$— Quare diameter o i ad i t eſt minoris rationis quam 2,017$\frac{1}{4}$ ad 66. Contraq; latus i t ad i o majoris eſt rationis quam 66 ad 2,017$\frac{1}{4}$. Et ſic quarta fuit anguli biſectio: ubi t o i eſt $\frac{1}{48}$ recti. & quidem $\frac{1}{96}$ peripherię. Nam u o i eſt $\frac{1}{12}$ ergo s o i eſt $\frac{1}{24}$ & r i o $\frac{1}{48}$ itaq; t o i $\frac{1}{96}$ totius peripheriæ. Quare recta i t eſt nonageſima ſexta pars inſcripti multanguli. Neq; modus ille duplicati anguli è prima parte huc repetitur: quia ſumitur tota diameter non radius. itaque fabrica

ſic

ſic eſt:per quam inventum latus nonagies ſexies inſcriptum totam peripheriam ſubtendet. Hinc jam ipſa ratio ita patet. cum latus inſcripti ſit 66. hoc per 96 multiplicatum, faciet perimetrum totam 6,336. At diameter eſt 2,017$\frac{1}{4}$. Quare ratio 6,336 ad 2,017$\frac{1}{4}$ eſt tripla major 284$\frac{1}{4}$ ipſius diametri ut ex Arithmetica inventione conſtat. Vel, ut ante, è perimetro tolle 2, 017 remanebunt 285. hinc tolle ter $\frac{1}{4}$ hoc eſt $\frac{3}{4}$ relinquentur 284$\frac{1}{4}$ ipſius diametri. Hæc reliqua majora ſunt $\frac{1}{7}\frac{0}{1}$ ergo quam $\frac{1}{8}$ majora exiſtunt. itaq; perimeter inſcripti multanguli ſuperat diametrum triplo & ſeſquioctavo. ideoq; peripheria major, quia continens, multo magis ſuperat.

Quare ut tota Archimedea demonſtratio concludatur: peripheria diametri eſt tripla & ferè ſeſquiſeptima. Minor enim eſt ſeſquiſeptima, major ſeſquioctava. ſed propior ſeſquiſrptimæ: illic enim deeſt unitas unius ſeptimæ: hic reliquum partibus plurimis majus quam $\frac{1}{7}\frac{0}{1}$ & ideo quam $\frac{1}{8}$. Quare vicinum magis & vero propinquum aſſumptum.

Et hoc Archimedeum ratiocinium tota poſteritas fermè retinuit: eoq; vulgo data diametro aut peripheria alterutrum inveſtigatur. ſtatuitur enim peripheria talium 22 qualium diameter eſt 7. Ergo ſi detur peripheria 88. erit ut

22 ad 7 ſic 88 ad 28 pro diametro quæſita. Et ſic deinceps.

Detur rurſus diameter 56. Quæritur in tali menſura peripheria. Erit ergo

ut 7 ad 22 ſic 56 ad 176 pro peripheria quæſita. Non eſt occluſa via perveniendi etiam Archimedeo modo ad accuratiorem rationem. poterit enim aſſumi multangulũ adſcribendum multorum laterum aliquot centenorum aut millenorum millium. & radius ſimiliter partium vel centies millies millenarum millium. Nam tum accuratius inveniri terminos rationis nullum eſt dubium. Hoc cuilibet Mathematum candidato tractandum relinquo. Et hinc jam geodæſia circuli eſt.

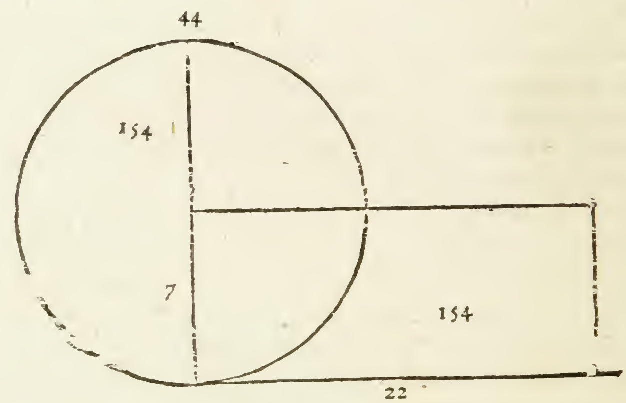
Itaq;

2. *Planus è radio & peripheriæ dimidio eſt area circuli.*

Archimedes

Archimedes in dimensione circuli triangulum è semisse perimetri tanquam altitudine & diametro tanquam basi circulo æquat. Hinc varia exciterunt axiomata, planum radii & peripheriæ duplum esse areæ circuli, Theon 2. c. 1. in Ptolem. & paulo post: Planum semissis radii & perimetri esse aream. Rursus: Planum diametri & quartæ partis peripheriæ esse aream. Et alia plura. Quæ tamen ex elemento jam posito cognita parallelogrammi doctrina non erunt ignota.

Esto jam peripheria data 44. etiam ignota diametro. e a enim per 1 e dabitur 14.



jam planus è 7 & 22 hoc est 154 est area circuli.

Rectanguli autem hujusmodi oblongi latera duo opposita sunt in diametro: & in peripheria duo reliqua opposita continentur. Quare binorum dimidia assumuntur tantum: cum ab iis rectangulum comprehendatur per 1.c.2.e.11.R.

Et

3. ut 14 ad 11, sic quadratum diametri ad circulum.

Proportio diametri quadratæ ad aream circuli ex Archimedea illa ratione est. Et dantur hic tres termini proportionis potestate. unde circulus invenitur.

Sit

Sit enim diameter 14 ejus quadratum est 196. jam ut 14 ad 11, sic 196 erit ad circuli aream: quæ propterea erit 154 ut ante. Et hoc è dimenso circulo & quadrato per analysin patet. Nãm ratio 196 ad 154 est ratio 14 ad 11.

Et est hic tetragonismus paulo à superiori diversus: Estq; Euclidis apud Heronem propositus. paulo tamen secus eo scilicet modo quo à sphæricis (sic enim elementarii Astronomi salutantur) vulgò proponitur. Si de quadrato diametri $\frac{3}{14}$ tollantur: relinquetur area circuli. ut in addito exemplo experiri potes. Nam $\frac{3}{14}$ de 196 sunt 42 quibus ablatis de 196 relinquuntur 154 pro area circuli.

Et

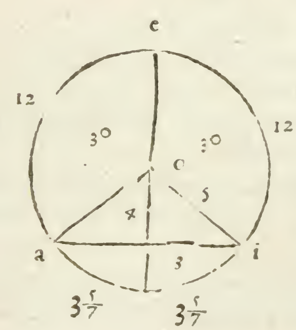
4. *Planus è radio & peripheriæ quadrante est area semicirculi.*

ut retento superiori exemplo planus è 7 radio per 11 quadrantem peripheriæ nempe 77 est semicirculus. Sed, ut ex definitione semicirculi patet, potest dimensio isthæc fieri sumpto dimidio dimensi jam circuli.

Et

5. *Planus è radio & basis dimidio est area sectoris in centro.*

Ex eodem illo fonte segmentorum geodæsia perficitur. Sint circuli a e i tres sectores quorum bases sint a e 12 pedum, e i itidem pedum 12, a i vero pedum $7\frac{3}{7}$. Erit ergo per 1 e radius 5. diameter enim invenitur pedum 10. jam multiplica 5 per 6. factus erit sector alteruter æqualiũ nempe pedum 30. Rursus multiplica 5 in $3\frac{5}{7}$ quotus erit $18\frac{4}{7}$ pro tertio sectore. Et ex his additis totus circulus restituitur $78\frac{4}{7}$. quantus inveniretur esse ex præcedentibus.



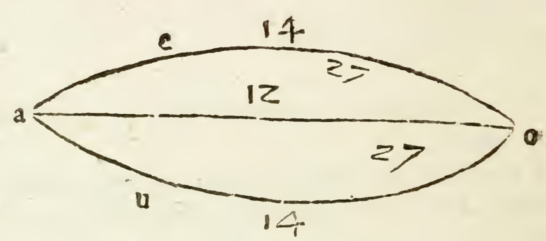
Et

6. Si triangulum è duobus radiis & basi majoris sectionis addatur duobus in ea sectoribus centri: totus erit area sectionis majoris: sin detrahatur suo sectori: reliquum erit area minoris.

ut in præmisso schemate sit triangulum a o i pedum 12 nempe si a i basis majoris sit pedum 6: erit altitudo 4. hinc 6 per 2 aut 4 per 3 faciunt 12 pro area trianguli. jam si hęc duob. sectoribus suis a o e, e o i nempe 60 addatur: totus 72 erit area majoris sectionis a e i. Rursus idem triangulum tollatur de sectore a o i $18\frac{4}{7}$ remanebit minor sectio $6\frac{4}{7}$.

Hinc oritur etiam geodæsia plani ovati, & ejus quod vulgò Lenticula vocatur.

Esto enim ovale planum a o e u. cujus arcus æquales sint sigillatim pedum 14. & a o pedum 12. Sigillatim igitur metire a e o, & a u o. tãquam sectiones circuli minores: areæ enim particulares 27. & 27 colligent totam pedum 54.

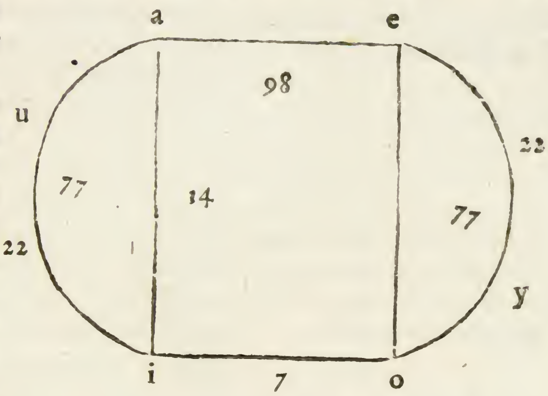


At unde longitudinem radii capies: cujus noticia in dimensione sectionis requiritur? non alio certè modo: quam a o es dimensus absoluto sectionis circulo.

Offeratur jam lenticula, figura in cujus medio quadrangulum est: utrinq; è duobus ejus lateribus semicirculi. ut figura a y o u.

Ejus sit latus a i vel e o pedum 14. i o vero pedum 7. Erit proinde per tetragonismum circuli e y o vel a u i pedum 22.

Hinc area figuræ oblatæ facile datur. Nam factus è 14 per 7



hoc eſt 98 eſt area oblongi a o per 6.e.11. R.

Et alterutrius ſemicirculi area eſt 77. Quare figuræ totius area eſt 252. Et ſimilis geodæſia fuerit in figuris aliis miſtis.

Atque ita Geometria circularis ſimplex & conjuncta fuit. Verum inde non deſcendemus ad Sphæram: antequam hujus Geometriæ theſauros auro nobili quovis precioſiores introſpexerimus.

TH. FINKII GEOMETRIAE ROTVNDI,

LIBER NONVS.

De canone triangulorum.

E Geometria circulari Canonem triangulorum in aſſumpta menſura tam peripheriæ quam diametri licet conficere.

Hoc nobis poſtulatum eſto: cujus fabrica ex ſuperioribus cuivis in promptu erit. Nam Canonem triangulorum vulgò vocant eum qui rectas ſemicirculi ad quadrantis partes deſcriptas continet. Talē huc transferre volui. Nihil à Geometriç ſublimitate cōmitti arbitror alienū: ſi termini circulares certa menſura definiantur. Sunt Mathematicorum hypotheſes multæ: licet iis qui regula falſi vulgari utuntur metam

ſibi præſcribere in aſſumptorum multiplicatione: quod ſic faciliтas quædam inſit. Et certè hypotheſi huic magna inerit facilitas. Sine enim ejuſmodi aſſumpta menſura præcepta calculi inſequentia difficillima & obſcura reddentur: cum propter exemplorum tractationem prolixam, tum etiam inveſtigationis tædium atq; laborem. Et poterit quælibet oblata alia ab aſſumpta diverſa menſura per regulam auream aſſumptæ accommodari. Retinebitur autem ea quæ Aſtrologis in cœlũ eſt derivata: quod ejus uſus non tantum cœli cardines cœleſtiumque domicilia diſtinxit: ſed & terram climatis diſtribuit: imò in omni re ſic menſuranda ſit adhibitus. Hypotheſis ergo menſuræ ſic erit:

2. Menſura peripheriæ aſſumpta ſic eſt: peripheria partibus 360 eſt diviſa: pars in 60 ſecatur ſcrupula prima: Scrupulum primum in 60 ſecunda: & ſic deinceps.

Pars vulgo gradus vocatur: ſed uſitatius & generalius hic eſt partis vocabulum. Diſtributio hæc Ptolemæi eſt: ante quem in uſu fuiſſe ex Eratoſthene & Hipparcho cognoſcimus menſuram peripheriæ partium 83. Vide Ptol. 10. c. 1. Pars deinceps quælibet in ſua eſt diviſa ſcrupula ſexageſima: quæ ſuis notis inſigniuntur hoc modo:

$\prime \quad \prime\prime \quad \prime\prime\prime \quad \prime\prime\prime\prime$

lineolis ad latus dextrum numeris adpoſitis. ut ſi ſcribas 10 prima 20 ſecunda 15 tertia. id hoc modo facies

$10'. \quad 20''. \quad 15'''.$

Vulgo etiam putantur habere peculiarem ut notationem ſic numerationem maximè conjunctam per tabulam Hexacontadon. At tabula hæc nil niſi reductionem continet factorum è datis terminis: nec ſemper ea nobis adeſt.

Retenta hæc fuit Ptolemæi diſtributio à poſteris tam partium peripheriæ quam partium minutorum: quod hi numeri 360 & 60 haberent plures menſores. itaq; & quotos viciſsim metientes: proindeq; eſſent ad numerationes aptiores & magis accommodati. Sunt enim numeri peripheriæ 360. menſores primi quidem

dem 1, 2, 3, 5. Compoſiti vero 4, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360, numero viginti quatuor: ut ex Arithmetica inventione patet.

Sic 60 invenies menſores duodecim 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60.

Sed typus utriuſq; exempli rem ob oculos ponet

1	360
5	72
3	120
15	24
9	40
2	180
10	36
6	60
30	12
18	20
90	4
45	8

Quo ſignificatur: ſi dividas 360 in 5 tum $\frac{1}{5}$ fore 72: ſi in 3 tum $\frac{1}{3}$ fore 120. Si in 90 tum $\frac{1}{90}$ fore 4. Vel contra ſi concipias 360 dividi in 72 tũ $\frac{1}{72}$ erit 5. ſi in 120 tum $\frac{1}{120}$ erit 3. ſi in 4 tũ $\frac{1}{4}$ erit 90.

Sic in menſura minutorum ac ſcrupulorum

1	60
3	20
5	12
15	4
2	30.
6	10

Si 60 in 3 dividas $\frac{1}{3}$ erit 20. ſi in 5. tum $\frac{1}{5}$ erit 12 vel contra: ut ex Arithmeticis inventionibus conſtat. Et hinc jam facile erit peripheriam in partes aſsis ſecare: quod utile eſt. Sæpe enim apud authores elegantiæ ſtudioſos aſsis nominibus anguli & arcus nominantur. Quare ut in promptu ſint hic accipe.

In partibus

uncia	30	Bes	240
Sextans	60	Dodrans	270
Quadrans	90	Dextans	300
Triens	120	Deunx	330
Quincunx	150	As	360.
Semissis	180		
Septunx	210		

unciæ partium nomina raro imposita hisce partibus leguntur.

Sic in scrupulis

As	60	Semissis	30	uncia	5
Deunx	55	Quincunx	25	Semuncia	2 30
Dextans	50	Triens	20	Duella	1 40
Dodrans	45	Quadrans	15		
Bes	40	Sextans	10		

Ergo hæc assumpta mensura esto: ad quā regula aurea omnes alias accommodabit. Sic Ptolemæus dicit Eratosthenem obseruasse tropicorum distantiam partium 11 qualium circulus in peripheria habet 83. Quæritur quanta fuerit in nostra mensura. Aurea regula respondebit. Si cum peripheria est partium 83 distantia est 11. Ergo si peripheria sit 360 distantia erit $47\ 42'\ 39''$ & paulo ultra. hoc est partiū 47 & partis plus besse, minus dodrāte.

3. Diameter assumitur partium 20,000,000.

Si ratiocinium Archimedeum sequamur ex assumpta peripheriæ mensura diameter esset $114\frac{6}{11}$. At Ptolemæus contentus ratione peripheriæ ad diametrum qualicunque diametrum ponit 120. Quod nulla habita ratione peripheriæ ad diametrum fieri potest. unde recentiores ut scrupulorum evitarent involutionem in multiplicationibus & divisionibus numerorum circa ipsas lineas, quæ ut plurimum sunt asymmetræ: alii aliter rationalem constituerunt diametrum. Arzahel eam posuit partium 300. Regiomontanus 12, 000, 000. item 20, 000,000 quæ mensura deinceps est retenta, à Copernico tamen Apiano & aliis circulis duobus imminuta. Nonnulli retinent Ptolemæi mensuram, & canones suos ad sexagenariam revocant numerationem.

Quæ

Quę ratio vel eo nomine eſt laborioſa: quod canone tali uti non poſsis niſi crebra inſpectione tabulæ numerationis aſtronomicæ: quæ non ſemper adeſt: & ſi adſit viſum nō mediocriter lædit.

Poſita jam hac diametro facile erit nobis canonem triangulorum ad ſingulos quadrantis gradus ſingulorumq; minuta perficere. Cujus uſus ut melius innoteſcat & altius hæreat: ejus ſyntaxin breviter è ſuperioribus repetam: primò quidem canonis ſinuum, via Ptolemæi ac Regiomontani & Purbachii. Latera itaq; inſcriptorum rectilineorum ex libro ſeptimo ſic habent:

Trianguli	120.	17,320, 508	14 e
Quadrati	90	14, 142, 136	4 e
Quinquanguli	72	11, 755, 704	20 e
Sexanguli	60	10,000,000	11 e
Decanguli	36	6, 180, 340	19 e
Quindecanguli	24	4, 158, 234	22 e

jam per 10 .e.5 erunt ſinus

p. 60	8, 660, 254
45	7, 071, 068
36	5, 877, 852
30	5,000,000
18	3, 090, 170
12	2, 079, 117

Et hinc per 9.e.5 ſinus colliguntur complementorum nempe

p. 54	8,090,170
72	9, 510, 565
78	9, 781, 476.

His jam ſinibus datis invenies per 20.e.5 ſinus ſemiſsium peripheriarum: & hinc tum per 9.e.5 ſinus complementorum ſemiſsium: & continuè in his deinceps. Exempli gratia datur ſinus 54 & ejus complementi 36. illic 8, 090, 170 hic 5, 877, 852. Quod ſi jam velis quærere ſinum arcus 27 ſemiſsis nempe 54. Quæres ſinũ ſecundũ p 54 per 17.e.5 nempe 10, 000,000 minus 5,877,852 hoc eſt 4,122,148. jam per 20.e.5 ut 5,000,000 ad ſinũ arcus 27. ſic ſinus 27 ad 4,122,148. Ergo ex proportione continua latus

latus facti à 5,000,000 per 4,122,148 nempe 4,539,905 eſt ſinus quæſitus nempe peripheriȩ gr.27.

Eodem ratiocinio invenies ſinum cuiuſcunq; dimidii arcus: ſic hujus ipſius nempe p.13,30' & rurſus hujus p, 6, 45' tum ſingulorũ complementorũ ſinus primo. 63. tum p.76,30' tũ p.83,15'. Et horum complementorũ dimidiorum ſinus ut p,31,30' rurſus p,15. 45' & 38,15'.Et horum rurſus complementa aſſumes: invenies ſinus p.58,30'.p.74,15' p.51,45'. Et horum ſumere potes ſemiſſes p.15,45'.p 29,15' & horum complem. p.74,15' & p.60,45'.

Sic examinatis reliquis arcubus multorum invenies ſinus. Sume gr.30.ejus ſinum habes.ergo per præmiſſum ratiocinium invenies ſinum gr.15.& hinc ſinum complementi gr.75. jam rurſus ex ſinibus gr.15 & gr.75 invenies ſinus gr.7,30',gr 37,30' gr.82, 30' gr. 52, 30.Rurſus ex his invenies ſinus gr.3, 45' gr.18,45' gr. 41, 15' gr.26,15' gr.86,15' gr.71,15' gr.48,45' gr.63,45'.

Sic ſi ſumas g.45 ſinum:invenies hinc ſimili inductione ſinus gr.22,30' gr.67,30' gr.11,15' gr.33,45' gr.78,45' gr.56,15'.

Sic ſi aſſumus ſinum gr.18 invenies methodo ſimili ſinus gr.9. gr.81.gr.4,30' gr.40,30'. gr.85,30' gr. 49,30'. gr.2,15' gr.20,15' gr. 42, 45' gr.24,45' gr.87,45' gr.69,45' gr.47,15' gr.65,15'.

Rurſus ſumas 12 ſinum.habebis hinc ſinum gr. 6 & 84. gr.3.gr. 42 gr.87 gr.48 gr.1,30' gr.21.gr. 43, 30' gr. 24 gr.88,30' gr. 69.gr.46, 30' gr.66.gr.0,45' gr.10,30' gr.21,45' gr.44,15' gr.34,30' gr.23,15' gr.33. gr.89,15' gr.79,30'. gr.68,15'.gr.45,45'.gr.55,30'.gr.66,45' gr.57.& ex his rurſus ſemiſsium, ſemiſsiumq; compleme͂torum. prout non tantum in his ſed & aliis numeris calcographus inductionem inſtituet & calculum adhibebit.Et ſic maxima pars canonis abſoluta eſt. Verum in iis ipſis compendium ſyntaxeos ſupra ad 15.e.7.propoſitum non eſt negligendum. Eo enim ſola numeratio ſimplex ſinum inveniet. Exempli gratia habes ſinum è ſuperioribus gr.17,15',habes etiam ſinum gr.42,45'.

Illic 2,965,416.
Hic 6,788,007

Ergo his additis 9,753,423 habes ſinum arcus compoſiti

politi è gr.60 & 17,15′ hoc eſt g.77,15′. Quia uterq; 42,45′ & gr.77,15′ eodẽ intervallo abeſt à partibus 60 nempe ſextante peripheriæ.

Sic habes ſinum 78 item ſinum gr.18.

Illic 9,781,476
Hic 3,090,170

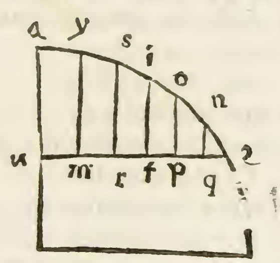
Ergo invicem ſublatis 6,691,306 erit pro ſinu gr.60—18 hoc eſt gr.42. quem laborioſius prædicto modo inveſtigaſſes.

Rurſus habes ſinus gr.66.& 54.

Illic 9,135,455
Hic 8,090,170

Ergo reliquus 1,045,285 erit pro ſinu gr. 6. quanto nempe intervallo alteruter datorum arcuum à gr.60 abeſt. Eodem modo in reliquis inducenti calcographo magnum ſyntaxeos compendium paratum eſt. jam vero pro complendo canone ſinus gr.1 & ſcrupuli 1 inveſtigandus eſt. idque ex collatione fiet. Si non præciſè inveniemus: inveniemus tamen ὡς ἔγγυσα.

Eſto in ſubjecta figura arcus a e gr.1. 30′. a i vero gr.0,45′. Et ſit a o gr.1 cujus ſinus quæritur. Et ſit ſinus a e quidem recta u e. arcus vero a i recta u t. quæritur ergo u p. dividatur a e in ſex æquales partes nempe quadrantes gradus unius. Datur autem ex inventione ſuperiori u t 130,896. u e vero 261,769. jam ſi dividas u t in tres æquales partes habebis 43,632 quę eſt major quam m r, vel r t, ideoque etiam major quam t p per 12.e.5. jam ſi addas u t



130,896
& 42, 622

habebis ſummam 174 520 majorem certe quam ſit u p ſinus unius gradus.

Rurſus quia u t eſt 130,896.& u e vero 261,769. erit t e 130,873. cujus tertia pars eſt $43,624\frac{1}{3}$ quam conſtat eodem prædicto modo eſſe minorem quam t p.

jam si addas ut 130,896
& 43,624

habebis summam 174,520 minorem sane quam sit u p gradus unius sinus.

Quare cum 174,528 sit major. 174,520 sit minor: medium certe eligamus oportet quo sine sensibili errore uti possimus nempe 174,524 pro sinu unius gradus.

Hinc jam ex superioribus viis ac modis innotescet sinus complementi 89. item sinus semissis 30′ & 44,30′ tum 89.30′ & 45 30′. rursus hinc sinus 15′ gr. 22,15′. gr. 44,45′ gr. 22,45′ gr. 89,45′ gr. 57,45. gr. 45,15′ gr. 67,15′. & sic deinceps.

Simili collatione maximè vicinorum colligetur sinus scrupuli unius 2,909 fere, parum enim deest. Hinc dabitur sinus duorum scrupulorum ex præcedenti methodo ut illic diminutionis sic hic auctionis viâ ex 20.e.5. Dehinc 4′ item 8′, 16′, 32′, & horum complementorum & semissium complementorum. Atq; simili inductione totus tandem perfectus canon est:, quem hisce constructum methodis scias.

Sequitur canonis hujus triangulorum pars altera, quæ vulgo canon fœcundus, nobis canon tangentium dicitur: & canon hypotenusarum Rhetico, nobis canon secantium vocatur.

Canon tangentium à Rheinholdo componitur per 24.e.5. nempe hoc ratiocinio: ut sinus arcus verbi gratia graduum 89 (hoc enim exemplo Erasmus utitur) nempe 9,998,477 ad sinum complementi nempe gradus unius 174,524, sic radius ad 174,551 tangentem gradus unius. Sed hoc tamen notandum est cum sinus gr. 1. non præcisè habeatur, nec hanc tangentem esse accuratam.

Canon secantium, seu ut à Rhetico vocatur canon hypotenusarum à Rheinholdo eodem loco componitur per 28.e.5. ut sinus arcus exempli gratia 89 nempe 9,998,477 ad radium sic radius ad 10,001,523 secantem gr. 1. ubi quod de tangente ante monui, idem hic non transeo.

Hæc

Hæc methodus eſt:ſed comparatio tangentis & ſecantis compendia largitur ea de quibus eſt 30 & 31.e.5.quæque ad ſyntaxin per ſolam numerationem primam plurimum conducunt:ſi ſuperiori methodo tangentes arcuum in promptu ſint. Sola enim tum ut dixi numeratio prima reliquam canonis triangulorum partem perficiet. Exempli gratia.

Tangens gr.1.eſt	174,550.	At ſemiſsis hujus
complementi eſt	9,826,974	tangens gr. $44\frac{1}{2}$
Ergo ſumma	10,001,524	eſt pro ſecante unius gradus.

Rurſus quia datur ſecans unius gradus jam inventa

	10,001,525.	& conſtat tan-
gens gr.1.	174,551	
Ergo ſumma	10,176,076	eſt pro tangente gr. $45\frac{1}{2}$

nempe datæ peripheriæ gr.1.auctæ ſemiſſe ſui complementi.

Et ſic ſimul canon tangentium canonem ſecantium componit,& ab eo viciſsim componitur:ſic neceſſe ſit. Sequitur de uſu qui tamen ex jam dictis notus eſſe poterat.

4. *Canonis exterior pars in margine ſupero partes peripheriæ quadrantis, in ſiniſtro partium ſcrupula prima continet : interea in area communi partis ſcrupuliq; rectas ſemicirculi.*

Et hoc elementum ſyntaxeos eſt. pars exterior nobis duobus membris perficitur margine ſupero & ſiniſtro. qui vulgo caput, frons,ſuprema pars tabulæ item limes & latus ſiniſtrum dicuntur.in hiſce itaq; marginibus canon ita continetur ut arcus in iis ſint totius quadrantis:ejuſq; gradus ſinguli nempe 90 ſingulorumq; minuta prima. gradus margine ſupero continentur,graduumq; ſcrupula ſiniſtro.

Interior pars vulgo venter tabulæ dicitur. ea areas continet lineis duplicibus diſtinctas tranſverſis & perpendicularibus. areæ illæ (quæ vulgo communis angulus, communis interſectio, profelis angularis dicuntur) continent in communi con-

cursu gradus scrupuliq; rectas semicirculi: nempe in canone sinuum sinus, in canone tangentium tangentes &c. ut si offeratur tibi arcus gr.20,30′ & cupias ejus quærere sinum. Quære in supero margine gr.20 in sinistro 30 jam area communis gr.20 & 30′ habet sinum 3,502,075. atq; hæc inventio vulgò laterali ingressu fieri dicitur. Quod si jam detur sinus 3,502,075 & cupias ejus arcum invenire: quod vulgò areali ingressu fieri dicitur. quæres in interiori parte donec invenias: nam inventus in supero margine ostendit gr.20 in sinistro 30 scrupula. Quare si datum in altera parte sit in reliqua erit quæsitum: si nempe præcisè detur statim prima inspectione quæsitum offertur. Sin minus sic quæres.

5. *Si datum in sua canonis parte præcisè non sit: differentiæ dati & proximè præcedentis è ratione differentiæ proximè vicinorum datorum ad differentiam suorũ quæsitorum conclusa pars proportionalis addita quæsito præcedenti, est dati quæsitum.*

Astrologi passim in suis libris de parte proportionali inquirenda præcepta inculcant: ejusq; prosthaphæresin variam docent pro ratione infinitarum suarum tabularum. Verum nobis sufficit ea proposuisse quæ præsenti canoni, tanquam reliquarum tabularum Phœbo inserviant. Esto igitur datus arcus gr. 20,30′,20″: Hujus quæritur sinus: quæritur tangẽs: quæritur secãs.

Datum in canone quæsitum, & quæsitum in sua parte nempe exteriori non invenitur præcisè. neq; igitur quæsita pręcisè aderunt. Quomodo ergo elicienda? Quæretur differentia dati & proximè ea in parte canonis pręcedentis. datum est gr. 20,30′,20″ proximè præcedens est gr.20,30′. Ergo differentia erit 20″.

Rursus quęratur differentia proximè vicinorum datorum. datum proximè sequens est. gr.20,31′. pręcedens gr.20,30′. Ergo differentia est 1 scrupuli primi: quanta semper in hoc canone est.

jam quærenda est differentia quæsitorum congruentium vicinis datis: & primo quidem in sinibus.

Sinus ergo ꝓximè sequẽtis dati nempe gr.20,31′ est 3,504,799. præce-

præcedentis nempe gr.20,30' est 3,502,075. Ergo differentia est 2,724. ut in sequenti typo patet.

Seq. dat. gr.20,31'	Datum ipsum gr. 20,30',20''	p.3,504,799
præ. dat gr.20,30	præced.dat. gr. 20,30	p.3,502,075
1'	20''	2,724

jam differentię 20'' est concludenda pars proportionalis è 2,724. Quare ut scrupulum 1 hoc est ut 60 secunda: nam terminos esse proximo genere homogeneos oportet: ut inquam 60 se habent ad 2,724: sic 20 se habebunt ad partem proportionalem quæsitam 908.

60 20 2,724 908

Hæc pars proportionalis addenda erit sinui proximè sequentis dati nempe 3,502,075 & summa 3,502,983 est sinus quæsitus peripheriæ nimirum gr.20,30',20''. Et sic sinus inventus est.

Tangens quoq; simili modo invenietur. retentis enim differentiis datorum: tangens grad. 20,31' est 3,742,164 at tangens gr.20,30. est 3,738,848.

Ergo differentia erit 3,316. ut sequens typus ob oculos ponit.

Seq.D. gr.20,31	Dat.gr. 20,30',20''	Q. 3,742,164
præ.D.gr.20,30	pręc.gr.20,30	Q. 3,738,848
1	20	3,316

Itaq; si 60 dant differentiam 3,316
tum 20 dabunt 1,105 $\frac{1}{3}$ proportionalem
partem addendam ad 3,738,848.
Et sic summa 3,739,953. erit pro tangente quęsita.

Eodem modo ex canone secantium dati arcus secans invenietur 10,676,483.

Cæterum in usu ii qui nec curiosi nimium nec nimis sunt scrupulosi radio contenti sunt 100,000. Quare & reliquis reliquorum arcuum sinibus, tangentibus, secantibus duas dextras fi-

guras sive notas præcidunt:ita tamen ut tertiæ addatur unitas si rejectæ duæ 50 superarint. Sic illis sinus gr. 20,30' est 35,021. nempe rejectis 7 & 5 à fine & addita ad o unitate.

Hoc ut in calculo non nimis accurato locum habere potest: ita tamen in sinibus non semper locum invenit: iis nempe qui radio non multo sunt minores. Fiet enim tum ut sinus idem diversis arcubus competat: exempli gratia radius 100,000 non tantum competet gr. 90 sed etiam gr. 89,59'. gr. 89,58'. grad. 89,57' usq; ad 50 scrupula prima. rursus sinus 99,999 non tantum erit gr. 89,49' sed & 89,42' atq; intermediorum.

Quare ut præcisio hæc admitti commode potest in tangentibus, secantibus, sinibus etiam quorum paulo major est differentia: sic in sinibus quorum differentiæ jam paulatim decrescendo ferè evanescunt locum habere non videtur.

Copernicus & Rheticus atque alii areales canonibus suis addiderunt differentias: addidit etiam Rheinholdus. Verum idem nobis esse faciendum non putavimus. Cum enim differentia non sit magna competens scilicet 1 scrupulo: non difficile erit eam prima inspectione invenire Arithmeticæ peritos. quales esse necesse est omnes eos qui hæc adire eorumque cognoscere usum volent. non, inquam, difficile erit: maximè si duæ præcidantur ab initio notæ. nec nimium chartarum excrescere cumulum voluimus. Quod si sunt qui omissas differentias non ferunt: ii ipsi differentias inquirant ac annotent. Et annotari quidem possunt inter ipsos numeros quorum sunt differentiæ tenui aliquo calamo: & sic semel inventæ semper esse usui possunt. Ac possunt hoc modo per ocium differentiæ inquiri. Quoties enim postulat exemplum inventionem sinuum: tum metho do jam dicta investigabis: & differentiam tum inventam annotabis. Sic successu temporis omnium omnino arearum differentias per lusum quasi cognosces & annotabis.

Atq; sic partis proportionalis inventio in ipsis areis quæsitum invenit: eadem erit ratio in marginibus.

Detur enim sinus 3,502,983.

Quæritur

Quæritur arcus ei competens

Sinus datus in canonis ſua parte nempe areis quæſitus non invenitur præciſè. Sed eo minor proximè eſt 3,502,075. Ergo differentia hujus à dato eſt 908.

Jam proximè ſequens ſinus eſt 3,504,799. ergo differentia hujus & proximè præcedentis eſt 2,724.

Tertio ſinui proximè ſequenti competit arcus gr. 20,31 præcedenti gr. 20,30. Ergo differentia eſt 1 ſcrup. primum. quanta ſemper hic eſt ut ſequens typus monſtrat.

Da. ſe. 3,504,799	Dat. 3,502,983	Q. 20,31
Da. pr. 3,502,075	pr. 3,502,075	Q. 20,30
2,724	908	1

Jam hinc concludenda eſt pars proportionalis congruens è 60 ſecundis differentiæ 908. Ergo ſi differentia datorum vicinorum 2,724 eſt ſecundorum 60, differentia 908 erit ſecundorum 20. pro parte p. quæſita.

2,724 908 60 20.

Jam ſi hæc addatur quæſito proximè præcedenti erit quæſitus arcus. grad. 20,30′,20″. Si modo arcus quærendus fuit quadrantis peripheria minor. Nam ſinus datus per 8. e. 5 competit etiam grad. 159,29′,40″ peripheriæ nempe reliquæ ad ſemiperipheriam.

Quod jam inſcriptarum & ſinuum ſecundorum inventionem attinet: ea his cognitis per 10, 17, & 18. e. 5 non eſt obſcura. Si enim inſcripta arcus gr. 41 & 40 ſecundorum quæratur: quæretur ſinus ſemiſsis nempe gr. 20,30′,20″. Sinus inventus 3,502,983 ſi duplicetur eſt quæſita inſcripta nempe 7,005,966.

Rurſus ſi quæratur arcus inſcriptæ 7,005,966 inſcriptæ dimidiæ tanquam ſinus 3,502,983 arcum quæro nempe gr. 20,30′,20″. is duplicatus eſt arcus quæſitus.

Quæ

Quæ sinus secundos attingunt: etiam facilia sunt. Sit quærendus sinus secundus arcus gr. 69,29′,40″. quæretur complementi nempe grad. 20,30′,20″ sinus 3,502,983. is enim de radio relinquit 6,497,017 pro sinu secundo quæsito.

Quod si sinus secundi arcus quæratur: sinus de radio relinquit sinum complementi arcus quæsiti.

Rursus si quęratur sinus secundus arcus grad. 110,30′,20″ per 18. e. 5. sinus complementi 3,502,983 cum radio est sinus secundus quæsitus.

Si sinus ejusmodi secundus radio minuatur relinquitur sinus arcus qui cum quadrante restituit arcum quæsitum.

Et hactenus Canonis syntaxis & usus: sequitur ipse Canon.

CANON

CANON SI-
nuum

RHETICO

Canon basis & perpendiculi
primus.

	0	1	2	3	4
0	0	174,524	348,995	523,360	697,565
1	2,909	177,433	351,902	526,265	700,467
2	5,818	180,341	354,809	529,170	703,369
3	8,727	183,250	357,716	532,075	706,270
4	11,636	186,158	360,623	534,980	709,172
5	14,544	189,066	363,530	537,884	712,073
6	17,453	191,975	366,437	540,789	714,975
7	20,362	194,883	369,344	543,694	717,876
8	23,271	197,792	372,251	546,598	720,777
9	26,180	200,700	375,158	549,503	723,678
10	29,088	203,608	378,064	552,407	726,579
11	31,997	206,517	380,971	555,312	729,480
12	34,906	209,425	383,878	558,216	732,381
13	37,815	212,333	386,785	561,120	735,282
14	40,724	215,241	389,692	564,024	738,183
15	43,632	218,149	392,598	566,928	741,084
16	46,541	221,057	395,505	569,832	743,985
17	49,450	223,965	398,412	572,736	746,886
18	52,359	226,873	401,318	575,640	749,787
19	55,268	229,781	404,225	578,544	752,688
20	58,177	232,689	407,131	581,448	755,588
21	61,086	235,597	410,038	584,352	758,489
22	63,995	238,505	412,944	587,256	761,389
23	66,904	241,413	415,851	590,160	764,290
24	69,813	244,321	418,757	593,064	767,180
25	72,721	247,229	421,663	595,967	770,090
26	75,630	250,137	424,570	598,871	772,991
27	78,539	253,045	427,476	601,775	775,891
28	81,448	255,953	430,382	604,678	778,791
29	84,357	258,861	433,288	607,582	781,691
30	87,265	261,769	436,194	610,485	784,591

	0	1	2	3	4
30	87,265	261,769	436,194	610,485	784,591
31	90,174	264,677	439,100	613,389	787,491
32	93,083	267,585	442,006	616,292	790,391
33	95,992	270,493	444,912	619,196	793,291
34	98,901	273,401	447,818	622,099	796,191
35	101,809	276,308	450,724	625,002	799,090
36	104,718	279,216	453,630	627,905	801,990
37	107,627	282,124	456,536	630,808	804,889
38	110,536	285,032	459,442	633,711	807,789
39	113,445	287,940	462,348	636,614	810,688
40	116,353	290,847	465,253	639,517	813,587
41	119,262	293,755	468,159	642,420	816,486
42	122,171	296,663	471,065	645,323	819,385
43	125,079	299,570	473,970	648,226	822,284
44	127,988	302,478	476,876	651,129	825,183
45	130,896	305,385	479,781	654,031	828,082
46	133,805	308,293	482,687	656,934	830,981
47	136,714	311,200	485,592	659,837	833,880
48	139,622	314,108	488,498	662,739	836,778
49	142,531	317,015	491,403	665,642	839,677
50	145,439	319,922	494,308	668,544	842,575
51	148,348	322,830	497,214	671,447	845,474
52	151,257	325,737	500,119	674,349	848,372
53	154,165	328,645	503,024	677,251	851,271
54	157,074	331,552	505,929	680,153	854,169
55	159,982	334,459	508,834	683,055	857,067
56	162,891	337,367	511,740	685,957	859,965
57	165,799	340,274	514,645	688,859	862,863
58	168,708	343,181	517,550	691,761	865,761
59	171,616	346,088	520,455	694,663	868,659
60	174,524	348,995	523,360	697,565	871,557

	5	6	7	8	9
0	871,557	1,045,285	1,218,693	1,391,731	1,564,345
1	874,455	1,048,178	1,221,580	1,394,612	1,567,218
2	877,353	1,051,071	1,224,467	1,397,492	1,570,091
3	880,250	1,053,964	1,227,354	1,400,373	1572,964
4	883,148	1,056,857	1,230,241	1,403,253	1,575,837
5	886,045	1,059,749	1,233,128	1,406,133	1,578,709
6	888,943	1,062,642	1,236,015	1,409,013	1,581,581
7	891,840	1,065,534	1,238,901	1,411,893	1,584,453
8	894,737	1,068,426	1,241,788	1,414,772	1,587,325
9	897,634	1,071,318	1,244,674	1,417,652	1,590,197
10	900,531	1,074,210	1,247,560	1,420,531	1,593,069
11	903,428	1,077,102	1,250,446	1,423,410	1,595,941
12	906,325	1,079,994	1,253,332	1,426,289	1,598,812
13	909,222	1,082,886	1,256,218	1.429,168	1,601,684
14	912,119	1,085,778	1,259,104	1,432,047	1,604,555
15	915,016	1,088,669	1,261,990	1,434,926	1,607,426
16	917,913	1,091,561	1,264,876	1,437,805	1,610,297
17	920,809	1,094,452	1,267,761	1,440,684	1,613,168
18	923,706	1,097,344	1,270,647	1,443,562	1,616,038
19	926,602	1,100,235	1,273,532	1,446,441	1,618,909
20	929,498	1,103,126	1,276,417	1,449,319	1,621,779
21	932,395	1,106,017	1,279,302	1,452,197	1,624,649
22	935,291	1,108,908	1,282,187	1,455,075	1,627,519
23	938,187	1,111,799	1,285,072	1,457,953	1,630,389
24	941,083	1,114,690	1,287,957	1,460,831	1,633,259
25	943,979	1,117,580	1,290,841	1,463,708	1,636,129
26	946,875	1,120,471	1,293,726	1,466,586	1,638,999
27	949,771	1,123,361	1,296,610	1,469,463	1,641,868
28	952,667	1,126,252	1,299,494	1,472,340	1,644,738
29	955,563	1,129,142	1,302,378	1,475,217	1,647,607
30	958,458	1,132,032	1,305,262	1,478,094	1,650,476

	5	6	7	8	9
30	958,458	1,132,032	1,305,262	1,478,094	1,650,476
31	961,354	1,134,922	1,308,146	1,480,971	1,653,345
32	964,249	1,137,812	1,311,030	1,483,848	1,656,214
33	967,144	1,140,702	1,313,914	1,486,724	1,659,082
34	970,039	1,143,592	1,316,798	1,489,601	1,661,951
35	972,934	1,146,482	1,319,681	1,492,477	1,664,819
36	975,829	1,149,372	1,322,564	1,495,353	1,667,687
37	978,724	1,152,261	1,325,447	1,498,229	1,670,555
38	981,619	1,155,151	1,328,330	1,501,105	1,673,423
39	984,514	1,158,040	1,331,213	1,503,981	1,676,291
40	987,408	1,160,929	1,334,096	1,506,857	1,679,159
41	990,303	1,163,818	1,336,979	1,509,733	1,682,027
42	993,198	1,166,707	1,339,862	1,512,608	1,684,894
43	996,092	1,169,596	1,342,744	1,515,484	1,687,761
44	998,987	1,172,485	1,345,627	1,518,359	1,690,628
45	1,001,881	1,175,374	1,348,509	1,521,234	1,693,495
46	1,004,775	1,178,263	1,351,392	1,524,109	1,696,362
47	1,007,669	1,181,151	1,354,274	1,526,984	1,699,229
48	1,010,563	1,184,040	1,357,156	1,529,859	1,702,095
49	1,013,457	1,186,928	1,360,038	1,532,734	1,704,962
50	1,016,351	1,189,816	1,362,920	1,535,608	1,707,828
51	1,019,245	1,192,704	1,365,802	1,538,482	1,710,694
52	1,022,139	1,195,592	1,368,683	1,541,356	1,713,560
53	1,025,032	1,198,480	1,371,564	1,544,230	1,716,426
54	1,027,926	1,201,368	1,374,446	1,547,104	1,719,292
55	1,030,819	1,204,255	1,377,327	1,549,978	1,722,157
56	1,033,713	1,207,143	1,380,208	1,552,852	1,725,022
57	1,036,606	1,210,031	1,383,089	1,555,725	1,727,887
58	1,039 499	1,212,918	1,385,970	1,558,599	1,730,752
59	1,042,392	1,215,806	1,388,851	1,561,472	1,733,617
60	1,045,285	1,218,693	1,391,731	1,564,345	1,736,482

	10	11	12	13	14.
0	1,736,482	1,908,090	2,079,117	2,249,511	2,419,219
1	1,739,347	1,910,945	2,081,962	2,252,345	2,422,041
2	1,742,211	1,913,800	2,084,807	2,255,179	2,424,863
3	1,745,075	1,916,655	2,087,652	2,258,013	2,427,685
4	1,747,939	1,919,510	2,090,497	2,260,847	2,430,507
5	1,750,803	1,922,365	2,093,342	2,263,680	2,433,329
6	1,753,667	1,925,220	2,096,180	2,266,513	2,436,150
7	1,756,531	1,928,074	2,099,030	2,269,346	2,438,971
8	1,759,394	1,930,928	2,101,874	2,272,179	2,441,792
9	1,762,258	1,933,782	2,104,718	2,275,012	2,444,613
10	1,765,121	1,936,636	2,107,562	2,277,844	2,447,434
11	1,767,984	1,939,490	2,110,405	2,280,676	2,450,254
12	1,770,847	1,942,344	2,113,248	2,283,508	2,453,074
13	1,773,710	1,945,197	2,116,091	2,286,340	2,455,894
14	1,776,573	1,948,050	2,118,934	2,289,172	2,458,714
15	1,779,435	1,950,903	2,121,777	2,292,004	2,461,533
16	1,782,298	1,953,756	2,124,620	2,294,835	2,464,352
17	1,785,160	1,956,609	2,127,462	2,297,666	2,467,171
18	1,788,022	1,959,462	2,130,304	2,300,497	2,469,990
19	1,790,884	1,962,314	2,133,146	2,303,328	2,472,809
20	1,793,746	1,965,166	2,135,988	2,306,159	2,475,628
21	1,796,608	1,968,018	2,138,830	2,308,949	2,478,446
22	1,799,469	1,970,870	2,141,671	2,311,819	2,481,264
23	1,802,331	1,973,722	2,144,512	2,314,649	2,484,082
24	1,805,192	1,976,574	2,147,353	2,317,479	2,486,900
25	1,808,053	1,979,425	2,150,194	2,320,309	2,489,717
26	1,810,914	1,982,276	2,153,035	2,323,138	2,492,534
27	1,813,774	1,985,127	2,155,876	2,325,967	2,495,351
28	1,816,634	1,987,978	2,158,716	2,328,799	2,498,168
29	1,819,495	1,990,829	2,161,556	2,331,625	2,500,984
30	1,822,355	1,993,679	2,164,396	2,334,454	2,503,800

	10	11	12	13	14
30	1,822,355	1,993,679	2,164,396	2,334,454	2,503,800
31	1,825,215	1,996,530	2,167,236	2,337,282	2,506,626
32	1,828,075	1,999,380	2,170,076	2,340,110	2,509,431
33	1,830,935	2,002,230	2,172,916	2,342,938	2,512,248
34	1,833,795	2,005,080	2,175,755	2,345,766	2,515,064
35	1,836,654	2,007,930	2,178,594	2,348,594	2,517,879
36	1,839,513	2,010,780	2,181,433	2,351,421	2,520,694
37	1,842,372	2,013,629	2,184,272	2,354,248	2,523,509
38	1,845,231	2,016,478	2,187,111	2,357,075	2,526,324
39	1,848,090	2,019,327	2,189,949	2,359,902	2,529,138
40	1,850,949	2,022,176	2,192,787	2,362,729	2,531,952
41	1,853,808	2,025,025	2,195,625	2,365,555	2,524,766
42	1,856,666	2,027,874	2,198,463	2,368,381	2,537,580
43	1,859,524	2,030,722	2,201,300	2,371,207	2,540,393
44	1,862,382	2,033,570	2,204,137	2,374,033	2,543,206
45	1,865,240	2,036,418	2,206,974	2,376,859	2,546,019
46	1,868,098	2,039,266	2,209,811	2,379,684	2,548,832
47	1,870,956	2,042,114	2,212,648	2,382,509	2,551,645
48	1,873,813	2,044,962	2,215,485	2,385,334	2,554,458
49	1,876,670	2,047,809	2,218,322	2;388,159	2,557,270
50	1,879,527	2,050,656	2,221,158	2,390,983	2,560,082
51	1,882,384	2,053,503	2,223,994	2,393,808	2,562,894
52	1,885,241	2,056,350	2,226,830	2,396,632	2,565,706
53	1,888,098	2,059,197	2,229,666	2,399,456	2,568,517
54	1,890,954	2,062,043	2,232,502	2,402,280	2,571,328
55	1,893,810	2,064,889	2,235,337	2,405,104	2,574,139
56	1,896,666	2,067,735	2,238,172	2,407,927	2,576,950
57	1,899,522	2,070,581	2,241,007	2,410,750	2,579,760
58	1,902,378	2,073,427	2,243,842	2,413,573	2,582,570
59	1,905,234	2,076,272	2,246,677	2,416,396	2,585,380
60	1,908,090	2,079,117	2,249,511	2,419,219	2,588,190

	15	16	17	18	19
0	2,588,190	2,756,373	2,923,717	3,090,170	3,255,682
1	2,591,000	2,759,169	2,926,499	3,092,936	3,258,432
2	2,593,809	2,761,965	2,929,280	3,095,702	3,261,182
3	2,596,618	2,764,761	2,932,061	3,098,468	3,263,931
4	2,599,427	2,767,556	2,934,842	3,101,234	3,266,681
5	2,602,236	2,770,351	2,937,623	3,103,999	3,269,430
6	2,605,045	2,773,146	2,940,403	3,106,764	3,272,179
7	2,607,853	2,775,941	2,943,183	3,109,529	3,274,927
8	2,610,661	2,778,735	2,945,963	3,112,294	3,277,675
9	2,613,469	2,781,529	2,948,743	3,115,058	3,280,423
10	2,616,277	2,784,323	2,951,523	3,117,822	3,283,171
11	2,619,084	2,787,117	2,954,302	3,120,586	3,285,918
12	2,621,891	2,789,911	2,957,081	3,123,349	3,288,665
13	2,624.698	2,792.704	2,959,860	3,126,112	3,291,412
14	2,627,505	2,795,497	2,962,630	3,128,875	3,294,159
15	2,630,312	2,798,290	2,965,416	3,131,638	3,296,906
16	2,633,118	2,801,082	2,968,194	3,134,400	3,299,652
17	2.635.924	2,803,874	2,970,972	3,137,162	3,302,398
18	2,638,730	2,806,666	2,973,750	3,139,924	3,305,144
19	2,641,536	2,809,458	2,976,527	3,142,686	3,307,889
20	2,644,342	2,812,250	2,979,304	3,145,448	3,310,634
21	2,647,147	2,815,041	2,982,081	3,148,209	3,313,379
22	2,649,952	2,817,832	2,984,857	3,150,970	3,316,123
23	2,652,757	2,820,623	2,987,633	3,153,731	3,318,867
24	2,655,562	2,823.414	2,990,409	3,156,491	3,321,611
25	2,658.366	2,826 244	2,993,185	3,159,251	3,324.355
26	2 661,170	2,828,994	2,995.960	3,162,011	3,327,098
27	2,663,974	2,831,784	2,998,735	3.164,770	3.329,841
28	2,666,777	2,834.574	3,001,510	3,167,529	3,332,585
29	2,669,580	2,837,364	3,004,284	3 170,288	3,335.327
30	2,672,383	2,840,153	3,007.058	3,173,047	3.338,069

	15	16	17	18	19
30	2,672,383	2,840,153	3,007,058	3,173,047	3,338,069
31	2,675,186	2,842,942	3,009,832	3,175,805	3,340,811
32	2,677,989	2,845,731	3,012,606	3,178,563	3,343,553
33	2,680,792	2,848,520	3,015,380	3,181,321	3,346,294
34	2,683,595	2,851,308	3,018,153	3,184,079	3,349,035
35	2,686,397	2,854,096	3,020,926	3,186,837	3,351,776
36	2,689,199	2,856,884	3,023,699	3,189,594	3,354,516
37	2,692,001	2,859,672	3,026,472	3,192,351	3,357,256
38	2,694,802	2,862,459	3,029,244	3,195,108	3,359,996
39	2,697,603	2,865,246	3,032,016	3,197,864	3,362,736
40	2,700,404	2,868,033	3,034,788	3,200,620	3,365,475
41	2,703,205	2,870,819	3,037,559	3,203,375	3,368,214
42	2,706,005	2,873,605	3,040,330	3,206,130	3,370,953
43	2,708,805	2,876,391	3,043,101	3,208,885	3,373,691
44	2,711,605	2,879,177	3,045,872	3,211,640	3,376,429
45	2,714,405	2,881,963	3,048,643	3,214,395	3,379,167
46	2,717,204	2,884,748	3,051,413	3,217,150	3,381,905
47	2,720,003	2,887,533	3,054,183	3,219,904	3,384,642
48	2,722,802	2,890,318	3,056,953	3,222,658	3,387,379
49	2,725,601	2,893,103	3,059,723	3,225,412	3,390,116
50	2,728,400	2,895,888	3,062,492	3,228,165	3,392,852
51	2,731,198	2,898,672	3,065,261	3,230,918	3,395,588
52	2,733,996	2,901,456	3,068,030	3,233,671	3,398,324
53	2,736,794	2,904,240	3,070,798	3,236,423	3,401,060
54	2,739,592	2,907,023	3,073,566	3,239,175	3,403,795
55	2,742,389	2,909,806	3,076,334	3,241,927	3,406,530
56	2,745,186	2,912,589	3,079,102	3,244,679	3,409,265
57	2,747,983	2,915,371	3,081,869	3,247,430	3,411,999
58	2,750,780	2,918,153	3,084,636	3,250,181	3,414,733
59	2,753,577	2,920,935	3,087,403	3,252,932	3,417,467
60	2,756,373	2,923,717	3,090,170	3,255,682	3,420,201

	20	21	22	23	24
0	3,420,201	3,583,679	3,746,066	3,907,311	4,067,366
1	3,422,934	3,586,395	3,748,763	3,909,989	4,070,023
2	3,425,667	3,589,110	3,751,460	3,912,666	4,072,680
3	3,428,400	3,591,825	3,754,156	3,915,343	4,075,337
4	3,431,133	3,594,540	3,756,852	3,918,020	4,077,993
5	3,433,865	3,597,254	3,759,548	3,920,696	4,080,649
6	3,436,597	3,599,968	3,762,243	3,923,372	4,083,305
7	3,439,329	3,602,682	3,764,938	3,926,048	4,085,960
8	3,442,060	3,605,395	3,767,633	3,928,723	4,088,615
9	3,444,791	3,608,108	3,770,327	3,931,398	4,091,269
10	3,447,522	3,610,821	3,773,021	3,934,072	4,093,923
11	3,450,253	3,613,533	3,775,715	3,936,746	4,096,577
12	3,452,983	3,616,245	3,778,408	3,939,420	4,099,231
13	3,455,713	3,618,957	3,781,101	3,942,093	4,101,884
14	3,458,442	3,621,669	3,783,794	3,944,766	4,104,537
15	3,461,171	3,624,380	3,786,486	3,947,439	4,107,189
16	3,463,900	3,627,091	3,789,178	3,950,112	4,109,841
17	3,466,629	3,629,802	3,791,870	3,952,784	4,112,493
18	3,469,357	3,632,512	3,794,562	3,955,456	4,115,144
19	3,472,085	3,635,222	3,797,253	3,958,128	4,117,795
20	3,474,813	3,637,932	3,799,944	3,960,799	4,120,446
21	3,477,540	3,640,642	3,802,635	3,963,470	4,123,096
22	3,480,267	3,643,351	3,805,325	3,966,140	4,125,746
23	3,482,994	3,646,060	3,808,015	3,968,810	4,128,395
24	3,485,721	3,648,768	3,810,704	3,971,480	4,131,044
25	3,488,447	3,651 476	3,813,393	3,974,149	4,133,693
26	3,491,173	3,654,184	3,816,082	3,976,818	4,136,341
27	3,493,899	3,656,892	3,818,771	3,979,487	4,138,989
28	3,496,624	3,659,599	3,821,459	3,982,155	4,141,637
29	3,499,343	3,662,306	3,824,147	3,984,823	4,144,285
30	3,502,075	3,665,012	3,826,834	3,987,491	4,146,932

	20	21	22	23	24
30	3,502,075	3,665,012	3,826,834	3,987,491	4,146,932
31	3,504,799	3,667,718	3,829,521	3,990,159	4,149,579
32	3,507,523	3,670,424	3,832,208	3,992,826	4,152,226
33	3,510,247	3,673,130	3,834,895	3,995,493	4,154,872
34	3,512,971	3,765,835	3,837,581	3,998,159	4,157,518
35	3,515,694	3,678,541	3,840,267	4,000,825	4,160,163
36	3,518,417	3,681,246	3,842,953	4,003,491	4,162,808
37	3,521,140	3,683,951	3,845,638	4,006,156	4,165,453
38	3,523,862	3,686,655	3,848,323	4,008,821	4,168,097
39	3,526,584	3,689,359	3,851,008	4,011,486	4,170,741
40	3,529,306	3,692,062	3,853,692	4,014,150	4,173,385
41	3,532,027	3,694,765	3,856,376	4,016,814	4,176,028
42	3,534,748	3,697,468	3,859,060	4,019,478	4,178,671
43	3,537,469	3,700,170	3,861,743	4,022,141	4,181,313
44	3,540,190	3,702,872	3,864,426	4,024,804	4,183,955
45	3,542,910	3,705,574	3,867,109	4,026,467	4,186,597
46	3,545,630	3,708,276	3,869,791	4,030,130	4,189,239
47	3,548,350	3,710,977	3,872,473	4,032,792	4,191,880
48	3,551,070	3,713,678	3,875,155	4,035,454	4,194,521
49	3,553,789	3,716,379	3,877,837	4,038,115	4,197,162
50	3,556,508	3,719,080	3,880,518	4,040,776	4,199,802
51	3,559,227	3,721,780	3,883,199	4,043,437	3,202,442
52	3,561,945	3,724,480	3,885,880	4,046,097	4,205,081
53	3,564,663	3,727,179	3,888,560	4,048,757	4,207,720
54	3,567,380	3,729,878	3,891,240	4,051,416	4,210,359
55	3,570,097	3,732,577	3,893,919	4,054,075	4,212,997
56	3,572,814	3,735,275	3,896,598	4,056,734	4,215,635
57	3,575,531	3,737,973	3,899,277	4,059,392	4,218,273
58	3,578,247	3,740,671	3,901,955	4,062,050	4,220,910
59	3,580,963	3,743,369	3,904,633	4,064,708	4,223,547
60	3,583,679	3,746,066	3,907,311	4,067,366	4,226,183

	25	26	27	28	29
0	4,226,183	4,383,712	4,539,905	4,694,716	4,848,090
1	4,228,819	4,386,326	4,542,497	4,697,284	4,850,640
2	4,231,455	4,388,940	4,545,088	4,699,852	4,853,184
3	4,234,090	4,391,554	4,547,679	4,702,419	4,855,727
4	4,236,725	4,394,167	4,540,270	4,704,986	4,858,270
5	4,239,360	4,396,780	4,552,860	4,707,553	4,860,812
6	4,241,994	4,399,392	4,555,450	4,710,119	4,863,354
7	4,244,628	4,402,004	4,558,039	4,712,685	4,865,895
8	4,247,262	4,404,616	4,560,628	4,715,250	4,868,436
9	4,249,895	4,407,227	4,563,216	4,717,815	4,870,977
10	4,252,528	4,409,838	4,565,804	4,720,380	4,873,517
11	4,255,161	4,412,449	4,568,392	4,722,944	4,876,057
12	4,257,793	4,415,059	4,570,979	4,725,508	4,878,596
13	4,260,425	4,417,669	4,573,566	4,728,071	4,881,135
14	4,263,056	4,420,278	4,576,153	4,730,634	4,883,674
15	4,265,687	4,422,887	4,578,739	4,733,197	4,886,212
16	4,268,318	4,425,496	4,581,325	4,735,759	4,888,750
17	4,270,949	4,428,104	4,583,911	4,738,321	4,891,287
18	4,273,579	4,430,712	4,586,496	4,740,882	4,893,824
19	4,276,209	4,433,320	4,589,081	4,743,443	4,896,361
20	4,278,838	4,435,927	4,591,665	4,746,004	4,898,897
21	4,281,467	4,438,534	4,594,249	4,748,564	4,901,433
22	4,284,096	4,441,140	4,596,833	4,751,124	4,903,968
23	4,286,724	4,443,746	4,599,416	4,753,683	4,906,503
24	4,289,352	4,446,352	4,601,999	4,756,242	4,909,037
25	4,291,979	4,448,957	4,604,581	4,758,801	4,911,571
26	4,294,606	4,451,562	4,607,163	4,761,359	4,914,105
27	4,297,233	4,454,167	4,609,744	4,763,917	4,916,638
28	4,299,859	4,456,771	4,612,325	4,766,474	4,919,171
29	4,302,485	4,459,375	4,614,906	4,769,031	4,921,703
30	4,305,111	4,461,978	4,617,486	4,771,588	4,924,235

	25	26	27	28	29
30	4,305,111	4,461,978	4,617,486	4,771,588	4,924,235
31	4,307,736	4,464,581	4,620,066	4,774,144	4,926,767
32	4,310,361	4,467,184	4,622,646	4,776,700	4,929,298
33	4,312,986	4,469,786	4,625,225	4,779,255	4,931,829
34	4,315,610	4,472,388	4,627,804	4,781,810	4,934,359
35	4,318,234	4,474,990	4,630,382	4,784,365	4,936,889
36	4,320,858	4,477,591	4,632,960	4,786,919	4,939,418
37	4,323,481	4,480,192	4,635,538	4,789,473	4,941,947
38	4,326,104	4,482,792	4,638,115	4,792,026	4,944,476
39	4,328,726	4,485,392	4,640,692	4,794,579	4,947,004
40	4,331,348	4,487,992	4,643,268	4,797,132	4,949,532
41	4,333,970	4,490,591	4,645,844	4,799,684	4,952,059
42	4,336,591	4,493,190	4,648,420	4,802,236	4,954,586
43	4,339,212	4,495,788	4,650,995	4,804,787	4,957,113
44	4,341,833	4,498,386	4,653,570	4,807,338	4,959,639
45	4,344,453	4,500,984	4,656,145	4,809,888	4,962,165
46	4,347,073	4,503,582	4,658,719	4,812,438	4,964,690
47	4,349,693	4,506,179	4,661,293	4,814,988	4,967,215
48	4,352,312	4,508,776	4,663,866	4,817,537	4,969,740
49	4,354,931	4,511,372	4,666,439	4,820,086	4,972,264
50	4,357,549	4,513,968	4,669,012	4,822,635	4,974,788
51	4,360,167	4,516,563	4,671,584	4,825,183	4,977,311
52	4,362,785	4,519,158	4,674,156	4,827,731	4,979,834
53	4,365,402	4,521,753	4,676,727	4,830,278	4,982,356
54	4,368,019	4,524,347	4,679,298	4,832,825	4,984,878
55	4,370,635	4,526,941	4,681,869	4,835,371	4,987,399
56	4,373,251	4,529,535	4,684,439	4,837,917	4,989,920
57	4,375,867	4,532,128	4,687,009	4,840,462	4,992,441
58	4,378,482	4,534,721	4,689,578	4,843,007	4,994,961
59	4,381,097	4,537,313	4,692,147	4,845,552	4,997,481
60	4,383,712	4,539,905	4,694,716	4,848,096	5,000,000

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	30	31	32	33.	34
0	5,000,000	5,150,381	5,299,192	5,446,390	5,591,929
1	5,002,519	5,152,874	5,301,659	5,448,829	5,594,340
2	5,005,038	5,155,367	5,304,125	5,451,268	5,596,751
3	5,007,556	5,157,859	5,306,591	5,453,707	5,599,161
4	5,010,074	5,150,351	5,309,056	5,456,145	5,601,571
5	5,012,591	5,162,843	5,311,521	5,458,583	5,603,981
6	5,015,108	5,165,334	5,313,985	5,461,020	5,606,390
7	5,017,624	5,167,825	5,316,449	5,463,456	5,608,798
8	5,020,140	5,170,315	5,318,913	5,465,802	5,611,206
9	5,022,656	5,172,805	5,321,376	5,468,328	5,613,614
10	5,025,171	5,175,294	5,323,839	5,470,763	5,616,021
11	5,027,686	5,177,783	5,326,301	5,473,198	5,618,427
12	5,030,200	5,180,271	5,328,763	5,475,632	5,620,833
13	5,032,714	5,182,759	5,331,224	5,478,066	5,623,239
14	5,035,227	5,185,246	5,333,685	5,480,499	5,625,644
15	5,037,740	5,187,733	5,336,145	5,482,932	5,628,049
16	5,040,253	5,190,220	5,338,605	5,485,364	5,630,453
17	5,042,765	5,192,706	5,341,065	5,487,796	5,632,857
18	5,045,277	5,195,192	5,343,524	5,490,228	5,635,260
19	5,047,788	5,197,667	5,345,983	5,492,659	5,637,663
20	5,050,299	5,200,162	5,348,441	5,495,090	5,640,066
21	5,052,809	5,202,646	5,350,898	5,497,520	5,642,468
22	5,055,319	5,205,130	5,353,355	5,499,950	5,644,869
23	5,057,829	5,207,614	5,355,812	5,502,379	5,647,270
24	5,060,338	5,210,097	5,358,268	5,504,808	5,649,670
25	5,062,847	5,212,580	5,360,724	5,507,236	5,652,070
26	5,065,355	5,215,062	5,363,179	5,509,664	5,654,469
27	5,067,863	5,217,544	5,365,634	5,512,091	5,656,868
28	5,070,370	5,220,025	5,368,088	5,514,518	5,659,266
29	5,072,877	5,222,506	5,370,542	5,516,944	5,661,664
30	5,075,384	5,224,986	5,372,996	5,519,370	5,664,062

	30	31	32	33	34
30	5,075,384	5,224,986	5,372,996	5,519,370	5,664,062
31	5,077,890	5,227,466	5,375,449	5,521,795	5,666,459
32	5,080,396	5,229,946	5,377,902	5,524,220	5,668,856
33	5,082,901	5,232,425	5,380,354	5,526,645	5,671,252
34	5,085,406	5,234,904	5,382,806	5,529,069	5,673,648
35	5,087,911	5,237,382	5,385,258	5,531,493	5,676,043
36	5,090,415	5,239,860	5,387,709	5,533,916	5,678.438
37	5,092,919	5,242,337	5,390,159	5,536,338	5,680,832
38	5,095,422	5,244,814	5,392,609	5,538,760	5,683,226
39	5,097,925	5,247,290	5,395,058	5,541,182	5,685,619
40	5,100,427	5,249,766	5,397,507	5,543,603	5,688,012
41	5,102,929	5,252,241	5,399,855	5,546,024	5,690.404
42	5,105,430	5,254,716	5,402,403	5,548,444	5,792,796
43	5,107,931	5,257,191	5,404,851	5,550,864	5,695,187
44	5,110,431	5,259,665	5,407,298	5,553,283	5,697,578
45	5,112,931	5,262,139	5,409,745	5,555,702	5,699,968
46	5,115,431	5,264,612	5,412,191	5,558,120	5,702,358
47	5,117,930	5,267,085	5,414,637	5,560,538	5,704,747
48	5,120,429	5,269,557	5,417,082	5,562,956	5,707,136
49	5,122,927	5,272,029	5,419,527	5,565,373	5,709,524
50	5,125,425	5,274,501	5,421,972	5,567,790	5,711,912
51	5,127,922	5,276,972	5,424,416	5,570,206	5,714,289
52	5,130,419	5,279,443	5,426,859	5,572,622	5,716,686
53	5,132,916	5,281,913	5,429,302	5,575,037	5,719,072
54	5,135,412	5,284,383	5,431,745	5,577,452	5,721,458
55	5,137,908	5,286,852	5.434 187	5,579,866	5,723,844
56	5,140,403	5,299,321	5,436,629	5,582,280	5,726,229
57	5,142,898	5,291,789	5,439,070	5,584,693	5,728,613
58	5,145,393	5,294,257	5,441,510	5,587,106	5,730,997
59	5,147,887	5,296,725	5,443,950	5,589,518	5,733,381
60	5,150,381	5,299,192	5.446.390	5,591,929	5,735,764

	35	36	37	38	39
0	5,735,764	5,877,852	6,018,150	6,156,615	6,293,204
1	5,738,147	5,880,205	6,020,473	6,158,907	6,295,464
2	5,740,529	5,882,558	6,022,796	6,161,198	6,297,724
3	5,742,911	5,884,910	6,025,118	6,163,489	6,299,983
4	5,745,292	5,887,262	6,027,439	6,165,781	6,302,242
5	5,747,672	5,889,613	6,029,760	6,168,070	6,304,501
6	5,750,052	5,891,964	6,032,080	6,170,259	6,306,759
7	5,752,432	5,894,314	6,034,400	6,172,648	6,309,016
8	5,754,811	5,896,664	6,036,719	6,174,936	6,311,273
9	5,757,190	5,899,013	6,039,038	6,177,224	6,313,529
10	5,759,568	5,901,361	6,041,357	6,179,512	6,315,784
11	5,761,946	5,903,709	6,043,675	6,181,799	6,318,039
12	5,764,323	5,906,056	6,045,992	6,184,085	6,320,293
13	5,766,700	5,908,403	6,048,309	6,186,371	6,322,547
14	5,769,076	5,910,750	6,050,625	6,188,656	6,324,800
15	5,771,452	5,913,096	6,052,940	6,190,940	6,327,053
16	5,773,827	5,915,442	6,055,255	6,193,224	6,329,305
17	5,776,202	5,917,787	6,057,570	6,195,508	6,331,557
18	5,778,576	5,920,132	6,059,884	6,197,791	6,333,808
19	5,780,950	5,922,476	6,062,198	6,200,074	6,336,059
20	5,783,324	5,924,820	6,064,511	6,202,356	6,338,310
21	5,785,697	5,927,163	6,066,824	6,204,638	6,340,560
22	5,788,069	5,929,505	6,069,136	6,206,919	6,342,809
23	5,790,441	5,931,847	6,071,448	6,209,199	6,345,058
24	5,792,812	5,934,189	6,073,759	6,211,479	6,347,309
25	5,795,183	5,936,530	6,076,069	6,213,758	6,349,553
26	5,797,553	5,938,871	6,078,379	6,216,037	6,351,800
27	5,799,923	5,941,211	6,080,688	6,218,315	6,354,046
28	5,802,292	5,943,551	6,082,997	6,220,593	6,356,292
29	5,804,661	5,945,890	7,085,306	6,222,870	6,358,537
30	5,807,030	5,948,228	6,087,614	6,225,146	6,360,782

	35	36	37	38	39
30	5,807,030	5,948,228	6,087,614	6,225,146	6,360,782
31	5,809,398	5,950,566	6,089,922	6,227,422	6,363,026
32	5,811,766	5,952,904	6,092,229	6,229,698	6,365,270
33	5,814,133	5,955,241	6,094,536	6,231,973	6,367,513
34	5,816,499	5,957,578	6,096,842	6,234,248	6,369,756
35	5,818,865	5,959,914	6,099,147	6,236,522	6,371,999
36	5,821,230	5,962,250	6,101,452	6,238,796	6,374,241
37	5,823,595	5,964,585	6,103,756	6,241,069	6,376,482
38	5,825,959	5,966,919	6,106,060	6,243,342	6,378,722
39	5,828,323	5,969,253	6,108,364	6,245,614	6,380,962
40	5,830,687	5,971,586	6,110,667	6,247,885	6,383,201
41	5,833,050	5,973,919	6,112,970	6,250,156	6,385,440
42	5,835,412	5,976,251	6,115,272	6,252,426	6,387,678
43	5,837,774	5,978,583	6,117,573	6,254,696	6,389,916
44	5,840,136	5,980,915	6,119,873	6,256,966	6,392,153
45	5,842,497	5,983,246	6,122,173	6,259,235	6,394,390
46	5,844,858	5,985,577	6,124,473	6,261,503	6,396,626
47	5,847,218	5,987,907	6,126,772	6,263,771	6,398,862
48	5,849,578	5,990,237	6,129,071	6,266,038	6,401,097
49	5,851,937	5,992,566	6,131,369	6,268,305	6,403,332
50	5,854,295	5,994,894	6,133,667	6,270,572	6,405,566
51	5,856,653	5,997,222	6,135,964	6,272,838	6,407,799
52	5,859,010	5,999,549	6,138,261	6,275,103	6,410,032
53	5,861,367	6,001,876	6,140,557	6,277,368	6,412,264
54	5,863,724	6,004,202	6,142,853	6,279,632	6,414,496
55	5,866,080	6,006,528	6,145,148	6,281,895	6,416,728
56	5,868,436	6,008,853	6,147,442	6,284,158	6,418,959
57	5,870,791	6,011,178	6,149,746	6,286,420	6,421,189
58	5,873,145	6,013,502	6,152,030	6,288,682	6,423,419
59	5,875,499	6,015,826	6,154,323	6,290,943	6,425,648
60	5,877,852	6,018,150	6,156,615	6,293,204	6,427,876

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	40	41	42	43	44
0	6,427,876	6,560,590	6,691,306	6,819,984	6,946,584
1	6,430,104	6,562,785	6,693,468	6,822,111	6,948,676
2	6,432,331	6,564,979	6,695,629	6,824,237	6,950,767
3	6,434,558	6,567,173	6,697,789	6,826,363	6,952,858
4	6,436,785	6,569,367	6,699,949	6,828,489	6,954,949
5	6,439,011	6,571,560	6,702,108	6,830,614	6,957,039
6	6,441,236	6,573,753	6,704,267	6,832,738	6,959,128
7	6,443,461	6,575,945	6,706,425	6,834,861	6,961,216
8	6,445,685	6,578,136	6,708,582	6,836,984	6,963,304
9	6,447,909	6,580,326	6,710,739	6,839,107	6,965,392
10	6,450,132	6,582,516	6,712,895	6,841,229	6,967,479
11	6,452,355	6,584,705	6,715,051	6,843,350	6,969,565
12	6,454,577	6,586,894	6,717,206	6,845,471	6,971,651
13	6,456,799	6,589,082	6,719,361	6,847,591	6,973,736
14	6,459,020	6,591,270	6,721,515	6,849,711	6,975,821
15	6,461,240	6,593,458	6,723,668	6,851,830	6,977,905
16	6,463,460	6,595,645	6,725,821	6,853,949	6,979,988
17	6,465,679	6,597,831	6,727,973	6,856,067	6,982,071
18	6,467,898	6,600,016	6,730,125	6,858,184	6,984,153
19	6,470,116	6,602,201	6,732,276	6,860,301	6,986,235
20	6,472,333	6,604,386	6,734,427	6,862,417	6,988,316
21	6,474,550	6,606,570	6,736,577	6,864,533	6,990,396
22	6,476,766	6,608,753	6,738,726	6,866,648	6,992,476
23	6,478,982	6,610,936	6,740,875	6,868,762	6,994,555
24	6,481,198	6,613,118	6,743,024	6,870,876	6,996,634
25	6,483,413	6,615,300	6,745,172	6,872,989	6,998,712
26	6,485,628	6,617,481	6,747,319	6,875,102	7,000,789
27	6,487,842	6,619,661	6,749,465	6,877,214	7,002,866
28	6,490,055	6,621,841	6,751,611	6,879,325	7,004,942
29	6,492,268	6,624,021	6,753,757	6,881,436	7,007,018
30	6,494,480	6,626,200	6,755,902	6,883,546	7,009,093

	40	41	42	43	44
30	6,494,480	6,626,200	6,755,902	6,883,546	7,009,093
31	6,496,692	6,628,379	6,758,047	6,885,656	7,011,167
32	6,498,903	6,630,557	6,760,191	6,887,765	7,013,241
33	6,501,114	6,632,734	6,762,334	6,889,874	7,015,314
34	6,503,324	6,634,911	6,764,477	6,891,982	7,017,387
35	6,505,533	6,637,087	6,766,619	6,894,089	7,019,459
36	6,507,742	6,639,263	6,768,760	6,896,196	7,021,530
37	6,509,950	6,641,438	6,770,901	6,898,302	7,023,601
38	6,512,158	6,643,612	6,773,041	6,900,408	7,025,671
39	6,514,365	6,645,786	6,775,181	6,902,513	7,027,741
40	6,516,572	6,647,959	6,777,320	6,904,617	7,029,810
41	6,518,778	6,650,132	6,779,459	6,906,721	7,031,879
42	6,520,984	6,652,304	6,781,597	6,908,824	7,033,947
43	6,523,189	6,654,476	6,783,734	6,910,927	7,036,014
44	6,525,394	6,656,647	6,785,971	6,913,029	7,038,081
45	6,527,598	6,658,817	6,788,007	6,915,131	7,040,147
46	6,529,801	6,660,987	6,790,143	6,917,232	7,042,213
47	6,532,004	6,663,156	6,792,278	6,919,332	7,044,278
48	6,534,206	6,665,325	6,794,413	6,921,432	7,046,342
49	6,536,408	6,667,493	6,796,547	6,823,531	7,048,406
50	6,538,609	6,669,661	6,798,681	6,925,630	7,050,469
51	6,540,809	6,671,828	6,800,814	6,927,728	7,052,532
52	6,543,009	6,673,994	6,802,946	6,929,825	7,054,594
53	6,545,208	6,676,160	6,805,078	6,931,922	7,056,655
54	6,547,407	6,678,326	6,807,209	6,934,018	7,058,716
55	6,549,606	6,680,491	6,809,340	6,936,114	7,060,776
56	6,551,804	6,682,655	6,811,470	6,938,209	7,062,836
57	6,554,001	6,684,818	6,813,599	6,940,303	7,064,895
58	6,556,198	6,686,981	6,815,728	6,942,397	7,066,953
59	6,558,394	6,689,144	6,817,856	6,944,491	7,069,011
60	6,560,590	6,691,306	6,819,984	6,946,584	7,071,068

	45	46	47	48	49
0	7,071,068	7,193,398	7,313,537	7,431,448	7,547,096
1	7,073,125	7,195,418	7,315,521	7,433,394	7,549,004
2	7,075,181	7,197,438	7,317,504	7,435,339	7,550,911
3	7,077,236	7,199,457	7,319,486	7,437,284	7,552,818
4	7,079,291	7,201,476	7,321,468	7,439,229	7,554,724
5	7,081,345	7,203,494	7,323,449	7,441,173	7,556,630
6	7,083,399	7,205,511	7,325,429	7,443,116	7,558,535
7	7,085,452	7,207,527	7,327,409	7,445,058	7,560,439
8	7,087,504	7,209,543	7,329,388	7,447,000	7,562,343
9	7,089,556	7,211,559	7,331,367	7,448,941	7,564,246
10	7,091,607	7,213,574	7,333,345	7,450,882	7,566,148
11	7,093,658	7,215,588	7,335,322	7,452,822	7,568,050
12	7,095,708	7,217,601	7,337,298	7,454,761	7,569,951
13	7,097,757	7,219,614	7,339,274	7,456,699	7,571,851
14	7,099,806	7,221,627	7,341,250	7,458,637	7,573,751
15	7,101,854	7,223,639	7,343,225	7,460,574	7,575,650
16	7,103,902	7,225,651	7,345,199	7,462,511	7,577,548
17	7,105,949	7,227,662	7,347,173	7,464,447	7,579,446
18	7,107,995	7,229,672	7,349,145	7,466,382	7,581,343
19	7,110,041	7,231,681	7,351,118	7,468,317	7,583,240
20	7,112,086	7,233,689	7,353,090	7,470,251	7,589,136
21	7,114,131	7,235,697	7,355,061	7,472,184	7,587,031
22	7,116,175	7,237,704	7,357,031	7,474,117	7,588,925
23	7,118,218	7,239,711	7,359,001	7,476,049	7,590,819
24	7,120,261	7,241,718	7,360,970	7,477,981	7,592,713
25	7,122,303	7,243,724	7,362,939	7,479,912	7,594,606
26	7,124,344	7,245,729	7,364,907	7,481,842	7,596,498
27	7,126,385	7,247,733	7,366,874	7,483,771	7,598,389
28	7,128,425	7,249,737	7,368,841	7,485,700	7,600,280
29	7,130,465	7,251,741	7,370,807	7,487,629	7,602,170
30	7,132,504	7,253,744	7,372,773	7,489,557	7,604,060

	45	46	47	48	49
30	7,132,504	7,253,744	7,372,773	7,489,557	7,604,060
31	7,134,543	7,255,746	7,374,738	7,491,484	7,605,949
32	7,136,581	7,257,747	7,376,702	7,493,410	7,607,837
33	7,138,618	7,259,748	7,378,666	7,495,336	7,609,725
34	7,140,655	7,261,749	7,380,629	7,497,262	7,611,612
35	7,142,691	7,263,749	7,382,592	7,499,187	7,613,498
36	7,144,727	7,265,748	7,384,554	7,501,111	7,615,384
37	7,146,762	7,267,746	7,386,515	7,503,034	7,617,269
38	7,148,796	7,269,744	7,388,475	7,504,957	7,619,153
39	7,150,830	7,271,741	7,390,435	7,506,879	7,621,037
40	7,152,863	7,273,737	7,392,394	7,508,801	7,622,920
41	7,154,895	7,275,733	7,394,353	7,510,722	7,624,802
42	7,156,927	7,277,728	7,396,311	7,512,642	7,626,683
43	7,158,958	7,279,722	7,398,268	7,514,561	7,628,564
44	7,160,989	7,281,716	7,400,225	7,516,840	7,630,445
45	7,163,019	7,283,710	7,402,181	7,518,398	7,632,325
46	7,165,049	7,285,703	7,404,137	7,520,316	7,634,204
47	7,167,078	7,287,695	7,406,092	7,522,233	7,636,082
48	7,169,106	7,289,687	7,408,046	7,524,149	7,637,960
49	7,171,134	7,291,678	7,410,000	7,526,065	7,639,838
50	7,173,161	7,293,668	7,411,953	7,527,980	7,641,715
51	7,175,187	7,295,658	7,413,905	7,529,894	7,643,591
52	7,177,213	7,297,647	7,415,856	7,531,808	7,645,466
53	7,179,238	7,299,635	7,417,807	7,533,721	7,647,341
54	7,181,263	7,301,623	7,419,758	7,535,634	7,649,215
55	7,183,287	7,303,610	7,421,708	7,537,546	7,651,088
56	7,185,310	7,305,597	7,423,657	7,539,457	7,652,961
57	7,187,333	7,307,583	7,425,605	7,541,367	7,654,833
58	7,189,355	7,309,568	7,427,553	7,543,277	7,656,704
59	7,191,377	7,311,553	7,429,501	7,545,187	7,658,575
60	7,193,398	7,313,537	7,431,448	7,547,096	7,660,445

	50	51	52	53	54
0	7,660,445	7,771,460	7,880,108	7,986,355	8,090,170
1	7,662,314	7,773,290	7,881,898	7,988,105	8,091,879
2	7,664,183	7,775,120	7,883,688	7,989,855	8,093,588
3	7,666,051	7,776,949	7,885,477	7,991,604	8,095,296
4	7,667,919	7,778,777	7,887,266	7,993,352	8,097,004
5	7,669,786	7,780,605	7,889,054	7,995,100	8,098,711
6	7,671,652	7,782,432	7,890,841	7,996,847	8,100,417
7	7,673,517	7,784,258	7,892,627	7,998,593	8,102,122
8	7,675,382	7,786,084	7,894,413	8,000,339	8,104,827
9	7,677,246	7,787,909	7,896,198	8,002,084	8,105,531
10	7,679,110	7,789,733	7,897,983	8,003,828	8,107,234
11	7,680,973	7,791,557	7,899,767	8,005,571	8,108,936
12	7,682,835	7,793,380	7,901,550	8,007,314	8,110,638
13	7,684,687	7,795,202	7,903,332	8,009,056	8,112,339
14	7,686,549	7,797,024	7,905,114	8,010,797	8,114,040
15	7,688,418	7,798,845	7,906,896	8,012,538	8,115,740
16	7,690,278	7,800,665	7,908,676	8,014,278	8,117,439
17	7,692,137	7,802,485	7,910,456	8,016,017	8,119,137
18	7,693,995	7,804,304	7,912,235	8,017,756	8,120,835
19	7,695,853	7,806,123	7,914,014	8,019,494	8,122,532
20	7,697,710	7,807,941	7,915,792	8,021,232	8,124,229
21	7,699,566	7,809,758	7,917,569	8,022,969	8,125,925
22	7,701,422	7,811,574	7,919,345	8,024,705	8,127,620
23	7,703,277	7,813,390	7,921,121	8,026,440	8,129,314
24	7,705,132	7,815,205	7,922,896	8,028,175	8,131,008
25	7,706,986	7,817,020	7,924,671	8,029,909	8,132,701
26	7,708,839	7,818,834	7,926,445	8,031,642	8,134,393
27	7,710,692	7,820,647	7,928,218	8,033,375	8,136,084
28	7,712,544	7,822,459	7,929,990	8,035,107	8,137,775
29	7,714,395	7,824,271	7,931,762	8,036,838	8,139,465
30	7,716,246	7,826,082	7,933,533	8,038,569	8,141,155

	50	51	52	53	54
30	7,716,246	7,826,082	7,933,533	8,038,569	8,141,155
31	7,718,096	7,827,892	7,935,303	8,040,299	8,142,844
32	7,719,945	7,829,702	7,937,073	8,042,028	8,144,532
33	7,721,794	7,831,511	7,938,842	8,043,757	8,146,220
34	7,723,642	7,833,320	7,940,611	8,045,485	8,147,907
35	7,725,490	7,835,128	7,942,379	8,047,212	8,149,593
36	7,727,337	7,836,935	7,944,146	8,048,938	8,151,278
37	7,729,183	7,838,741	7,945,912	8,050,664	8,152,963
38	7,731,028	7,840,547	7,947,678	8,052,389	8,154,647
39	7,732,872	7,842,352	7,949,443	8,054,114	8,156,330
40	7,734,716	7,844,157	7,951,208	8,055,838	8,158,013
41	7,736,559	7,845,961	7,952,972	8,057,561	8,159,695
42	7,738,402	7,847,764	7,954,735	8,059,283	8,161,376
43	7,740,244	7,849,566	7,956,497	8,061,005	8,163,057
44	7,742,085	7,851,368	7,958,259	8,062,726	8,164,737
45	7,743,926	7,853,169	7,960,020	8,064,446	8,166,416
46	7,745,766	7,854,970	7,961,780	8,066,166	8,168,094
47	7,747,606	7,856,770	7,963,540	8,067,885	8,169,772
48	7,749,445	7,858,569	7,965,299	8,069,603	8,171,449
49	7,751,283	7,860,368	7,967,057	8,071,321	8,173,126
50	7,753,121	7,862,166	7,968,815	8,073,038	8,174,802
51	7,754,958	7,863,963	7,970,572	8,074,754	8,176,477
52	7,756,794	7,865,759	7,972,328	8,076,470	8,178,151
53	7,758,630	7,867,555	7,974,084	8,078,185	8,179,825
54	7,760,465	7,869,350	7,975,838	8,079,899	8,181,498
55	7,762,299	7,871,145	7,977,593	8,081,613	8,183,170
56	7,764,132	7,872,939	7,979,347	8,083,326	8,184,841
57	7,765,965	7,874,732	7,981,100	8,085,038	8,186,512
58	7,767,797	7,876,525	7,982,852	8,086,749	8,188,182
59	7,769,629	7,878,317	7,984,604	8,089,460	8,189,851
60	7,771,460	7,880,108	7,986,355	8,090,170	8,191,520

	55	56	57	58	59
0	8,191,520	8,290,376	8,386,706	8,480,481	8,571,673
1	8,193,188	8,292,002	8,388,290	8,482,022	8,573,171
2	8,194,855	8,293,628	8,389,873	8,483,562	8,574,668
3	8,196,522	8,295,253	8,301,456	8,485,102	8,576,164
4	8,198,188	8,296,877	8,393,038	8,486,641	8,577,660
5	8,199,854	8,298,501	8,394,619	8,488,180	8,579,155
6	8,201,519	8,300,124	8,396,199	8,489,718	8,580,649
7	8,203,183	8,301,746	8,397,778	8,491,255	8,582,142
8	8,204,846	8,303,367	8,399,357	8,492,791	8,583,635
9	8,206,508	8,304,987	8,400,935	8,494,326	8,585,127
10	8,208,170	8,306,607	8,402,513	8,495,860	8,586,619
11	8,209,831	8,308,226	8,404,090	8,497,394	8,588,110
12	8,211,491	8,309,844	8,405,666	8,498,927	8,589,600
13	8,213,151	8,311,462	8,407,241	8,500,459	8,591,089
14	8,214,810	8,313,079	8,418,816	8,501,991	8,592,577
15	8,216,469	8,314,696	8,410,390	8,503,522	8,594,064
16	8,218,127	8,316,312	8,411,963	8,505,052	8,595,551
17	8,219,784	8,317,927	8,413,536	8,506,582	8,597,037
18	8,221,440	8,319,541	8,415,108	8,508,111	8,598,523
19	8,223,096	8,321,155	8,416,679	8,509,639	8,600,008
20	8,224,751	8,322,768	8,418,250	8,511,167	8,601,492
21	8,226,405	8,324,380	8,419,820	8,512,694	8,602,975
22	8,228,058	8,325,991	8,421,389	8,514,220	8,604,457
23	8,229,711	8,327,602	8,422,957	8,515,745	8,605,939
24	8,231,363	8,329,212	8,424,525	8,517,270	8,607,420
25	8,233,015	8,330,822	8,426,092	8,518,794	8,608,901
26	8,234,666	8,332,431	8,427,658	8,520,317	8,610,381
27	8,236,316	8,334,039	8,429,223	8,521,839	8,611,860
28	8,237,965	8,335,646	8,430,788	8,523,361	8,613,338
29	8,239,614	8,337,252	8,432,352	8,524,882	8,614,815
30	8,241,262	8,338,858	8,433,915	8,526,402	8,616,292

	55	56	57	58	59
30	8,241,262	8,338,858	8,433,915	8,526,402	8,616,292
31	8,242,909	8,340,463	8,435,477	8,527,921	8,617,768
32	8,244,556	8,342,067	8,437,039	8,529,440	8,619,243
33	8,246,202	8,343,671	8,438,600	8,530,958	8,620,718
34	8,247,847	8,345,274	8,440,161	8,532,476	8,622,192
35	8,249,492	8,346,877	8,441,721	8,533,993	8,623,665
36	8,251,136	8,348,479	8,443,280	8,535,509	8,625,137
37	8,252,779	8,350,080	8,444,838	8,537,024	8,626,608
38	8,254,421	8,351,680	8,446,396	8,538,538	8,628,079
39	8,256,062	8,353,279	8,447,953	8,540,052	8,629,549
40	8,257,703	8,354,878	8,449,509	8,541,565	8,631,019
41	8,249,343	8,356,476	8,451,064	8,543,077	8,732,488
42	8,260,982	8,358,073	8,452,618	8,544,588	8,633,956
43	8,262,621	8,359,670	8,454,172	8,546,096	8,635,423
44	8,264,259	8,361,256	8,455,725	8,547,609	8,636,889
45	8,265,897	8,362,862	8,457,278	8,549,119	8,638,355
46	8,267,534	8,364,457	8,458,830	8,550,628	8,639,820
47	8,269,170	8,366,051	8,460,381	8,552,136	8,641,284
48	8,270,806	8,367,644	8,461,932	8,553,643	8,642,748
49	8,272,441	8,369,236	8,463,482	8,555,149	8,644,211
50	8,274,075	8,370,828	8,465,031	8,556,655	8,645,673
51	8,275,708	8,372,419	8,466,579	8,558,160	8,647,134
52	8,277,340	8,374,009	8,468,126	8,559,664	8,648,595
53	8,278,972	8,375,599	8,469,673	8,561,168	8,650,055
54	8,280,603	8,377,188	8,471,219	8,562,671	8,651,514
55	8,282,234	8,378,776	8,472,765	8,564,173	8,652,973
56	8,283,864	8,380,363	8,474,310	8,565,675	8,654,431
57	8,285,493	8,381,950	8,475,854	8,567,176	8,655,888
58	8,287,121	8,383,536	8,477,297	8,568,676	8,657,344
59	8,288,794	8,385,121	8,478,939	8,570,175	8,658,799
60	8,290,376	8,386,706	8.480,481	8,571,673	8,660,254

	60	61	62	63	64
0	8,660,254	8,746,197	8,829,476	8,910,065	8,987,946
1	8,661,708	8,747,607	8,830,841	8,911,385	8,989,215
2	8,663,162	8,749,016	8,832,205	8,912,704	8,999,489
3	8,664,615	8,750,425	8,833,569	8,914,023	8,991,762
4	8,666,067	8,751,833	8,834,932	8,915,341	8,993,035
5	8,667,518	8,753,240	8,836,295	8,916,659	8,994,307
6	8,668,968	8,754,646	8,837,657	8,917,976	8,995,578
7	8,670,417	8,756,051	8,839,018	8,919,292	8,996,848
8	8,671,866	8,757,456	8,840,378	8,920,607	8,998,117
9	8,673,314	8,758,860	8,841,737	8,921,921	8,999,386
10	8,674,762	8,760,263	8,843,095	8,923,234	9,000,654
11	8,676,209	8,761,665	8,844,452	8,924,546	9,001,921
12	8,677,655	8,763,068	8,845,809	8,925,858	9,003,187
13	8,679,100	8,764,468	8,847,165	8,927,169	9,004,453
14	8,680,544	8,765,868	8,848,521	8,928,479	9,005,718
15	8,681,988	8,767,267	8,849,876	8,929,789	9,006,982
16	8,683,431	8,768,667	8,851,230	8,931,098	9,008,245
17	8,684,873	8,670,065	8,852,583	8,932,406	9,009,508
18	8,686,316	8,771,462	8,852,936	8,933,717	9,010,770
19	8,687,757	8,772,859	8,855,288	8,935,021	9,012,031
20	8,689,197	8,774,255	8,856,639	8,936,327	9,013,292
21	8,690,636	8,775,650	8,857,989	8,937,632	9,014,552
22	8,692,074	8,777,044	8,859,338	8,938,936	9,015,811
23	8,693,512	8,778,437	8,860,687	8,940,240	9,017,069
24	8,694,949	8,779,830	8,862,035	8,941,543	9,018,326
25	8,696,386	8,781,222	8,863,383	8,942,845	9,019,582
26	8,697,822	8,782,613	8,864,730	8,944,146	9,020,838
27	8,699,257	8,784,003	8,866,076	8,945,446	9,022,093
28	8,700,691	8,785,393	8,867,421	8,946,746	9,023,347
29	8,702,124	8,786,782	8,868,765	8,948,045	9,024,600
30	8,703,557	8,788,171	8,870,108	8,949,344	9,025,853

	60	61	62	63	64
30	8,703,557	8,788,171	8,870,108	8,949,344	9,025,853
31	8,704,989	8,789,559	8,871,451	8,950,642	9,027,105
32	8,706,420	8,790,946	8,872,793	8,951,939	9,028,356
33	8,707,851	8,792,332	8,874,134	8,953,235	9,029,606
34	8,709,281	8,793,717	8,875,475	8,954,530	9,030,856
35	8,710,710	8,795,102	8,876,815	8,955,824	9,032,105
36	8,712,138	8,796,486	8,878,154	8,957,117	9,033,353
37	8,713,565	8,797,869	8,879,492	8,958,410	9,034,600
38	8,714,992	8,799,251	8,880,830	8,959,702	9,035,847
39	8,716,418	8,800,633	8,882,167	8,960,994	9,037,093
40	8,717,844	8,802,014	8,883,503	8,962,285	9,038,338
41	8,719,269	8,803,394	8,884,838	8,963,575	9,039,582
42	8,720,693	8,804,773	8,886,172	8,964,864	9,040,825
43	8,722,116	8,806,152	8,887,506	8,966,152	9,042,068
44	8,723,538	8,807,530	8,888,839	8,967,440	9,043,310
45	8,724,960	8,808,907	8,890,171	8,968,727	9,044,551
46	8,726,381	8,810,285	8,891,502	8,970,013	9,045,791
47	8,727,801	8,811,659	8,892,833	8,971,299	9,047,031
48	8,729,221	8,813,034	8,894,163	8,972,584	9,048,270
49	8,730,640	8,814,408	8,895,492	8,973,868	9,049,508
50	8,732,058	8,815,783	8,896,821	8,975,151	9,050,746
51	8,733,475	8,817,155	8,898,149	8,976,433	9,051,983
52	8,734,891	8,818,527	8,899,476	8,977,715	9,053,219
53	8,736,307	8,819,898	8,900,802	8,978,996	9,054,454
54	8,737,722	8,821,268	8,902,127	8,980,276	9,055,688
55	8,739,137	8,822,638	8,903,452	8,981,555	9,056,922
56	8,740,551	8,824,007	8,904,776	8,982,833	9,058,155
57	8,741,964	8,825,375	8,906,099	8,984,111	9,059,387
58	8,743,376	8,826,743	8,907,422	8,985,388	9,060,618
59	8,744,787	8,828,110	8,908,744	8,986,664	9,061,848
60	8,746,197	8,829,476	8,910,065	8,987,940	9,063,078

	65	66	67	68	69
0	9,063,078	9,135,455	9,205,049	9,271,839	9,335,804
1	9,064,307	9,136,638	9,206,185	9,272,928	9,336,846
2	9,065,535	9,137,820	9,207,321	9,274,017	9,337,887
3	9,066,763	9,139,001	9,208,456	9,275,105	9,338,928
4	9,067,990	9,140,181	9,209,590	9,276,192	9,339,968
5	9,069,216	9,141,361	9,210,723	9,277,278	9,341,007
6	9,070,441	9,142,540	9,211,855	9,278,363	9,342,045
7	9,071,665	9,143,718	9,212,986	9,279,448	9,343,082
8	9,072,889	9,144,895	9,214,117	9,280,532	9,344,119
9	9,074,112	9,146,072	9,215,247	9,281,615	9,345,155
10	9,075,334	9,147,248	9,216,376	9,282,697	9,346,190
11	9,076,555	9,148,423	9,217,504	9,283,778	9,347,224
12	9,077,775	9,149,597	9,218,631	9,284,859	9,348,257
13	9,078,995	9,150,770	9,219,758	9,285,939	9,349,289
14	9,080,214	9,151,943	9,220,884	9,287,018	9,350,321
15	9,081,432	9,153,115	9,222,010	9,288,096	9,351,352
16	9,082,649	9,154,286	9,223,135	9,289,173	9,352,382
17	9,083,866	9,155,457	9,224,259	9,290,250	9,353,411
18	9,085,082	9,156,627	9,225,382	9,291,326	9,354,440
19	9,086,297	9,157,796	9,226,504	9,292,401	9,355,468
20	9,087,512	9,158,964	9,227,625	9,293,476	9,356,495
21	9,088,726	9,160,131	9,228,746	9,294,550	9,357,521
22	9,089,939	9,161,297	9,229,866	9,295,623	9,358,546
23	9,091,151	9,162,463	9,230,985	9,296,695	9,359,571
24	9,092,362	9,163,628	9,232,103	9,297,766	9,360,595
25	9,093,572	9,164,792	9,233,220	9,298,836	9,361,618
26	9,094,781	9,165,955	9,234,337	9,299,905	9,362,640
27	9,095,990	9,167,117	9,235,453	9,300,974	9,363,662
28	9,097,198	9,168,279	9,236,568	9,302,042	9,364,683
29	9,098,406	9,169,440	9,237,682	9,303,109	9,365,703
30	9,099,613	9,170,601	9,238,795	9,304,176	9,366,722

	65	66	67	68	69
30	9,099,613	9,170,601	9,238,795	9,304,176	9,366,722
31	9,100,819	9,171,761	9,239,908	9,305,242	9,367,740
32	9,102,024	9,172,920	9,241,020	9,306,307	9,368,758
33	9,103,228	9,174,078	9,242,131	9,307,371	9,369,775
34	9,134,432	9,175,235	9,243,242	9,308,434	9,370,791
35	9,105,635	9,176,391	9,244,352	9,309,497	9,371,806
36	9,106,837	9,177,547	9,245,461	9,310,559	9,372,820
37	9,108,038	9,178,702	9,246,569	9,311,620	9,373,834
38	9,109,238	9,179,856	9,247,676	9,312,680	9,374,847
39	9,110,438	9,181,009	9,248,782	9,313,739	9,375,859
40	9,111,637	9,182,161	9,249,888	9,314,798	9,376,870
41	9,112,835	9,183,313	9,250,993	9,315,856	9,377,880
42	9,114,032	9,184,464	9,252,097	9,316,913	9,378,889
43	9,115,229	9,185,614	9,253,200	9,317,969	9,379,898
44	9,116,425	9,186,763	9,254,303	9,319,024	9,380,906
45	9,117,620	9,187,912	9,255,405	9,320,079	9,381,913
46	9,118,814	9,189,060	9,256,506	9,321,133	9,382,919
47	9,120,007	9,190,207	9,257,606	9,322,186	9,383,925
48	9,121,200	9,191,353	9,258,706	9,323,238	9,384,930
49	9,122,392	9,192,499	9,259,805	9,324,290	9,385,934
50	9,123,584	9,193,644	9,260,903	9,325,341	9,386,937
51	9,124,775	9,194,788	9,262,000	9,326,391	9,387,939
52	9,125,965	9,195,931	9,263,096	9,327,440	9,388,941
53	9,127,154	9,197,073	9,264,192	9,328,488	9,389,942
54	9,128,342	9,198,215	9,265,287	9,329,535	9,390,942
55	9,129,529	9,199,356	9,266,381	9,330,582	9,391,941
56	9,130,716	9,200,496	9,267,474	9,331,628	9,392,940
57	9,131,902	9,201,635	9,268,566	9,332,673	9,393,938
58	9,133,087	9,202,774	9,269,658	9,333,717	9,394,935
59	9,134,271	9,203,912	9,270,749	9,334,761	9,395,931
60	9,135,455	9,205,049	9,271,839	9,335,804	9,396,926

	70	71	72	73	74
0	9,396,926	9,455,186	9,510,565	9,563,048	9,612,617
1	9,397,921	9,456,133	9,511,464	9,563,898	9,613,418
2	9,398,915	9,457,079	9,512,362	9,564,747	9,614,219
3	9,399,908	9,458,024	9,513,259	9,565,596	9,615,019
4	9,400,900	9,458,968	9,514,155	9,566,444	9,615,818
5	9,401,891	9,459,911	9,515,050	9,567,291	9,616,616
6	9,402,882	9,460,854	9,515,944	9,568,137	9,617,413
7	9,403,872	9,461,796	9,516,838	9,568,982	9,618,209
8	9,404,861	9,462,737	9,517,731	9,569,826	9,619,005
9	9,405,849	9,463,677	9,518,623	9,570,670	9,619,800
10	9,406,836	9,464,616	9,519,514	9,571,513	9,620,594
11	9,407,822	9,465,555	9,520,404	9,572,355	9,621,387
12	9,408,808	9,466,493	9,521,294	9,573,196	9,622,179
13	9,409,793	9,467,430	9,522,183	9,574,036	9,622,971
14	9,410,777	9,468,366	9,523,071	9,574,875	9,623,762
15	9,411,760	9,469,301	9,523,958	9,575,714	9,624,552
16	9,412,742	9,470,236	9,524,844	9,576,552	9,625,341
17	9,413,724	9,471,170	9,525,730	9,577,389	9,626,129
18	9,414,705	9,472,103	9,526,615	9,578,225	9,626,917
19	9,415,685	9,473,035	9,527,499	9,579,061	9,627,704
20	9,416,665	9,473,967	9,528,382	9,579,896	9,628,490
21	9,417,644	9,474,898	9,529,264	9,580,730	9,629,275
22	9,418,622	9,475,828	9,530,146	9,581,563	9,630,059
23	9,419,599	9,476,757	9,531,027	9,582,395	9,630,843
24	9,420,575	9,477,685	9,531,907	9,583,226	9,631,626
25	9,421,550	9,478,612	9,532,786	9,584,057	9,632,408
26	9,422,525	9,479,539	9,533,664	9,584,887	9,633,189
27	9,423,499	9,480,465	9,534,541	9,585,716	9,633,969
28	9,424,472	9,481,390	9,535,418	9,586,544	9,634,748
29	9,425,444	9,482,314	9,536,294	9,587,371	9,635,527
30	9,426,415	9,483,237	9,537,169	9,588,197	9,636,305

	70	71	72	73	74
30	9,426,415	9,483,237	9,537,169	9,588,197	9,636,305
31	9,427,386	9,484,160	9,538,043	9,589,023	9,637,082
32	9,428,356	9,485,082	9,538,917	9,589,848	9,637,858
33	9,429,325	9,486,003	9,539,790	9,590,672	9,638,633
34	9,430,293	9,486,923	9,540,662	9,591,495	9,639,408
35	9,431,260	9,487,842	9,541,533	9,592,318	9,640,182
36	9,432,227	9,488,761	9,542,403	9,593,140	9,640,955
37	9,433,193	9,489,679	9,543,272	9,593,961	9,641,727
38	9,434,158	9,490,596	9,544,141	9,594,781	9,642,498
39	9,435,122	9,491,512	9,545,009	9,595,600	9,643,268
40	9,436,085	9,492,427	9,545,876	9,596,419	9,644,038
41	9,437,048	9,493,341	9,546,742	9,597,237	9,644,807
42	9,438,010	9,494,255	9,547,607	9,598,054	9,645,575
43	9,438,971	9,495,168	9,548,472	9,598,870	9,646,342
44	9,439,931	9,496,080	9,549,336	9,599,685	9,647,108
45	9,440,890	9,496,991	9,550,199	9,600,499	9,647,873
46	9,441,849	9,497,902	9,551,061	9,601,313	9,648,638
47	9,442,807	9,498,812	9,551,922	9,602,126	9,649,402
48	9,443,764	9,499,721	9,552,783	9,602,938	9,650,165
49	9,444,720	9,500,629	9,553,643	9,603,749	9,650,927
50	9,445,676	9,501,536	9,554,502	9,604,559	9,651,689
51	9,446,631	9,502,443	9,555,360	9,605,368	9,652,450
52	9,447,585	9,503,349	9,556,217	9,606,177	9,653,210
53	9,448,538	9,504,254	9,557,074	9,606,985	9,653,969
54	9,449,490	9,505,158	9,557,930	9,607,792	9,654,727
55	9,450,441	9,506,061	9,558,785	9,608,598	9,655,484
56	9,451,392	9,506,963	9,559,639	9,609,403	9,656,240
57	9,452,342	9,507,865	9,560,492	9,610,208	9,656,996
58	9,453,291	9,508,766	9,561,345	9,611,012	9,657,751
59	9,454,239	9,509,666	9,562,197	9,611,815	9,658,505
60	9,455,186	9,510,565	9,563,048	9,612,617	9,659,268

	75	76	77	78	79
0	9,659,258	9,702,957	9,743,700	9,781,476	9,816,272
1	9,660,011	9,703,660	9,744,355	9,782,080	9,816,827
2	9,660,763	9,704,363	9,745,008	9,782,684	9,817,381
3	9,661,514	9,705,065	9,745,660	9,783,287	9,817,934
4	9,662,264	9,705,766	9,746,312	9,783,889	9,818,486
5	9,663,013	9,706,466	9,746,963	9,784,490	9,819,037
6	9,663,761	9,707,165	9,747,613	9,785,090	9,819,587
7	9,664,508	9,707,863	9,748,262	9,785,689	9,820,137
8	9,665,255	9,708,561	9,748,910	9,786,288	9,820,686
9	9,666,001	9,709,258	9,749,557	9,786,886	9,821,234
10	9,666,746	9,709,954	9,750,203	9,787,483	9,821,781
11	9,667,490	9,710,649	9,750,849	9,788,079	9,822,227
12	9,668,233	9,711,343	9,751,494	9,788,674	9,822,872
13	9,668,976	9,712,036	9,752,138	9,789,268	9,823,417
14	9,669,718	9,712,729	9,752,781	9,789,862	9,823,961
15	9,670,459	9,713,421	9,753,423	9,790,455	9,824,504
16	9,671,199	9,714,112	9,754,065	9,791,047	9,825,046
17	9,671,938	9,714,802	9,754,706	9,791,638	9,825,587
18	9,672,677	9,715,491	9,755,346	9,792,228	9,826,128
19	9,673,415	9,716,180	9,755,985	9,792,818	9,826,668
20	9,674,152	9,716,868	9,756,623	9,793,407	9,827,207
21	9,674,888	9,717,555	9,757,260	9,793,995	9,827,745
22	9,675,623	9,718,241	9,757,897	9,794,582	9,728,282
23	9,676,357	9,718,926	9,758,533	9,795,168	9,828,818
24	9,677,091	9,719,610	9,759,168	9,795,753	9,829,354
25	9,677,824	9,720,294	9,759,802	9,796,337	9,829,889
26	9,678,556	9,720,977	9,760,435	9,796,921	9,830,423
27	9,679,287	9,721,659	9,761,067	9,797,504	9,830,956
28	9,680,017	9,722,340	9,761,699	9,798,086	9,831,488
29	9,680,747	9,723,020	9,762,330	9,798,667	9,832,019
30	9,681,476	9,723,699	9,762,960	9,799,247	9,832,549

	75	76	77	78	79
30	9,681,476	9,723,699	9,762,960	9,799,247	9,832,549
31	9,682,204	9,724,378	9,763,589	9,799,827	9,833,079
32	9,682,931	9,725,056	9,764,217	9,800,406	9,833,608
33	9,683,657	9,725,733	9,764,845	9,800,984	9,834,136
34	9,684,383	9,726,409	9,765,472	9,801,561	9,834,663
35	9,685,108	9,727,085	9,766,098	9,802,137	9,835,189
36	9,685,832	9,727,760	9,766,723	9,802,712	9,835,714
37	9,686,555	9,728,434	9,767,347	9,803,287	9,836,239
38	9,687,277	9,729,107	9,767,970	9,803,861	9,836,763
39	9,687,998	9,729,779	9,768,593	9,804,434	9,837,286
40	9,688,719	9,730,450	9,769,215	9,805,006	9,837,808
41	9,689,439	9,731,120	9,769,836	9,805,577	9,838,329
42	9,690,158	9,731,789	9,770,456	9,806,147	9,838,850
43	9,690,876	9,732,458	9,771,075	9,806,716	9,839,370
44	9,691,593	9,733,126	9,771,693	9,807,285	9,839,889
45	9,692,309	9,733,793	9,772,311	9,807,853	9,840,407
46	9,693,025	9,734,459	9,772,928	9,808,420	9,840,924
47	9,693,740	9,735,124	9,773,544	9,808,986	9,841,440
48	9,694,454	9,735,789	9,774,159	9,809,551	9,841,956
49	9,695,167	9,736,453	9,774,773	9,810,116	9,842,471
50	9,695,879	9,737,116	9,775,387	9,810,680	9,842,985
51	9,696,590	9,737,778	9,776,000	9,811,243	9,843,498
52	9,697,301	9,738,439	9,776,612	9,811,850	9,844,010
53	9,698,011	9,739,099	9,777,223	9,812,366	9,844,521
54	9,698,720	9,739,759	9,777,833	9,812,926	9,845,032
55	9,699,428	9,740,418	9,778,442	9,813,486	9,845,542
56	9,700,135	9,741,076	9,779,050	9,814,045	9,846,051
57	9,700,842	9,741,733	9,779,658	9,814,603	9,846,559
58	9,701,548	9,742,389	9,780,265	9,815,160	9,847,066
59	9,702,253	9,743,045	9,780,871	9,815,716	9,847,572
60	9,702,957	9,743,700	9,781,476	9,816,272	9,848,078

	80	81	82	83	84
0	9,848,078	9,876,883	9,902,681	9,925,461	9,945,219
1	9,848,583	9,877,338	9,903,085	9,925,816	9,945,523
2	9,849,087	9,877,792	9,903,489	9,926,169	9,945,826
3	9,849,590	9,878,245	9,903,892	9,926,521	9,946,128
4	9,850,092	9,878,697	9,904,294	9,926,873	9,946,429
5	9,850,593	9,879,148	9,904,695	9,927,224	9,946,729
6	9,851,093	9,879,598	9,905,095	9,927,574	9,947,028
7	9,851,593	9,880,048	9,905,494	9,927,923	9,947,327
8	9,852,092	9,880,497	9,905,893	9,928,271	9,947,625
9	9,852,590	9,880,945	9,906,291	9,928,618	9,947,922
10	9,853,087	9,881,392	9,906,688	9,918,965	9,948,218
11	9,853,583	9,881,838	9,907,084	9,929,311	9,948,513
12	9,854,079	9,882,283	9,907,479	9,929,656	9,948,807
13	9,854,574	9,882,728	9,907,873	9,930,000	9,949,100
14	9,855,068	9,883,172	9,908,266	9,930,343	9,949,393
15	9,855,561	9,883,615	9,908,659	9,930,685	9,949,685
16	9,856,053	9,884,057	9,909,051	9,931,026	9,949,976
17	9,856,544	9,884,498	9,909,442	9,931,367	9,950,266
18	9,857,035	9,884,938	9,909,832	9,931,707	9,950,555
19	9,857,525	9,885,378	9,910,221	9,932,046	9,950,844
20	9,858,014	9,885,817	9,910,610	9,932,384	9,951,132
21	9,858,502	9,886,255	9,910,998	9,932,721	9,951,419
22	9,858,989	9,886,692	9,911,385	9,933,057	9,951,705
23	9,859.475	9,887,128	9,911,771	9,933,393	9,951,990
24	9,859,961	9,887,564	9,912,156	9,933,728	9,952,274
25	9,860,446	9,887,999	9,912,540	9,934,062	9,952,557
26	9,860,930	9,888,433	9,912,923	9,934,395	9,952,840
27	9,861.413	9,888.866	9,913,306	9,934,727	9,953,122
28	9,861,895	9,889,298	9,913,688	9,935,058	9,953,403
29	9,862,376	9,889.729	9,914.069	9,935,389	9,953,683
30	9,862,856	9,890,159	9,914,449	9,935,719	9,953,962

	80	81	82	83	84
30	9,862,856	9,890,159	9,914,449	9,935,719	9,953,962
31	9,863,336	9,890,588	9,914,828	9,936,048	9,954,240
32	9,863,815	9,891,017	9,915,206	9,936,376	9,954,518
33	9,864,293	9,891,445	9,915,584	9,936,703	9,954,795
34	9,864,770	9,891,872	9,915,961	9,937,029	9,955,071
35	9,865,246	9,892,298	9,916,337	9,937,355	9,955,346
36	9,865,722	9,892,723	9,916,712	9,937,680	9,955,620
37	9,866,197	9,893,147	9,917,086	9,938,004	9,955,893
38	9,866,671	9,893,571	9,917,459	9,938,327	9,956,165
39	9,867,144	9,893,994	9,917,832	9,938,649	9,956,437
40	9,867,616	9,894,416	9,918,204	9,938,970	9,956,708
41	9,868,087	9,894,837	9,918,575	9,939,290	9,956,978
42	9,868,557	9,895,257	9,918,945	9,939,609	9,957,247
43	9,869,027	9,895,677	9,919,314	9,939,928	9,957,515
44	9,869,496	9,896,096	9,919,682	9,940,246	9,957,782
45	9,869,964	9,896,514	9,920,049	9,940,563	9,958,049
46	9,870,431	9,896,931	9,920,416	9,940,879	9,958,315
47	9,870,897	9,897,347	9,920,782	9,941,194	9,958,580
48	9,871,362	9,897,762	9,921,147	9,941,509	9,958,844
49	9,871,827	9,898,177	9,921,511	9,941,823	9,959,107
50	9,872,291	9,898,591	9,921,874	9,942,136	9,959,370
51	9,872,754	9,899,004	9,922,236	9,942,448	9,959,632
52	9,873,216	9,899,416	9,922,598	9,942,759	9,959,893
53	9,873,677	9,899,827	9,922,959	9,943,069	9,960,153
54	9,874,137	9,900,237	9,923,319	9,943,379	9,960,412
55	9,874,597	9,900,646	9,923,678	9,943,688	9,960,670
56	9,875,056	9,901,055	9,924,036	9,943,996	9,960,927
57	9,875,514	9,901,463	9,924,393	9,944,303	9,961,183
58	9,875,971	9,901,870	9,924,750	9,944,609	9,961,438
59	9,876,427	9,902,276	9,925,106	9,944,914	9,961,693
60	9,876 883	9,902,681	9,925,461	9,945,219	9,961,947

	85	86	87	88	89
0	9,961,947	9,975,640	9,986,295	9,993,908	9,998,477
1	9,962,200	9,975,843	9,986,447	9,994,009	9,998,527
2	9,962,452	9,976,045	9,986,598	9,994,109	9,998,577
3	9,962,703	9,976,246	9,986,748	9,994,208	9,998,625
4	9,962,954	9,976,446	9,986,897	9,994,307	9,998,673
5	9,963,204	9,976,645	9,987,045	9,994,405	9,998,720
6	9,963,453	9,976,843	9,987,193	9,994,502	9,998,766
7	9,963,701	9,977,040	9,987,340	9,994,598	9,998,811
8	9,963,948	9,977,237	9,987,486	9,994,693	9,998,856
9	9,964,194	9,977,433	9,987,631	9,994,787	9,998,900
10	9,964,440	9,977,628	9,987,775	9,994,881	9,998,942
11	9,964,685	9,977,822	9,987,918	9,994,974	9,998,984
12	9,964,929	9,978,015	9,988,061	9,995,066	9,999,025
13	9,965,172	9,978,207	9,988,203	9,995,157	9,999,065
14	9,965,414	9,978,398	9,988,344	9,995,247	9,999,105
15	9,965,655	9,978,589	9,988,484	9,995,336	9,999,143
16	9,965,895	9,978,779	9,988,623	9,995,424	9,999,181
17	9,966,135	9,978,968	9,988,761	9,995,512	9,999,218
18	9,966,374	9,979,156	9,988,899	9,995,599	9,999,254
19	9,966,612	9,979,343	9,989,036	9,995,685	9,999,289
20	9,966,849	9,979,530	9,989,172	9,995,770	9,999,323
21	9,967,085	9,979,716	9,989,307	9,995,854	9,999,357
22	9,967,320	9,979,901	9,989,441	9,995,937	9,999,389
23	9,967,555	9,980,085	9,989,574	9,996,019	9,999,421
24	9,967,789	9,980,268	9,989,706	9,996,101	9,999,452
25	9,968,022	9,980,450	9,989,837	9,996,182	9,999,482
26	9,968,254	9,980,631	9,989,968	9,996,262	9,999,511
27	9,968,485	9,980,811	9,990,098	9,996,341	9,999,539
28	9,968,715	9,980,991	9,990,227	9,996,419	9,999,566
29	9,968,944	9,981,170	9,990,355	9,996,496	9,999,593
30	9,969,173	9,981,348	9,990,482	9,996,573	9,999,619

	85	86	87	88	89
30	9,969,173	9,981,348	9,990,482	9,996,573	9,999,619
31	9,969,401	9,981,525	9,990,608	9,996,649	9,999,644
32	9,969,628	9,981,701	9,990,734	9,996,724	9,999,668
33	9,969,854	9,981,877	9,990,859	9,996,798	6,999,692
34	9,970,079	9,982,052	9,990,983	9,996,871	9,999,714
35	9,970,304	9,982,226	9,991,106	9,996,943	9,999,736
36	9,970,528	9,982,399	9,991,228	9,997,014	9,999,756
37	9,970,751	9,982,571	9,991,349	9,997,085	9,999,776
38	9,970,973	9,982,742	9,991,470	9,997,155	9,999,795
39	9,971,194	9,982,912	9,991,590	9,997,224	9,999,813
40	9,971,414	9,983,082	9,991,709	9,997,292	9,999,831
41	9,971,633	9,983,251	9,991,827	9,997,359	9,999,847
42	9,971,851	9,983,419	9,991,944	9,997,425	9,999,863
43	9,972,069	9,983,586	9,992,060	9,997,491	9,999,878
44	9,972,286	9,983,752	9,992,175	9,997,556	9,999,892
45	9,972,502	9,983,917	9,992,290	9,997,620	9,999,905
46	9,972,717	9,984,081	9,992,404	9,997,683	9,999,917
47	9,972,931	9,984,245	9,992,517	9,997,745	9,999,928
48	9,973,145	9,984,408	9,992,629	9,997,806	9,999,940
49	9,973,358	9,984,570	9,992,740	9,997,867	9,999,950
50	9,973,570	9,984,731	9,992,850	9,997,927	9,999,959
51	9,973,781	9,984,891	9,992,960	9,997,986	9,999,967
52	9,973,991	9,985,050	9,993,069	9,998,044	9,999,974
53	9,974,200	9,985,209	9,993,177	9,998,101	9,999,980
54	9,974,408	9,985,367	9,993,284	9,998,157	9,999,986
55	9,974,615	9,985,524	9,993,390	9,998,212	9,999,989
56	9,974,822	9,985,680	9,993,495	9,998,267	9,999,993
57	9,975,028	9,985,835	9,993,599	9,998,321	9,999,996
58	9,975,233	9,985,989	9,993,703	9,998,374	9,999,998
59	9,975,437	9,986,143	9,993,806	9,998,426	10,000,000
60	9,975,640	9,986,295	9,993,908	9,998,477	10,000,000

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	0	1	2	3
0	0,000	174,550	349,207	524,078
1	2,909	177,459	352,120	526,995
2	5,818	180,369	355,033	529,911
3	8,727	183,279	357,945	532,828
4	11,636	186,189	360,858	535,745
5	14,544	189,100	363,770	538,663
6	17,452	192,010	366,683	541,580
7	20,361	194,920	369,596	544,498
8	23,270	197,830	372,508	547,415
9	26,179	200,740	375,421	550,333
10	29,088	203,650	378,334	553,251
11	31,996	206,561	381,247	556,169
12	34,905	209,471	384,160	559,087
13	37,814	212,381	387,073	562,005
14	40,723	215,291	389,987	564,923
15	43,632	218,201	392,900	567,841
16	46,541	221,111	395,814	570,759
17	49,450	224,022	398,727	573,678
18	52,359	226,932	401,641	576,596
19	55,268	229,842	404,554	579,514
20	58,177	232,752	407,468	582,433
21	61,086	235,663	410,382	585,352
22	63,995	238,574	413,295	588,270
23	66,904	241,485	416,209	591,189
24	69,813	244,395	419,123	594,108
25	72,722	247,306	422,037	597,028
26	75,631	250,217	424,951	599,947
27	78,540	253,128	427,866	602,866
28	81,450	256,038	430,780	605,786
29	84,359	258,949	433,694	608,705
30	87,268	261,859	436,609	611,625

	0	1	2	3
30	87,268	261,859	436,609	611,625
31	90,177	264,770	439,523	614,544
32	93,086	267,681	442,438	617,464
33	95,995	270,592	445,353	620,384
34	98,904	273,503	448,267	623,304
35	101,814	276,414	451,182	626,225
36	104,723	279,325	454,097	629,145
37	107,632	282,237	457,012	632,066
38	110,541	285,148	459,927	634,986
39	113,450	288,059	462,842	637,907
40	116,360	290,970	465,757	640,828
41	119,269	293,882	468,672	643,749
42	122,178	296,794	471,588	646,671
43	125,088	299,705	474,503	649,592
44	127,997	302,617	477,419	652,514
45	130,906	305,528	480,335	655,435
46	133,816	308,439	483,251	658,357
47	136,725	311,351	486,166	661,278
48	139,635	314,262	489,082	664,200
49	142,544	317,174	491,997	667,121
50	145,454	320,085	494,913	670,043
51	148,363	322,997	497,829	672,965
52	151,273	325,909	500,745	675,888
53	154,182	328,821	503,662	678,810
54	157,092	331,733	506,578	681,733
55	160,001	334,645	509,495	684,656
56	162,911	337,558	512,411	687,578
57	165,820	340,470	515,328	690,501
58	168,730	343,382	518,244	693,423
59	171,640	346,295	521,161	696,346
60	174,550	349,207	524,078	699,269

	4	5	6	7
0	699,269	874,886	1,051,042	1,227,846
1	702,193	877,817	1,053,983	1,230,798
2	705,116	880,748	1,056,924	1,233,751
3	708,039	883,680	1,059,866	1,236,704
4	710,962	886,611	1,062,808	1,239,658
5	713,886	889,543	1,065,750	1,242,612
6	716,809	892,475	1,068,692	1,245,566
7	719,733	895,407	1,071,634	1,248,520
8	722,657	898,339	1,074,576	1,251,474
9	725,580	901,271	1,077,518	1,254,428
10	728,504	904,204	1080,461	1,257,383
11	731,428	907,137	1,083,404	1,260,338
12	734,353	910,070	1,086,347	1,263,293
13	737,277	913,003	1,089,291	1,266,249
14	740,202	915,936	1,092,234	1,269,205
15	743,127	918,870	1,095,178	1,272,161
16	746,052	921,804	1,098,122	1,275,117
17	748,978	924,738	1,101,066	1,278,073
18	751,903	927,671	1,104,010	1,281,029
19	754,829	930,605	1,106,954	1,283,986
20	757,754	933,539	1,109,899	1,286,943
21	760,680	936,473	1,112,844	1,289,900
22	763,606	939,407	1,115,789	1,292,857
23	766,532	942,342	1,118,734	1,295,815
24	769,459	945,277	1,121,680	1,298,773
25	772,385	948,212	1,124,625	1,301,731
26	775,311	951,147	1,127,571	1,304,689
27	778,238	954,083	1,130,517	1,307,648
28	781,164	957,019	1,133,463	1,310,607
29	784,091	959,954	1,136,409	1,313,566
30	787,017	962,890	1,139,355	1 316,525

	4	5	6	7
30	787,017	962,890	1,139,355	1,316,525
31	789,944	965,826	1,142,302	1,319,485
32	792,871	968,763	1,145,249	1,322,445
33	795,799	971,699	1,148,196	1,325,405
34	798,726	974,636	1,151,144	1,328,365
35	801,653	977,573	1,154,092	1,331,325
36	804,581	980,509	1,157,040	1,334,285
37	807,509	983,446	1,159,988	1,337,246
38	810,437	986,383	1,162,936	1,340,207
39	813,365	989,320	1,165,884	1,343,168
40	816,293	992,257	1,168,832	1,346,129
41	819,221	995,195	1,171,781	1,349,091
42	822,150	998,133	1,174,730	1,352,053
43	825,079	1,001,072	1,177,679	1,355,015
44	828,008	1,004,010	1,180,628	1,357,977
45	830,937	1,006,949	1,183,577	1,360,940
46	833,866	1,009,887	1,186,527	1,363,903
47	836,795	1,012,825	1,189,477	1,366,866
48	839,724	1,015,763	1,192,427	1,369,830
49	842,653	1,018,702	1,195,377	1,372,793
50	845,583	1,021,641	1,198,328	1,375,757
51	848,513	1,024,580	1,201,279	1,378,721
52	851,443	1,027,519	1,204,230	1,381,686
53	854,374	1,030,459	1,207,181	1,384,650
54	857,304	1,033,399	1,210,132	1,387,615
55	860,234	1,036,339	1,213,084	1,390,580
56	863,164	1,039,279	1,216,036	1,393,545
57	866,095	1,042,219	1,218,988	1,396,510
58	869,025	1,045,160	1,221,940	1,399,476
59	871,956	1,048,101	1,224,892	1,402,442
60	874,886	1,051,042	1,227,845	1,405,408

	8	9	10	11
0	1,465,408	1,583,844	1,763,268	1,943,803
1	1,408,374	1,586,826	1,766,268	1,946,822
2	1,411,341	1,589,808	1,769,268	1,949,841
3	1,414,308	1,592,791	1,772,268	1,952,861
4	1,417,275	1,595,774	1,775,269	1,955,881
5	1,420,242	1,598,757	1,778,270	1,958,901
6	1,423,210	1,601,740	1,781,271	1,961,922
7	1,426,178	1,604,723	1,784,272	1,964,943
8	1,429,146	1,607,707	1,787,274	1,967,964
9	1,432,115	1,610,691	1,790,276	1,970,985
10	1,435,084	1,613,675	1,793,278	1,974,007
11	1,438,053	1,616,660	1,796,281	1,977,029
12	1,441,022	1,619,645	1,799,284	1,980,052
13	1,443,992	1,622,630	1,802,287	1,983,075
14	1,446,961	1,625,615	1,805,291	1,986,098
15	1,449,931	1,628,601	1,808,295	1,989,122
16	1,452,901	1,631,587	1,811,299	1,992,146
17	1,455,871	1,634,573	1,814,303	1,995,171
18	1,458,842	1,637,560	1,817,308	1,998,196
19	1,461,813	1,640,547	1,820,313	2,001,221
20	1,464,784	1,643,534	1,823,318	2,004,247
21	1,467,755	1,646,522	1,826,324	2,007,273
22	1,470,727	1,649,510	1,829,329	2,010,299
23	1,473,699	1,652,499	1,832,335	2,013,326
24	1,476,671	1,655,488	1,835,342	2,016,353
25	1,479,644	1,658,477	1,838,349	2,019,380
26	1,482,617	1,661,466	1,841,357	2,022,408
27	1,485,590	1,664,456	1,844,365	2,025,436
28	1,488,563	1,667,446	1,847,373	2,028,464
29	1,491,536	1,670,436	1,850,382	2,031,493
30	1,494,510	1,673,426	1,853,391	2,034,522

	8	9	10	11
30	1,494,510	1,673,426	1,853,391	2,034,522
31	1,497,484	1,676,417	1,856,400	2,037,552
32	1,500,458	1,679,408	1,859,409	2,040,582
33	1,503,433	1,682,399	1,862,419	2,043,612
34	1,500,408	1,685,390	1,865,429	2,046,643
35	1,509,383	1,688,382	1,868,439	2,049,674
36	1,512,358	1,691,374	1,871,449	2,052,705
37	1,515,334	1,694,366	1,874,460	2,055,737
38	1,518,310	1,697,358	1,877,471	2,058,769
39	1,521,286	1,700,351	1,880,482	2,061,801
40	1,524,262	1,703,344	1,883,494	2,064,834
41	1,527,239	1,706,337	1,886,506	2,067,867
42	1,530,216	1,709,331	1,889,518	2,070,900
43	1,533,193	1,712,325	1,892,531	2,073,934
44	1,536,170	1,715,319	1,895,544	2,076,968
45	1,539,148	1,718,313	1,898,558	2,080,002
46	1,542,126	1,721,308	1,901,572	2,083,037
47	1,545,104	1,724,304	1,904,586	2,086,073
48	1,548,082	1,727,300	1,907,601	2,089,109
49	1,551,061	1,730,296	1,910,616	2,092,145
50	1,554,040	1,733,292	1,913,632	2,095,182
51	1,557,019	1,736,287	1,916,648	2,098,219
52	1,559,999	1,739,284	1,999,664	2,101,256
53	1,562,979	1,742,281	1,922,680	2,104,293
54	1,565,959	1,745,278	1,925,697	2,107,331
55	1,568,440	1,748,275	1,928,714	2,110,369
56	1,571,920	1,751,273	1,931,731	2,113,407
57	1,574,901	1,754,271	1,934,749	2,116,446
58	1,577,882	1,757,270	1,937,767	2,119,485
59	1,580,863	1,760,269	1,940,785	2,122,525
60	1,583,844	1,763,268	1,943,803	2,125,565

	12	13	14	15
0	2,125,565	2,308,682	2,493,280	2,679,492
1	2,128,605	2,311,746	2,496,370	2,682,610
2	2,131,646	2,314,810	2,499,461	2,685,728
3	2,134,687	2,317,875	2,502,552	2,688,847
4	2,137,729	2,320,940	2,505,643	2,691,966
5	2,140,771	2,324,006	2,508,735	2,695,086
6	2,143,814	2,327,072	2,511,827	2,698,206
7	2,146,857	2,330,139	2,514,920	2,701,327
8	2,149,900	2,333,206	2,518,013	2,704,448
9	2,152,944	2,336,273	2,521,106	2,707,570
10	2,155,988	2,339,341	2,524,200	2,710,693
11	2,159,032	2,342,419	2,527,294	2,713,816
12	2,162,077	2,345,478	2,530,389	2,716,940
13	2,165,122	2,348,547	2,533,484	2,720,064
14	2,168,167	2,351,616	2,536,580	2,723,189
15	2,171,213	2,354,686	2,539,676	2,726,314
16	2,174,259	2,357,757	2,542,773	2,729,439
17	2,177,306	2,360,828	2,545,870	2,732,565
18	2,180,353	2,363,899	2,548,968	2,735,691
19	2,183,400	2,366,971	2,552,066	2,738,818
20	2,186,448	2,370,043	2,555,165	2,741,945
21	2,189,496	2,373,116	2,558,264	2,745,073
22	2,192,544	2,376,189	2,561,364	2,748,201
23	2,195,593	2,379,263	2,564,464	2,751,330
24	2,198,642	2,382,337	2,567,564	2,754,459
25	2,201,692	2,385,411	2,570,665	2,757,589
26	2,204,742	2,388,486	2,573,766	2,760,729
27	2,207,792	2,391,561	2,576,868	2,763,856
28	2,210,843	2,394,636	2,579,970	2,766,981
29	2,213,894	2,397,712	2,583,073	2,770,113
30	2,216,946	2,400,788	2,586,176	2,773,245

	12	13	14	15
30	2,216,946	2,400,788	2,586,176	2,773,245
31	2,219,998	2,403,865	2,589,280	2,776,378
32	2,223,051	2,406,942	2,592,384	2,779,511
33	2,226,104	2,410,020	2,595,489	2,782,645
34	2,229,157	2,413,098	2,598,594	2,785,779
35	2,232,211	2,416,176	2,601,700	2,788,914
36	2,235,265	2,419,255	2,604,806	2,792,050
37	2,238,319	2,422,334	2,607,912	2,795,186
38	2,241,374	2,425,414	2,611,019	2,798,323
39	2,244,429	2,428,494	2,614,126	2,801,460
40	2,247,485	2,431,574	2,617,234	2,804,597
41	2,250,541	2,434,655	2,620,342	2,807,735
42	2,253,597	2,437,736	2,623,451	2,810,873
43	2,256,654	2,440,818	2,626,560	2,814,012
44	2,259,711	2,443,900	2,629,670	2,817,151
45	2,262,769	2,446,983	2,632,780	2,820,291
46	2,265,827	2,450,066	2,635,891	2,823,432
47	2,268,885	2,453,150	2,639,002	2,826,573
48	2,271,944	2,456,234	2,642,114	2,829,714
49	2,275,003	2,459,319	2,645,226	2,832,856
50	2,278,063	2,462,404	2,648,339	2,835,999
51	2,281,123	2,465,490	2,651,452	2,839,142
52	2,284,183	2,468,576	2,654,566	2,842,286
53	2,287,244	2,471,662	2,657,680	2,845.430
54	2,290,305	2,474,749	2,660,795	2,848,575
55	2,293,367	2,477,836	2,663,910	2,851,720
56	2,296,429	2,480,924	2,667,026	2,854,866
57	2,299.492	2.484,012	2,670,142	2,858,012
58	2,302,555	2,487,101	2,673,258	2,861,159
59	2,305,618	2,490,191	2,676,375	2,864,306
60	2,308,682	2,493,280	2,679,492	2,867,453

	16	17	18	19
0	2,897,453	3,057,307	3,249,197	3,443,276
1	2,870,601	3,060,487	3,252,413	3,446,530
2	2,873,749	3,063,669	3,255,630	3,449,785
3	2,876,898	3,066,851	3,258,848	3,453,040
4	2,880,048	3,070,034	3,262,066	3,456,296
5	2,883,198	3,073,218	3,265,285	3,459,553
6	2,886,349	3,076,402	3,268,504	3,462,810
7	2,889,501	3,076,587	3,271,724	3,466,068
8	2,892,653	3,082,772	3,274,944	3,469,326
9	2,895,806	3,085,958	3,278,165	3,472,585
10	2,898,960	3,089,144	3,281,387	3,475,845
11	2,902,114	3,692,331	3,284,609	3,479,105
12	2,905,268	3,095,518	3,287,832	3,482,366
13	2,908,423	3,098,706	3,291,055	3,485,628
14	2,911,578	3,101,895	3,294,280	3,488,891
15	2,914,734	3,105,084	3,297,505	3,492,154
16	2,917,890	3,108,274	3,300,731	3,495,418
17	2,921,047	3,111,464	3,303,957	3,498,683
18	2,924,204	3,114,655	3,307,184	3,501,949
19	2,927,362	3,117,846	3,310,411	3,505,215
20	2,930,520	3,121,038	3,313,639	3,598,482
21	2,933,679	3,124,230	3,316,868	3,511,749
22	2,936,839	3,127,423	3,320,097	3,515,017
23	2,939,999	3,130,617	3,323,327	3,518,286
24	2,943,160	3,133,811	3,326,558	3,521,555
25	2,946,321	3,137,006	3,329,789	3,524,825
26	2,949,483	3,140,201	3,333,020	3,528,096
27	2,952,645	3,143,397	3,336,252	3,531,368
28	2,955,808	3,146,594	3,339,485	3,534,640
29	2,958,971	3,149,791	3,342,719	3,537,913
30	2,962,135	3,152,989	3,345,953	3,541,186

	16	17	18	19
30	2,962,135	3,152,989	3,345,953	3,541,186
31	2,965,299	3,156,187	3,349,188	3,544,460
32	2,968,464	3,159,386	3,352,423	3,547,735
33	2,971,629	3,162,585	3,355,659	3,551,010
34	2,974,795	3,165,785	3,358,896	3,554,286
35	2,977,962	3,168,986	3,362,133	3,557,563
36	2,981,129	3,172,187	3,365,371	3,560,840
37	2,984,297	3,175,389	3,368,610	3,564,118
38	2,987,465	3,178,591	3,371,850	3,567,397
39	2,990,634	3,181,794	3,375,090	3,570,676
40	2,993,804	3,184,998	3,378,331	3,573,956
41	2,996,973	3,188,202	3,381,572	3,577,237
42	3,000,143	3,191,407	3,384,814	3,580,519
43	3,003,314	3,194,613	3,388,057	3,583,801
44	3,006,486	3,197,819	3,391,300	3,587,084
45	3,009,658	3,201,026	3,394,544	3,590,367
46	3,012,831	3,204,233	3,397,798	3,593,651
47	3,016,004	3,207,441	3,401,033	3,596,936
48	3,019,178	3,210,649	3,404,279	3,600,221
49	3,022,353	3,213,858	3,407,525	3,603,507
50	3,025,528	3,217,067	3,410,772	3,606,794
51	3,028,703	3,220,277	3,414,020	3,610,082
52	3,031,879	3,223,488	3,417,268	3,613,370
53	3,035,055	3,226,699	3,420,517	3,616,659
54	3,038,232	3,229,911	3,423,766	3,619,949
55	3,041,410	3,233,124	3,427,016	3,623,239
56	3,044,588	3,236,337	3,430,267	3,626,530
57	3,047,757	3,239,551	3,433,518	3,629,822
58	3,050,946	3,242,766	3,436,770	3,633,115
59	3,054,126	3,245,981	3,440,023	3,636,408
60	3,057,307	3,249,197	3,443,276	3,639,702

	02	21	22	32
0	3,639,702	3,838,640	4,040,262	4,244,748
1	3,642,997	3,841,978	4,043,647	4,248,182
2	3,646,293	3,845,316	4,047,031	4,251,617
3	3,649,589	3,848,655	4,050,416	4,255,052
4	3,652,886	3,851,995	4,053,802	4,258,488
5	3,656,183	3,855,336	4,057,189	4,261,925
6	3,659,481	3,858,678	4,060,577	4,265,363
7	3,662,780	3,862,020	4,063,966	4,268,801
8	3,666,079	3,865,363	4,067,356	4,272,240
9	3,669,379	3,868,707	4,070,747	4,275,680
10	3,672,680	3,872,052	4,074,139	4,279,121
11	3,675,982	3,875,397	4,077,531	4,282,563
12	3,679,284	3,878,743	4,080,924	4,286,006
13	3,682,587	3,882,090	4,084,318	4,289,450
14	3,685,891	3,885,438	4,087,713	4,292,895
15	3,689,195	3,888,787	4,091,109	4,296,340
16	3,692,500	3,892,136	4,094,506	4,299,786
17	3,695,806	3,895,486	4,097,903	4,303,233
18	3,699,113	3,898,837	4,101,301	4,306,681
19	3,702,420	3,902,188	4,104,699	4,310,130
20	3,705,728	3,905,540	4,108,097	4,313,580
21	3,709,037	3,908,893	4,111,497	4,317,031
22	3,712,347	3,912,247	4,114,898	4,320,482
23	3,715,657	3,915,601	4,118,300	4,323,934
24	3,718,968	3,918,956	4,121,704	4,327,387
25	3,722,279	3,922,312	4,125,107	4,330,841
26	3,725,591	3,925,669	4,128,511	4,334,296
27	3,728,904	3,929,027	4,131,916	4,337,752
28	3,732,218	3,932,385	4,135,322	4,341,209
29	3,735,533	3,935,744	4,138,728	4,344,666
30	3,738,848	3,939,104	4,142,135	4,348,124

	20	21	22	23
30	3,738,848	3,939,104	4,142,135	4,348,124
31	3,742,164	3,942,465	4,145,544	4,351,583
32	3,745,480	3,945,826	4,148,953	4,355,043
33	3,748,797	3,949,188	4,152,363	4,358,504
24	3,752,115	3,952,551	4,155,773	4,361,966
35	3,755,434	3,955,915	4,159,184	4,365,429
36	3,758,753	3,959,280	4,162,596	4,368,893
37	3,762,073	3,962,646	4,166,009	4,372,357
38	3,765,394	3,966,012	4,169,423	4,375,822
39	3,768,716	3,969,379	4,172,838	4,379,288
40	3,772,038	3,972,746	4,176,255	4,382,755
41	3,775,361	3,976,114	4,179,672	4,386,223
42	3,778,685	3,979,483	4,183,090	4,389,692
43	3,782,010	3,982,853	4,186,509	4,393,162
44	3,785,335	3,986,224	4,189,928	4,396,633
45	3,788,661	3,989,596	4,193,348	4,400,105
46	3,791,988	3,992,969	4,196,769	4,403,578
47	3,795,315	3,996,342	4,200,191	4,407,051
48	3,798,643	3,999,716	4,203,613	4,410,525
49	3,801,972	4,003,090	4,207,036	4,414,000
50	3,805,302	4,006,465	4,210,460	4,417,476
51	3,808,632	4,009,841	4,213,885	4,420,953
52	3,811,963	4,013,217	4,217,311	4,424,431
53	3,815,295	4,016,594	4,220,738	4,427,910
54	3,818,628	4,019,972	4,224,165	4,431,390
55	3,821,961	4,023,351	4,227,593	4,434,871
56	3,825,295	4,026,731	4,231,022	4,438,352
57	3,828,630	4,030,112	4,234,452	4,441,834
58	3,831,966	4,033,494	4,237,883	4,445,317
59	3,835,303	4,036,877	4,241,315	4,448,801
60	3,838,640	4,040,262	4,244,748	4,452,286

	24	25	26	27
0	4,452,286	4,663,081	4,877,328	5,095,254
1	4,455,772	4,666,623	4,880,930	5,098,919
2	4,459,259	4,670,166	4,884,533	5,102,585
3	4,462,747	4,673,710	4,888,137	5,106,252
4	4,466,236	4,677,255	4,891,742	5,109,920
5	4,469,726	4,680,801	4,895,347	5,113,589
6	4,473,216	4,684,348	4,898,953	5,117,259
7	4,476,707	4,687,896	4,902,560	5,120,930
8	4,480,199	4,691,444	4,206,168	5,124,602
9	4,483,692	4,694,993	4,909,777	5,128,275
10	4,487,186	4,698,543	4,913,387	5,131,949
11	4,490,681	4,702,094	4,916,998	5,135,625
12	4,494,177	4,705,646	4,920,610	5,139,302
13	4,497,674	4,709,199	4,924,223	5,142,980
14	4,501,172	4,712,753	4,927,838	5,146,659
15	4,504,671	4,716,308	4,931,454	5,150,339
16	4,508,171	4,719,864	4,935,071	5,154,020
17	4,511,672	4,723,422	4,938,689	5,157,702
18	4,515,173	4,726,981	4,942,308	5,161,385
19	4,518,675	4,730,541	4,945,928	5,165,069
20	4,522,178	4,734,102	4,949,549	5,168,755
21	4,525,682	4,737,664	4,953,171	5,172,442
22	4,529,187	4,741,227	4,956,794	5,176,130
23	4,532,693	4,744,790	4,960,418	5,179,819
24	4,536,200	4,748,354	4,964,043	5,183,509
25	4,539,708	4,751,919	4,967,669	5,187,200
26	4,543,217	4,755,485	4,971,296	5,190,892
27	4,546,727	4,759,052	4,974,924	5,194,585
28	4,550,238	4,762,620	4,978,553	5,198,279
29	4,553,750	4,766,189	4,982,184	5,201,974
30	4,557,264	4,769,759	4,985,816	5,205,670

	24	25	26	27
30	4,557,264	4,769,759	4,985,816	5,205,670
31	4,560,778	4,773,330	4,989,448	5,209,368
32	4,564,293	4,776,902	4,993,081	5,213,067
33	4,567,809	4,780,475	4,996,716	5,216,767
34	4,571,326	4,784,049	5,000,352	5,220,868
35	4,574,843	4,787,624	5,003,989	5,224,170
36	4,578,361	4,791,200	5,007,627	5,227,873
37	4,581,880	4,794,777	5,011,266	5,231,577
38	4,585,400	4,798,355	5,014,906	5,235,283
39	4,588,921	4,801,934	5,018,547	5,238,990
40	4,592,443	4,805,515	5,022,189	5,242,698
41	4,595,966	4,809,096	5,025,832	5,246,407
42	4,599,490	4,812,678	5,029,476	5,250,117
43	4,603,015	4,816,261	5,033,121	5,253,828
44	4,606,541	4,819,845	5,036,767	5,257,540
45	4,610,068	4,823,430	5,040,414	5,261,254
46	4,613,596	4,827,016	5,044,062	5,264,969
47	4,617,125	4,830,603	5,047,712	5,268,685
48	4,620,654	4,834,191	5,051,363	5,272,402
49	4,624,184	4,837,780	5,055,015	5,276,120
50	4,627,715	4,841,371	5,058,668	5,279,839
51	4,631,247	4,844,962	5,062,322	5,283,959
52	4,634,780	4,848,554	5,065,977	5,287,280
53	4,638,314	4,852,147	5,069,633	5,291,003
54	4,641,849	4,855,741	5,073,290	5,294,727
55	4,645,385	4,859,336	5,076,948	5,298,452
56	4,648,922	4,862,932	5,080,607	5,302,168
57	4,652,460	4,866,529	5,084,267	5,305,905
58	4,655,999	4,870,127	5,087,928	5,309,633
59	4,659,540	4,873,727	5,091,590	5,313,363
60	4,663,081	4,877,328	5,095,254	5,317,094

	28	29	30	31
0	5,317,094	5,543,090	5,773,502	6,008,606
1	5,320,826	5,546,893	5,777,381	6,012,566
2	5,324,559	5,550,697	5,781,262	6,016,528
3	5,328,293	5,554,503	5,785,144	6,020,491
4	5,332,028	5,558,310	5,789,027	6,024,455
5	5,335,765	5,562,118	5,792,911	6,028,420
6	5,339,503	5,565,927	5,796,797	6,032,387
7	5,343,242	5,569,738	5,800,684	6,036,355
8	5,346,982	5,573,550	5,804,572	6,040,324
9	5,350,723	5,577,363	5,808,462	6,044,295
10	5,354,465	5,581,177	5,812,353	6,048,267
11	5,358,209	5,584,993	5,816,245	6,052,241
12	5,361,954	5,588,810	5,820,139	6,056,216
13	5,365,700	5,592,628	5,824,034	6,060,193
14	5,369,447	5,596,447	5,827,930	6,064,171
15	5,373,195	5,600,268	5,831,828	6,068,150
16	5,376,944	5,604,090	5,835,727	6,072,131
17	5,380,694	5,607,913	5,839,627	6,076,113
18	5,384,445	5,611,737	5,843,528	6,080,096
19	5,388,198	5,615,562	5,847,431	6,084,081
20	5,391,952	5,619,388	5,851,335	6,088,067
21	5,395,707	5,623,216	5,855,241	6,092,055
22	5,399,463	5,627,045	5,859,148	6,096,044
23	5,403,221	5,630,875	5,863,056	6,100,035
24	5,406,980	5,634,707	5,866,966	6,104,027
25	5,410,740	5,638,540	5,870,877	6,108,020
26	5,414,501	5,642,374	5,874,789	6,112,015
27	5,418,263	5,646,210	5,878,702	6,116,011
28	5,422,026	5,650,047	5,882,617	6,120,009
29	5,425,791	5,653,885	5,886,533	6,124,008
30	5,429,557	5,657,725	5,890,450	6,128,008

	28	29	30	31
30	5,429,557	5,657,725	5,890,450	6,128,008
31	5,433,324	5,661,566	5,894,369	6,132,010
32	5,437,092	5,665,408	5,898,289	6,136,013
33	5,440,861	5,669,251	5,902,211	6,140,018
34	5,444,632	5,673,096	5,906,134	6,144,024
35	5,448,404	5,676,942	5,910,058	6,148,032
36	5,452,177	5,680,789	5,913,984	6,152,041
37	5,455,951	5,684,637	5,917,911	6,156,052
38	5,459,726	5,688,486	5,921,839	6,160,064
39	5,463,503	5,692,337	5,925,769	6,164,077
40	5,467,281	5,696,189	5,929,700	6,168,092
41	5,471,060	5,700,043	5,933,633	6,172,108
42	5,474,840	5,703,898	5,937,567	6,176,126
43	5,478,621	5,707,754	5,941,502	6,180,147
44	5,482,404	5,711,611	5,945,438	6,184,168
45	5,486,188	5,715,469	5,949,376	6,188,190
46	5,489,973	5,719,329	5,953,315	6,192,213
47	5,493,759	5,723,190	5,957,255	6,196,237
48	5,497,546	5,727,052	5,961,197	6,200,263
49	5,501,335	5,730,916	5,965,140	6,204,290
50	5,505,125	5,734,781	5,969,084	6,208,319
51	5,508,916	5,738,647	5,973,030	6,212,350
52	5,512,708	5,742,515	5,976,979	6,216,382
53	5,516,501	5,746,384	5,980,926	6,220,416
54	5,520,296	5,750,254	5,984,876	6,224,451
55	5,524,092	5,754,125	5,988,827	6,228,488
56	5,527,889	5,757,998	5,992,780	6,232,526
57	5,531,687	5,761,872	5,996,734	6,236,566
58	5,535,487	5,765,747	6,000,690	6,240,607
59	5,539,288	5,769,624	6,004,647	6,244,649
60	5,543,090	5,773,502	6,008,606	6,248,693

	32	33	34	35
0	6,248,693	6,494,076	6,745,085	7,002,075
1	6,252,738	6,498,212	6,749,318	7,006,411
2	6,256,785	6,502,350	6,753,553	7,010,749
3	6,260,834	6,506,489	6,757,789	7,015,088
4	6,264.884	6,510,630	6,762,027	7,019,429
5	6,268,935	6,514,773	6,766,267	7,023,772
6	6,272,988	6,518,917	6,770,508	7,028,117
7	6,277,042	6,523,063	6,774,751	7,032,463
8	6,281,098	6,527,200	6,778,996	7,036,811
9	6,285,155	6,531,359	6,783,243	7,041,161
10	6,289,214	6,535,510	6,787,401	7,045,513
11	6,293,274	6,539,662	6,791,741	7,049,867
12	6,297,336	6,543,816	6,795,993	7,054,223
13	6,301,399	6,547,971	6,800,246	7,058,581
14	6,305,464	6,552,128	6,804,501	7,062,940
15	6,309,530	6,556,287	6,808,758	7,067,301
16	6,313,598	6,560,447	6,813,016	7,071,664
17	6,317,667	6,564,609	6,817,276	7,076,029
18	6,321,738	6,568,772	6,821,538	7,070,395
19	6,325,810	6,572,937	6,825,801	7,084,763
20	6,329,883	6,577,103	6,830,066	7,089,133
21	6,333,958	6,581,271	6,834,333	7,093,505
22	6,338,034	6,585,440	6,838,602	7,097,879
23	6,342,112	6,589,611	6,842,872	7,102,254
24	6,346,191	6,593,784	6,847,144	7,106,631
25	6,350,272	6,597,958	6,851,417	7,111,010
26	6,354,355	6,602,134	6,855,692	7,115,391
27	6,358,439	6,606,312	6,859,969	7,119,773
28	6,362,525	6,610,491	6,864,247	7,124,167
29	6,366,613	6,614,672	6,868,527	7,128,543
30	6,370,702	6,618,855	6,872,809	7,132,931

	32	33	34	35
30	6,370,702	6,618,855	6,872,809	7,132,931
31	6,374,792	6,623,039	6,877,093	7,137,321
32	6,378,884	6,627,225	6,881,379	7,141,713
33	6,382,977	6,631,413	6,885,666	7,146,106
34	6,387,072	6,635,603	6,889,955	7,150,501
35	6,391,169	6,639,792	6,894,246	7,154,878
36	6,395,267	6,643,984	6,898,539	7,159,298
37	6,399,366	6,648,178	6,902,833	7,163,698
38	6,403,467	6,652,373	6,907,129	7,168,100
39	6,407,569	6,656,570	6,911,426	7,172,504
40	6,411,673	6,660,768	6,915,725	7,176,910
41	6,415,779	6,664,968	6,920,026	7,181,318
42	6,419,886	6,669,170	6,924,329	7,185,728
43	6,423,995	6,673,373	6,928,634	7,190,140
44	6,428,105	6,677,578	6,932,940	7,194,554
45	6,432,216	6,681,785	6,937,248	7,198,970
46	6,436,329	6,685,994	6,941,558	7,203,387
47	6,440,444	6,690,204	6,945,869	7,207,806
48	6,444,560	6,694,416	6,950,182	7,212,227
49	6,448,678	6,698,630	6,954,497	7,216,650
50	6,452,798	6,702,845	6,958,813	7,221,075
51	6,456,919	6,707,062	6,963,131	7,225,502
52	6,461,042	6,711,281	6,967,451	7,229,931
53	6,465,166	6,715,501	6,971,773	7,234,362
54	6,469,292	6,719,723	6,976,097	7,238,794
55	6,473,419	6,723,946	6,980,423	7,243,228
56	6,477,548	6,728,171	6,984,750	7,247,664
57	6,481,678	6,732,397	6,989,079	7,252,102
58	6,485,809	6,736,625	6,993,409	7,256,541
59	6,489,942	6,740,854	6,997,741	7,260,982
60	6,494,976	6,745,084	7,002,075	7,265,424

	36	37	38	39
0	7,265,424	7,535,541	7,812,856	8,097,840
1	7,269,869	7,540,103	7,817,542	8,102,658
2	7,274,316	7,544,667	7,822,230	8,107,478
3	7,278,765	7,549,233	7,826,920	8,112,300
4	7,283,216	7,553,801	7,831,612	8,117,124
5	7,287,669	7,558,371	7,836,306	8,121,951
6	7,292,124	7,562,943	7,841,002	8,126,780
7	7,296,581	7,567,517	7,845,700	8,131,611
8	7,301,040	7,572,093	7,850,400	8,136,444
9	7,305,501	7,576,670	7,855,102	8,141,280
10	7,309,963	7,581,249	7,859,807	8,146,118
11	7,314,427	7,585,830	7,864,514	8,150,958
12	7,318,893	7,590,413	7,869,223	8,155,801
13	7,323,361	7,594,999	7,873,934	8,160,646
14	7,327,831	7,599,587	7,878,647	8,165,493
15	7,332,303	7,604,177	7,883,363	8,170,343
16	7,336,777	7,608,769	7,888,081	8,175,195
17	7,341,253	7,613,363	7,892,801	8,180,049
18	7,345,731	7,617,959	7,897,523	8,184,905
19	7,350,210	7,622,557	7,902,247	8,189,764
20	7,354,691	7,627,157	7,906,973	8,194,625
21	7,359,174	7,631,759	7,911,702	8,199,488
22	7,363,659	7,636,363	7,916,433	8,204,354
23	7,368,146	7,640,969	7,921,166	8,209,222
24	7,372,635	7,645,577	7,925,901	8,214,092
25	7,377,126	7,650,187	7,930,638	8,218,965
26	7,381,619	7,654,799	7,935,378	8,223,840
27	7,386,114	7,659,413	7,940,120	8,228,717
28	7,390,611	7,664,030	7,944,864	8,233,597
29	7,395,110	7,668,649	7,949,610	8,238,479
30	7,399,610	7,673,270	7,954,358	8,243,363

	36	37	38	39
30	7,399,610	7,673,270	7,954,358	8,243,363
31	7,404,112	7,677,893	7,959,109	8,248,250
32	7,408,616	7,682,518	7,963,862	8,253,139
33	7,413,122	7,687,145	7,968,617	8,258,031
34	7,417,630	7,691,774	7,973,374	8,262,925
35	7,422,140	7,696,405	7,978,133	8,267,821
36	7,426,652	7,701,038	7,982,895	8,272,720
37	7,431,167	7,705,673	7,987,659	8,277,621
38	7,435,684	7,710,310	7,992,425	8,282,524
39	7,440,203	7,714,949	7,997,193	8,287,429
40	7,444,724	7,719,590	8,001,963	8,292,337
41	7,449,246	7,724,233	8,006,736	8,297,247
42	7,453,770	7,728,878	8,011,511	8,302,160
43	7,458,296	7,733,525	8,016,288	8,307,075
44	7,462,824	7,738,175	8,021,067	8,311,992
45	7,467,354	7,742,827	8,025,849	8,316,912
46	7,471,886	7,747,481	8,030,633	8,321,834
47	7,476,420	7,752,137	8,035,419	8,326,759
48	7,480,956	7,756,795	8,040,207	8,331,686
49	7,485,494	7,761,455	8,044,997	8,336,615
50	7,490,033	7,766,117	8,049,790	8,341,547
51	7,494,574	7,770,781	8,054,585	8,346,481
52	7,499,117	7,775,447	8,059,382	8,351,418
53	7,503,663	7,780,116	8,064,181	8,356,357
54	7,508,211	7,784,787	8,068,983	8,361,298
55	7,512,761	7,789,460	8,073,787	8,366,242
56	7,517,313	7,794,135	8,078,593	8,371,188
57	7,521,867	7,798,812	8,083,401	8,376,136
58	7,526,423	7,803,491	8,088,212	8,381,087
59	7,530,981	7,808,172	8,093,025	8,386,040
60	7,535,541	7,812,856	8,097,840	8,390,996

	40	41	42	43
0	8,390,996	8,692,867	9,004,040	9,325,151
1	8,395,954	8,697,075	9,009,308	9,330,591
2	8,400,915	8,703,085	9,014,579	9,336,034
3	8,405,878	8,708,198	9,019,853	9,341,480
4	8,410,844	8,713,344	9,025,130	9,346,929
5	8,415,812	8,718,433	9,030,410	9,352,381
6	8,420,782	8,723,555	9,035,693	9,357,835
7	8,425,754	8,728,679	9,040,978	9,363,292
8	8,430,729	8,733,806	9,046,266	9,368,752
9	8,435,706	8,738,935	9,051,557	9,374,215
10	8,440,686	8,744,067	9,056,850	9,379,682
11	8,445,668	8,749,201	9,062,146	9,385,152
12	8,450,653	8,754,338	9,067,445	9,390,625
13	8,455,640	8,759,478	9,072,747	9,596,101
14	8,460,630	8,764,620	9,078,052	9,401,580
15	8,465,622	8,769,764	9,083,360	9,407,062
16	8,470,617	8,774,911	9,088,670	9,412,547
17	8,475,614	8,780,061	9,093,983	9,418,034
18	8,480,614	8,785,214	9,099,299	9,423,524
19	8,485,617	8,790,369	9,104,618	9,429,017
20	8,490,622	8,795,527	9,109,940	9,434,513
21	8,495,629	8,800,688	9,115,265	9,440,012
22	8,500,639	8,805,851	9,120,593	9,445,514
23	8,505,651	8,811,017	9,125,923	9,451,019
24	8,510,666	8,816,186	9,131,256	9,456,528
25	8,515,683	8,821,357	9,136,592	9,462,040
26	8,520,703	8,826,531	9,141,930	9,467,555
27	8,525,725	8,831,708	9,147,271	9,473,073
28	8,530,750	8,836,887	9,152,615	9,478,594
29	8,535,777	8,842,069	9,157,962	9,484,118
30	8,540,806	8,847,253	9,163,312	9,489,645

	40	41	42	43
30	8,540,806	8,847,253	9,163,312	9,489,645
31	8,545,838	8,852,440	9,168,665	9,495,175
32	8,550,872	8,857,630	9,174,021	9,400,708
33	8,555,909	8,862,822	9,179,380	9,506,244
34	8,560,949	8,868,017	9,184,741	9,511,783
35	8,565,991	8,873,215	9,190,105	9,517,325
36	8,571,036	8,878,415	9,195,472	9,522,870
37	8,576,083	8,883,628	9,200,842	9,528,419
38	8,581,133	8,888,824	9,206,215	9,533,971
39	8,586,185	8,894,033	9,211,590	9,539,526
40	8,591,239	8,899,244	9,216,968	9,545,084
41	8,596,296	8,904,458	9,222,349	9,550,645
42	8,601,355	8,909,675	9,227,733	9,556,209
43	8,606,417	8,914,894	9,233,120	9,561,776
44	8,611,482	8,920,116	9,238,510	9,567,346
45	8,616,549	8,925,341	9,243,903	9,572,919
46	8,621,619	8,930,568	9,249,399	9,578,495
47	8,626,692	8,935,798	9,254,698	9,584,074
48	8,631,767	8,941,031	9,260,100	9,589,656
49	8,636,845	8,946,267	9,265,505	9,595,241
50	8,641,926	8,951,506	9,270,913	9,600,830
51	8,647,009	8,956,747	9,276,324	9,606,422
52	8,652,095	8,961,991	9,281,738	9,612,017
53	8,657,183	8,967,238	9,287,155	9,617,615
54	8,662,273	8,972,487	9,292,574	9,623,216
55	8,667,366	8,977,739	9,297,996	9,628,820
56	8,672,461	8,982,994	9,303,421	9,634,427
57	8,677,559	8,988,252	9,308,849	9,640,037
58	8,682,659	8,993,512	9,314,280	9,645,651
59	8,687,762	8,998,775	9,319,714	9,651,268
60	8,692,867	9.004,040	9,325,151	9,656,888

	44	45	46	47
0	9,656,888	10,000,000	10,355,302	10,723,686
1	9,662,511	10,005,820	10,361,332	10,729,942
2	9,668,137	10,011,643	10,367,365	10,736,202
3	9,673,766	10,017,469	10,373,402	10,742,466
4	9,679,398	10,023,299	10,379,443	10,748,734
5	9,685,034	10,029,132	10,385,487	10,755,006
6	9,690,674	10,034,968	10,391,535	10,707,282
7	9,696,315	10,040,808	10,397,587	10,768,562
8	9,701,960	10,046,651	10,403,643	10,773,845
9	9,707,609	10,052,497	10,409,702	10,780,132
10	9,713,261	10,058,347	10,415,765	10,786,423
11	9,718,916	10,064,201	10,421,832	10,792,718
12	9,724,574	10,070,058	10,427,902	10,799,017
13	9,730,235	10,075,918	10,433,976	10,805,320
14	9,735,900	10,081,782	10,440,054	10,811,627
15	9,741,568	10,087,649	10,446,135	10,817,938
16	9,747,239	10,093,520	10,452,220	10,824,253
17	9,752,913	10,099,394	10,458,309	10,830,572
18	9,758,591	10,105,272	10,464,401	10,836,895
19	9,764,272	10,111,153	10,470,407	10,843,222
20	9,769,956	10,117,038	10,476,597	10,849,554
21	9,775,643	10,122,926	10,482,701	10,855,889
22	9,781,334	10,128,818	10,488,808	10,862,228
23	9,787,028	10,134,713	10,494,919	10,868,571
24	9,792,725	10,140,611	10,501,034	10,874,918
25	9,798,425	10,146,513	10,507,153	10,881,269
26	9,804,128	10,152,418	10,513,275	10,887,624
27	9,809,835	10,158,327	10,519,401	10,893,983
28	9,815,545	10,164,239	10,525,531	10,900,346
29	9,821,258	10,170,154	10,531,664	10,906,713
30	9,826,974	10,176,073	10,537,801	10,913,084

	44	45	46	47
30	9,826,974	10,176,073	10,537,801	10,913,084
31	9,832,694	10,181,996	10,543,942	10,919,459
32	9,838,417	10,187,922	10,550,087	10,925,838
33	9,844,143	10,193,852	10,556,235	10,932,221
34	9,849,872	10,199,785	10,562,387	10,938,608
35	9,855,605	10,205,722	10,568,543	10,945,000
36	9,861,341	10,211,663	10,574,703	10,951,396
37	9,867,080	10,217,607	10,580,867	10,957,796
38	9,872,822	10,223,555	10,587,034	10,964,200
39	9,878,968	10,229,506	10,593,205	10,970,608
40	9,884,317	10,235,460	10,599,280	10,977,020
41	9,890,070	10,241,418	10,605,559	10,983,436
42	9,895,826	10,247,380	10,611,742	10,989,856
43	9,901,585	10,253,345	10,617,929	10,996,280
44	9,907,347	10,259,314	10,624,119	11,002,708
45	9,913,113	10,265,286	10,630,313	11,009,140
46	9,918,882	10,271,262	10,636,511	11,015,577
47	9,924,654	10,277,242	10,642,713	11,022,028
48	9,930,430	10,283,225	10,648,919	11,028,463
49	9,936,209	10,289,212	10,655,128	11,034,912
50	9,941,991	10,295,202	10,661,341	11,041,365
51	9,947,777	10,301,196	10,667,558	11,047,822
52	9,953,566	10,307,193	10,673,779	11,054,283
53	9,959,359	10,313,194	10,680,004	11,060,748
54	9,965,155	10,319,199	10,686,233	11,067,218
55	9,970,954	10,325,207	10,692,466	11,073,692
56	9,976,756	10,331,219	10,698,702	11,080,170
57	9,982,562	10,337,234	10,704,942	11,086,652
58	9,988,371	10,343,253	10,711,186	11,093,138
59	9,994,184	10,349,276	10,717,434	11,099,629
60	10,000,000	10,355,302	10,723,686	11,106,124

	48	49	50	51
0	11,106,124	11,503,684	11,917,537	12,348,972
1	11,112,623	11,510,444	11,924,580	12,356,320
2	11,119,126	11,517,208	11,931,628	12,363,673
3	11,125,634	11,523,977	11,938,680	12,371,031
4	11,132,146	11,530,751	11,945,737	12,378,394
5	11,138,662	11,537,529	11,952,799	12,385,762
6	11,145,182	11,544,312	11,959,866	12,393,136
7	11,151,706	11,551,100	11,966,938	12,400,515
8	11,158,235	11,557,893	11,974,015	12,407,999
9	11,164,768	11,564,691	11,981,097	12,415,288
10	11,171,305	11,571,494	11,988,183	12,422,683
11	11,177,846	11,578,301	11,995,274	12,430,083
12	11,184,392	11,585,112	12,002,370	12,437,489
13	11,190,942	11,591,928	12,009,471	12,444,900
14	11,197,496	11,598,748	12,016,578	12,452,317
15	11,204,054	11,605,572	12,023,690	12,459,739
16	11,210,617	11,612,401	12,030,807	12,467,167
17	11,217,184	11,619,234	12,037,929	12,474,600
18	11,223,755	11,626,072	12,045,056	12,482,039
19	11,230,330	11,632,915	12,052,188	12,489,484
20	11,236,910	11,639,763	12,059,325	12,496,934
21	11,243,494	11,646,615	12,066,457	12,504,389
22	11,250,082	11,653,472	12,073,614	12,511,850
23	11,256,675	11,660,334	12,080,766	12,519,316
24	11,263,272	11,667,200	12,087,923	12,526,787
25	11,269,873	11,674,071	12,095,085	12,534,264
26	11,276 478	11,680,947	12,102,252	12,541,746
27	11,283,038	11,687,827	12,109,424	12,549,233
28	11,289,702	11,694,712	12,116,601	12,556,725
29	11,296,321	11,701,602	12,123,783	12,564,222
30	11,302,944	11,708,497	12,130,970	12,571,724

	48	49	50	51
30	11,302,944	11,708,497	12,130,970	12,571,724
31	11,309,571	11,715,396	12,138,162	12,579,232
32	11,316,203	11,722,300	12,145,359	12,586,746
33	11,322,899	11,729,208	12,152,561	12,594,265
34	11,329,480	11,736,121	12,159,768	12,601,790
35	11,336,125	11,743,039	12,166,981	12,609,321
36	11,342,774	11,749,962	12,174,199	12,616,858
37	11,349,428	11,756,889	12,181,422	12,624,400
38	11,356,086	11,763,821	12,188,650	12,631,948
39	11,362,748	11,770,758	12,195,883	12,639,501
40	11,369,415	11,777,700	12,203,121	12,647,060
41	11,376,086	11,784,646	12,210,364	12,654,624
42	11,382,762	11,791,597	12,217,613	12,662,294
43	11,389,442	11,798,553	12,224,867	12,669,769
44	11,396,126	11,805,514	12,232,126	12,677,350
45	11,402,815	11,812,479	12,239,390	12,684,937
46	11,409,508	11,819,449	12,246,659	12,692,530
47	11,416,206	11,826,424	12,253,933	12,700,128
48	11,422,908	11,833,404	12,261,212	12,707,732
49	11,429,615	11,840,388	12,268,496	12,715,341
50	11,436,326	11,847,377	12,275,786	12,722,956
51	11,443,042	11,854,371	12,283,081	12,730,577
52	11,449,762	11,861,370	12,290,381	12,738,203
53	11,456,487	11,868,374	12,297,687	12,745,835
54	11,463,216	11,875,383	12,304,998	12,753,473
55	11,469,950	11,882,397	12,312,314	12,761,116
56	11,476,688	11,889,417	12,319,635	12,768,765
57	11,483,431	11,896,438	12,326,961	12,776,420
58	11,490,178	11,903,466	12,334,293	12,784,080
59	11,496,929	11,910,499	12,341,630	12,791,745
60	11,503,684	11,917,537	12,348,972	12,799,416

	52	53	54	55
0	12,799,416	13,270,448	13,763,820	14,281,480
1	12,807,093	13,278,483	13,772,243	14,290,325
2	12,814,776	13,286,524	13,780,673	14,299,177
3	12,822,465	13,294,571	13,789,109	14,308,037
4	12,830,159	13,302,624	13,797,552	14,316,905
5	12,837,859	13,310,683	13,806,002	14,325,780
6	12,845,565	13,318,749	13,814,459	14,334,662
7	12,853,277	13,326,821	13,822,922	14,343,552
8	12,860,994	13,334,899	13,831,392	14,352,449
9	12,868,717	13,342,984	13,839,869	14,361,354
10	12,876,445	13,351,075	13,848,352	14,370,266
11	12,884,179	13,359,172	13,856,842	14,379,186
12	12,891,919	13,367,276	13,865,339	14,388,113
13	12,899,665	13,375,386	13,873,843	14,297,048
14	12,907,417	13,383,502	13,882,354	14,405,990
15	12,915,175	13,391,624	13,890,872	14,414,939
16	12,922,939	13,399,753	13,899,397	14,423,896
17	12,930,709	13,407,888	13,907,930	14,432,861
18	12,938,485	13,416,029	13,016,470	14,441,833
19	12,946,267	13,424,177	13,925,017	14,450,812
20	12,954,055	13,432,331	13,933,571	14,459,799
21	12,961,848	13,440,492	13,942,131	14,468,794
22	12,969,647	13,448,659	13,950,698	14,477,797
23	12,977,457	13,456,832	13,959,272	14,486,807
24	12,985,263	13,465,011	13,967,853	14,495,825
25	12,993,080	13,473,197	13,976,441	14,504,850
26	13,000,903	13,481,390	13,985,035	14,513,883
27	13,008,732	13,489,589	13,993,636	14,522,924
28	13,016,567	13,497,794	14,002,244	14,531,972
29	13,024,407	13,506,006	14,010,859	14,541,028
30	13,032,253	13,514,224	14,019,481	14,550,0[illegible]1

	52	53	54	55
30	13,032,253	13,514,224	14,019,481	14,550,091
31	13,040,105	13,522,449	14,028,110	14,559,162
32	13,047,965	13,530,680	14,036,746	14,568,241
33	13,055,827	13,538,918	14,045,389	14,577,327
34	13,063,697	13,547,162	14,054,040	14,586,421
35	13,071,573	13,555,413	14,062,698	14,595,523
36	13,079,455	13,563,670	14,071,363	14,604,633
37	13,087,343	13,571,934	14,080,035	14,613,750
38	13,095,237	13,580,204	14,088,715	14,622,875
39	13,103,138	13,588,481	14,097,402	14,632,007
40	13,111,045	13,596,764	14,106,097	14,641,146
41	13,118,958	13,605,054	14,114,798	14,650,293
42	13,126,877	13,613,350	14,123,506	14,659,449
43	13,134,802	13,621,653	14,132,221	14,668,613
44	13,142,732	13,629,963	14,140,923	14,677,785
45	13,150,668	13,638,279	14,149,672	14,686,965
46	13,158,610	13,646,602	14,158,409	14,696,153
47	13,166,558	13,654,932	14,167,153	14,705,349
48	13,174,512	13,663,268	14,175,904	14,714,553
49	13,182,472	13,671,610	14,184,663	14,723,765
50	13,190,438	13,679,959	14,193,429	14,732,985
51	13,198,411	13,688,315	14,202,202	14,742,212
52	13,206,390	13,696,677	14,210,982	14,751,447
53	13,214,375	13,705,046	14,219,769	14,760,690
54	13,222,367	13,713,422	14,228,563	14,769,941
55	13,230,365	13,721,805	14,237,365	14,779,200
56	13,238,369	13,730,194	14,246,174	14,788,466
57	13,246,379	13,738,590	14,254,990	14,797,740
58	13,254,396	13,746,993	14,263,813	14,807,022
59	13,262,419	13,755,403	14,272,643	14,816,312
60	13,270,448	13,763,820	14,281,480	14,825,610

	56	57	58	59
0	14,825,610	15,398,651	16,003,347	16,642,794
1	14,834,916	15,408,461	16,013,710	16,653,766
2	14,844,230	15,418,280	16,024,083	16,664,749
3	14,853,553	15,428,108	16,034,466	16,675,743
4	14,862,884	15,437,945	16,044,859	16,686,746
5	14,872,223	15,447,791	16,055,261	16,697,760
6	14,881,570	15,457,646	16,065,673	16,708,785
7	14 890,925	15,467,510	16,076,095	16,719,820
8	14,909,288	15,477,382	16,086,527	16,730,866
9	14,909,659	15,487,263	16,096,968	16,741,922
10	14,919,038	15,497,153	16,107,419	16,752,989
11	14,928,426	15,507,052	16,117,880	16,764,067
12	14,937,822	15,516,960	16,128,351	16,775,156
13	14,947,226	15,526,877	16,138,832	16,786,256
14	14,956,638	15,536,803	16,149,322	16,797,367
15	14 966,058	15,546,738	16,159,822	16,808,489
16	14,975,486	15,556,682	16,170,332	16,819,621
17	14,984,923	15,566,636	16,180,852	16,830,764
18	14,994,368	15,576,599	16,191,381	16,841,918
19	15,003,821	15,586,571	16,201,920	16,853,083
20	15,013,283	15,596,552	16,212,469	16,864 259
21	15,022,753	15,606,542	16,223,028	16.875,446
22	15,032,231	15,616,541	16,233,597	16,886,644
23	15,041,717	15,626,549	16,244,176	16,897,853
24	15,051,211	15,636,566	16,254 766	16,909,074
25	15,060,714	15,646,592	16,265,366	16,920,306
26	15,070,225	15,656,627	16,275,976	16,931,549
27	15,079,744	15,666,671	16,286,596	16,942,803
28	15,089,271	15,676,724	16,297,226	16,954,068
29	15,098,807	15,686,786	16,307,866	16,965,344
30	15,108,351	15,696,857	16,318,516	16,976,631

	56	57	58	59
30	15,108,351	15,696,857	16,318,516	16,976,631
31	15,117,903	15,706,938	16,329,176	16,987,929
32	15,127,464	15,717,028	16,339,847	16,999,239
33	15,137,034	15,727,127	16,350,528	17,010,560
34	15,146,612	15,737,235	16,361,219	17,021,892
35	15,156,199	15,747,353	16,371,920	17,033,236
36	15,165,794	15,757,480	16,382,631	17,044,591
37	15,175,398	15,767,616	16,393,352	17,055,957
38	15,185,011	15,777,761	16,404,083	17,067,325
39	15,194,632	15,787,915	16,414,824	17,078,714
40	15,204,261	15,798,078	16,425,575	17,080,115
41	15,213,899	15,808,251	16,436,337	17,101,527
42	15,223,545	15,818,433	16,447,109	17,112,950
43	15,233,200	15,828,625	16,457,892	17,124,384
44	15,242,863	15,838,827	16,468,685	17,135,829
45	15,252,535	15,849,038	16,479,488	17,147,285
46	15,262,216	15,859,259	16,490,302	17,158,752
47	15,271,905	15,869,489	16,501,126	17,170,231
48	15,281,603	15,879,729	16,511,960	17,181,721
49	15,291,309	15,889,979	16,522,805	17,193,222
50	15,301,024	15,900,238	16,533,660	17,204,734
51	15,310,748	15,910,507	16,544,526	17,216,258
52	15,320,481	15,920,785	16,555,402	17,227,794
53	15,330,222	15,931,073	16,566,289	17,239,342
54	15,339,972	15,941,370	16,577,186	17,250,902
55	15,349,730	15,951,676	16,588,094	17,262,473
56	15,359,497	15,961,992	16,599,013	17,274,056
57	15,369,273	15,972,317	16,609,942	17,285,651
58	15,379,057	15,982,651	16,620,882	17,297,258
59	15,388,850	15,992,994	16,631,833	17,388,877
60	15,398,651	16,003,347	16,642,794	17,320,508

	60	61	62	63
0	17,320,508	18,040,478	18,807,265	19,626,104
1	17,332,150	18,052,860	18,820,471	19,640,225
2	17,343,804	18,065,255	18,833,691	19,654,362
3	17,355,469	18,077,663	18,846,925	19,668,516
4	17,367,146	18,090,084	18,860,174	19,682,686
5	17,378,834	18,102,518	18,873,437	19,696,872
6	17,390,534	18,114,966	18,886,715	19,711,074
7	17,402,246	18,127,427	18,900,007	19,725,293
8	17,413,969	18,139,901	18,913,314	19,739,528
9	17,425,704	18,152,388	18,926,636	19,753,780
10	17,437,451	18,164,889	18,939,972	19,768,048
11	17,449,210	18,177,403	18,953,323	19,782,333
12	17,460,981	18,189,930	18,966,689	19,796,634
13	17,472,764	18,202,470	18,980,070	19,810,951
14	17,484,559	18,215,024	18,993,466	19,825,285
15	17,496,366	18,227,591	19,006,876	19,839,635
16	17,508,185	18,240,171	19,020,301	19,854,002
17	17,520,026	18,252,765	19,033,741	19,868,386
18	17,531,869	18,265,372	19,047,196	19,882,786
19	17,543,724	18,277,992	19,060,665	19,897,203
20	17,555,591	18,290,626	19,074,149	19,911,637
21	17,567,470	18,303,273	19,087,648	19,926,088
22	17,579,362	18,315,934	19,101,162	19,940,555
23	17,591,266	18,328,608	19,114,691	19,955,039
24	17,603,182	18,341,296	19,128,235	19,969,540
25	17,615,111	18,353,997	19,141,795	19,984,057
26	17,627,052	18,366,712	19,155,370	19,998,591
27	17,639,006	18,379,440	19,168,960	20,013,142
28	17,650,972	18,392,182	19,182,565	20,027,709
29	17,662,951	18,404,938	19,196,185	20,042,297
30	17,674,942	18,417,707	19,209,821	20,056,898

	60	61	62	63
30	17,674,942	18,417,707	19,209,821	20,056,898
31	17,686,945	18,430,490	19,223,472	20,071,516
32	17,698,960	18,443,287	19,237,138	20,086,152
33	17,710,987	18,456,098	19,250,819	20,100,805
34	17,723,027	18,468,922	19,264,516	20,115,475
35	17,735,079	18,481,760	19,278,228	20,130,163
36	17,747,143	18,494,612	19,291,955	20,144,868
37	17,759,220	18,507,478	19,305,698	20,159,590
38	17,771,309	18,520,357	19,319,456	20,174,329
39	17,783,410	18,533,250	19,333,230	20,189,086
40	17,795,524	18,546,157	19,347,019	20,203,860
41	17,808,651	18,559,078	19,360,824	20,218,651
42	17,819,790	18,572,013	19,374,644	20,233,460
43	17,831,942	18,584,962	19,388,480	20,248,286
44	17,844,107	18,597,925	19,402,331	20,263,130
45	17,856,285	18,610,902	19,416,198	20,277,991
46	17,868,475	18,623,894	19,430,081	20,292,870
47	17,880,678	18,636,900	19,443,980	20,307,767
48	17,892,894	18,649,920	19,457,894	20,322,681
49	17,905,123	18,662,954	19,471,824	20,337,613
50	17,917,364	18,676,002	19,485,770	20,352,563
51	17,929,618	18,689,064	19,499,732	20,365,531
52	17,941,885	18,702,140	19,513,710	20,382,516
53	17,954,164	18,715,231	19,527,704	20,397,519
54	17,966,456	18,628,335	19,541,714	20,412,539
55	17,978,761	18,741,454	19,555,739	20,427,577
56	17,991,079	18,754,587	19,569,780	20,442,633
57	18,003,410	18,767,735	19,583,837	20,457,706
58	18,015,753	18,780,897	19,597,910	20,472,797
59	18,028,109	18,794,074	19,611,999	20,487,906
60	18,040,478	18,807,265	19,626,104	20,503,034

	64	65	66	67
0	20,503,034	21,445,067	22,460,371	23,558,529
1	20,518,180	21,461,364	22,477,965	23,577,595
2	20,533,344	21,477,681	22,495,582	23,596,687
3	20,548,526	21,494,019	22,513,222	23,615,805
4	20,563,726	21,510,377	22,530,885	23,634,950
5	20,578,945	21,526,756	22,548,571	23,654,121
6	20,594,182	21,543,155	22,566,281	23,673,318
7	20,609,437	21,559,575	22,584,014	23,692,542
8	20,624,711	21,576,015	22,601,771	23,711,793
9	20,640,003	21,592,475	22,619,551	23,731,071
10	20,655,313	21,608,956	22,637,355	23,750,375
11	20,670,642	21,625,458	22,655,183	23,769,706
12	20,685,989	21,641,981	22,673,034	23,789,064
13	20,701,355	21,658,525	22,690,909	23,808,448
14	20,716,739	21,675,090	22,708,808	23,827,859
15	20,732,142	21,691,776	22,726,730	23,847,297
16	20,747,564	21,708,283	22,744,676	23,866,762
17	20,763,004	21,724,911	22,762,646	23,886,254
18	20,778,463	21,741,559	22,780,639	23,905,773
19	20,793,941	21,758,228	22,798,656	23,925,320
20	20,809,438	21,774,918	22,816,696	23,944,895
21	20,824,953	21,791,629	22,834,760	23,964,496
22	20,840,487	21,808,362	22,852,848	23,984,124
23	20,856,040	21,825,116	22,870,960	24,003,779
24	20,871,612	21,841,892	22,889,096	24,023,462
25	20,887,202	21,858,689	22,907,256	24,043,172
26	20,902,811	21,875,508	22,925,441	24,062,910
27	20,918,439	21,892,348	22,943,650	24,082,675
28	20,934,086	21,909,210	22,961,883	24,102,468
29	20,949,752	21,926,094	22,980,141	24,122,289
30	20,965,436	21,943,000	22,998,424	24,142,137

	64	65	66	67
30	20,965,436	21,943,000	22,998,424	24,142,137
31	20,981,140	21,959,926	23,016,731	24,162,013
32	20,996,863	21,976,874	23,035,062	24,181,917
33	21,012,605	21,993,843	23,053,418	24,201,849
34	21,028,367	22,010,834	23,071,798	24,221,809
35	21,044,148	22,027,846	23,090,203	24,241,798
36	21,059,949	22,044,879	23,108,632	24,261,815
37	21,075,769	22,061,934	23,127,086	24,281,860
38	21,091,609	22,079,011	23,145,565	24,301,934
39	21,107,468	22,096,109	23,164,068	24,322,037
40	21,123,347	22,113,229	23,182,597	24,342,169
41	21,139,246	22,130,372	23,201,151	24,362,329
42	21,155,164	22,147,537	23,219,730	24,382,518
43	21,171,102	22,164,725	23,238,335	24,402,735
44	21,187,059	22,181,935	23,256,965	24,422,981
45	21,203,036	22,199,168	23,275,621	24,443,256
46	21,219,032	22,216,424	23,294,302	24,463,559
47	21,235,048	22,233,703	23,313,008	24,483,891
48	21,251,083	22,251,004	23,331,740	24,504,252
49	21,267,138	22,268,328	23,350,498	24,524,642
50	21,283,213	22,285,675	23,369,282	24,545,061
51	21,299,308	22,303,044	23,388,092	24,565,509
52	21,315,423	22,320,435	23,406,927	24,585,986
53	21,331,558	22,337,848	23,425,788	24,606,492
54	21,347,713	22,355,284	23,444,674	24,627,028
55	21,363,888	22,372,742	23,463,586	24,647,594
56	21,380,083	22,390,223	23,482,523	24,668,189
57	21,396,298	22,407,726	23,501,486	24,688,814
58	21,412,534	22,425,252	23,520,475	24,709,469
59	21,428,790	22,442,800	23,539,489	24,730,154
60	21,445,067	22,460,371	23,558,529	24,750,869

	68	69	70	71
0	24,750,869	26,050,893	27,474,777	29,042,105
1	24,771,613	26,073,559	27,499,665	29,069,569
2	24,792,387	26,096,260	27,524,592	29,097,080
3	24,813,191	26,118,996	27,549,559	29,124,638
4	24,834,024	26,141,766	27,574,565	29,152,243
5	24,854,887	26,164,571	27,599,612	29,179,895
6	24,875,780	26,187,411	27,624,699	29,207,595
7	24,896,704	26,210,286	27,649,827	29,235,343
8	24,917,659	26,233,196	27,674,995	29,263,139
9	24,938,644	26,256,141	27,700,204	29,290,982
10	24,959,659	26,279,120	27,725,453	29,318,873
11	24,980,705	26,302,135	27,750,742	29,346,811
12	25,001,782	26,325,185	27,776,072	29,374,797
13	25,022,890	26,348,270	27,801,443	29,402,831
14	25,044,029	26,371,390	27,826,855	29,430,913
15	25,065,198	26,394,546	27,852,308	29,459,043
16	25,086,398	26,417,738	27,877,803	29,487,221
17	25,107,629	26,440,966	27,903,339	29,515,446
18	25,128,891	26,464,229	27,928,917	29,543,719
19	25,150,183	26,487,528	27,954,536	29,572,041
20	25,171,506	26,510,863	27,980,196	29,600,411
21	25,192,861	26,534,234	28,005,898	29,628,831
22	25,214,248	26,557,641	28,031,642	29,657,301
23	25,235,666	26,581,084	28,057,429	29,685,820
24	25,257,116	26,604,563	28,083,258	29,714,388
25	25,278,597	26,628,079	28,109,129	29,743,006
26	25,300,110	26,651,631	28,135,043	29,771,674
27	25,321,655	26,675,220	28,160,999	29,800,392
28	25,343,232	26,698,845	28,186,998	29,829,160
29	25,364,841	26,722,507	28,213,040	29,857,978
30	25,386,482	26,746,206	28,239,125	29,886,847

	68	69	70	71
30	25,386,482	26,746,206	28,239,125	29,886,847
31	25,408,154	26,769,942	28,265,253	29,915,765
32	25,429,858	26,793,716	28,291,424	29,944,734
33	25,451,594	26,817,527	28,317,638	29,973,753
34	25,473,362	26,841,375	28,343,895	30,002,823
35	25,495,162	26,865,260	28,360,195	30,031,943
36	25,516,995	26,889,183	28,396,539	30,061,113
37	25,538,860	26,913,143	28,422,926	30,090,334
38	25,560,758	26,937,141	28,449,357	30,119,605
39	25,582,688	26,961,177	28,475,832	30,148,927
40	25,604,651	26,985,251	28,502,350	30,178,299
41	25,626,647	27,009,362	28,528,913	30,207,723
42	25,648,675	27,033,511	28,555,520	30,237,200
43	25,670,736	27,057,698	28,582,172	30,266,730
44	25,692,830	27,081,922	28,608,868	30,296,312
45	25,714,957	27,106,184	28,635,608	30,325,947
46	25,737,118	27,130,484	28,662,393	30,355,635
47	25,759,312	27,154,823	28,689,222	30,385,375
48	25,781,540	27,179,200	28,716,096	30,415,169
49	25,803,801	27,203,616	28,743,015	30,445,015
50	25,826,096	27,228,070	28,769,979	30,474,915
51	25,848,424	27,252,563	28,796,987	30,504,867
52	25,870,786	27,277,095	28,824,040	30,534,872
53	25,893,181	27,301,667	28,851,139	30,564,930
54	25,915,610	27,326,278	28,878,283	30,595,041
55	25,938,073	27,350,929	28,905,472	30,625,205
56	25,960,569	27,375,620	28,932,707	30,655,423
57	25,983,099	27,400,350	28,959,988	30,685,695
58	26,005,663	27,425,120	28,987,315	30,716,020
59	26,028,261	27,449,929	29,014,687	30,746,400
60	26,050,893	27,474,777	29,042,105	30,776,834

	72	73	74	75
0	30,776,834	32,708,528	34,874,151	37,320,517
1	30,807,323	32,742,586	34,912,477	37,363,987
2	30,837,866	32,776,709	34,950,881	37,407,551
3	30,868,465	32,810,898	34,989,364	37,451,210
4	30,899,119	32,845,153	35,027,925	37,494,964
5	30,929,828	32,879,477	35,066,565	37,538,814
6	30,960,593	32,913,862	35,105,283	37,582,760
7	30,991,413	32,948,317	35,144,080	37,626,803
8	31,022,289	32,982,839	35,182,956	37,670,943
9	31,053,221	33,017,427	35,221,911	37,715,180
10	31,084,208	33,052,082	35,260,945	37,759,515
11	31,115,252	33,086,802	35,300,059	37,803,948
12	31,146,352	33,121,588	35,339,253	37,848,479
13	31,177,508	33,156,441	35,378,528	37,893,109
14	31,208,720	33,191,362	35,417,883	37,937,838
15	31,239,989	33,226,351	35,457,320	37,982,666
16	31,271,315	33,261,408	35,496,838	38,027,592
17	31,302,698	33,296,534	35,536,438	38,072,616
18	31,334,138	33,331,728	35,576,121	38,117,740
19	31,365,636	33,366,990	35,615,888	38,162,963
20	31,397,191	33,402,321	35,655,739	38,208,285
21	31,428,805	33,437,720	35,695,672	38,253,708
22	31,460,476	33,473,188	35,735,689	38,299,232
23	31,492,205	33,508,725	35,775,789	38,344,857
24	31,523,992	33,544,330	35,815,973	38,390,584
25	31,555,838	33,580,005	35,856,241	38,436,414
26	31,587,742	33,615,750	35,896,593	38,482,347
27	31,619,705	33,651,566	35,937,029	38,528,384
28	31,651,727	33,687,453	35,977,550	38,574,525
29	31,683,807	33,723,410	36,018,156	38,620,772
30	31,715,946	33,759,438	36,058,848	38,667,125

	72	73	74	75
30	31,715,946	33,759,438	36,058,848	38,667,125
31	31,748,144	33,795,535	36,099,623	38,713,580
32	31,780,401	33,831,703	36,140,483	38,760,139
33	31,812,717	33,867,942	36,181,427	38,806,801
34	31,845,093	33,904,252	36,222,456	38,853,567
35	31,877,528	33,940,634	36,263,570	38,900,438
36	31,910,024	33,977,088	36,304,771	38,947,416
37	31,942,580	34,013,615	36,346,060	38,994,501
38	31,975,197	34,050,215	36,387,437	39,041,695
39	32,007,875	34,086,888	36,428,903	39,088,998
40	32,040,613	34,123,634	36,470,459	39,136,409
41	32,073,413	34,160,453	36,512,103	39,183,929
42	32,106,275	34,197,345	36,553,836	39,231,557
43	32,139,200	34,234,310	36,595,659	39,279,294
44	32,172,187	34,271,348	36,637,572	39,327,139
45	32,205,237	34,308,459	36,679,574	39,375,094
46	32,238,349	34,345,644	36,721,666	39,423,158
47	32,271,524	34,382,903	36,763,849	39,471,331
48	32,304,762	34,420,237	36,806,121	39,519,614
49	32,338,064	34,457,647	36,848,483	39,568,006
50	32,371,430	34,495,132	36,890,936	39,616,509
51	32,404,858	34,532,692	36,933,479	39,665,124
52	32,438,348	34,570,327	36,976,114	39,713,852
53	32,471,901	34,608,038	37,018,840	39,762,695
54	32,505,517	34,645,824	37,061,659	39,811,654
55	32,539,196	34,683,686	37,104,570	39,860,729
56	32,572,937	34,721,625	37,147,574	39,909,917
57	32,606,741	34,759,640	37,190,670	39,959,218
58	32,640,607	34,797,733	37,233,859	40,008,633
59	32,674,536	34,835,903	37,277,141	40,058,103
60	32,708,528	34,874,151	37,320,517	40,107,808

	76	77	78	79
0	40,107,808	43,314,742	47,046,295	51,445,543
1	40,157,569	43,372,301	47,113,680	51,525,561
2	40,207,446	43,430,006	47,181,249	51,605,820
3	40,257,440	43,487,857	47,249,003	51,686,321
4	40,307,552	43,545,855	47,316,942	51,767,065
5	40,357,781	43,604,000	47,385,067	51,848,053
6	40,408,129	43,662,293	47,453,380	51,929,285
7	40,458,596	43,720,733	47,521,882	52,010,762
8	40,509,183	43,779,321	47,590,575	52,092,485
9	40,559,890	43,838,057	47,659,460	52,174,455
10	40,610,718	43,896,942	47,728,538	52,256,673
11	40,661,665	43,955,977	47,797,809	52,339,140
12	40,712,731	44,015,163	47,867,274	52,421,857
13	40,763,917	44,074,501	47,936,934	52,504,826
14	40,815,224	44,133,992	48,006,790	52,588,048
15	40,866,652	44,193,637	48,076,841	52,671,525
16	40,918,201	44,253,435	48,147,088	52,755,259
17	40,969,871	44,313,387	48,217,531	52,839,251
18	41,021,663	44,373,494	48,288,171	52,923,503
19	41,073,577	44,433,756	48,359,008	53,008,016
20	41,125,614	44,494,174	48,430,043	53,092,792
21	41,177,775	44,554,749	48,501,278	53,177,831
22	41,230,062	44,615,481	48,572,714	53,263,134
23	41,282,475	44,676,371	48,644,352	53,348,702
24	41,335,015	44,737,419	48,716,193	53,434,536
25	41,387,683	44,798,626	48,788,238	53,520,637
26	41,440,480	44,859,993	48,860,488	53,607,006
27	41,493,407	44,921,521	48,932,945	53,693,644
28	41,546,464	44,983,211	49,005,610	53,780,552
29	41,599,653	45,045,065	49,078,483	53,867,731
30	41,652,974	45,107,083	49,151,565	53,955,183

	76	77	78	79
30	41,652,974	45,107,083	49,151,565	53,955,183
31	41,706,424	45,169,263	49,224,856	54,042,909
32	41,760,003	45,231,607	49,298,357	54,130,911
33	41,813,712	45,294,114	49,372,069	54,219,190
34	41,867,550	45,356,785	49,445,993	54,307,748
35	41,921,518	45,419,621	49,520,130	54,396,586
36	41,975,617	45,482,623	49,594,481	54,485,705
37	42,029,848	45,545,790	49,669,047	54,575,107
38	42,084,211	45,609,123	49,743,829	54,664,793
39	42,138,706	45,672,623	49,818,827	54,754,764
40	42,193,334	45,736,291	49,894,042	54,845,022
41	42,248,096	45,800,128	49,969,475	54,935,569
42	42,302,993	45,864,135	50,045,127	55,026,406
43	42,358,025	45,928,314	50,120,999	55,117,535
44	42,413,193	45,992,666	50,197,092	55,208,958
45	42,468,497	46,057,192	50,273,407	55,300,676
46	42,523,937	46,121,892	50,349,945	55,392,692
47	42,579,514	46,186,767	50,426,707	55,485,007
48	42,635,228	46,251,817	50,503,695	55,577,622
49	42,691,080	46,317,043	50,580,910	55,670,539
50	42,747,070	46,382,445	50,558,353	55,763,759
51	42,803,199	46,448,023	50,736,025	55,857,283
52	42,859,468	46,513,778	50,813,927	55,951,112
53	42,915,878	46,579,711	50,892,060	56,045,247
54	42,972,429	46,645,823	50,970,425	56,139,689
55	43,029,122	46,712,115	51,049,023	56,234,439
56	43,085,958	46,778,587	51,127,855	56,329,498
57	43,142,937	46,845,240	51,206,922	56,424,868
58	43,200,060	46,912,075	51,286,225	56,520,550
59	43,257,328	46,979,093	51,365,765	56,616,545
60	43,314,742	47,046,295	51,445,543	56,712,854

	80	81	82	83
0	56,712,854	63,137,478	71,153,707	81,445,302
1	56,809,480	63,256,564	71,304,198	81,639,821
2	56,906,425	63,376,089	71,455,313	81,837,074
3	57,003,690	63,496,056	71,607,058	82,035,268
4	57,101,277	63,616,468	71,759,440	82,234,410
5	57,199,188	63,737,327	71,912,459	82,434,508
6	57,297,425	63,858,635	72,066,117	82,635,570
7	57,395,990	63,980,394	72,220,422	82,837,603
8	57,494,885	64,102,607	72,375,376	83,040,614
9	57,594,111	64,225,276	72,530,983	83,244,610
10	57,693,670	64,348,404	72,687,247	83,449,598
11	57,793,564	64,471,994	72,844,173	83,655,585
12	57,893,795	64,596,049	73,001,766	83,862,572
13	57,994,366	64,720,571	73,160,031	84,070,565
14	58,095,279	64,845,563	73,318,972	84,279,571
15	58,196,536	64,971,028	73,478,593	84,489,598
16	58,298,138	65,096,069	73,638,898	84,700,687
17	58,400,087	65,223,388	73,799,892	84,912,817
18	58,502,385	65,350,287	73,961,579	85,125,995
19	58,605,034	65,477,669	74,123,964	85,340,229
20	58,708,035	65,605,537	74,287,052	85,555,525
21	58,811,388	65,733,894	74,450,847	85,771,891
22	58,915,095	65,862,743	74,615,354	85,989,335
23	59,019,157	65,992,087	74,780,577	86,207,866
24	59,123,576	66,121,928	74,946,521	86,427,493
25	59,228,353	66,252,268	75,113,189	86,648,225
26	59,333,490	66,383,110	75,280,586	86,870,072
27	59,438,989	66,514,457	75,448,716	87,093,043
28	59,544,852	66,646,313	75,617,584	87,317,150
29	59,651,081	66,778,681	75,787,195	87,542,404
30	59,757,678	66,911,564	75,957,554	87,768,816

	80	81	82	83
30	59,757,678	66,911,564	75,957,554	87,768,816
31	59,864,646	67,044,965	76,128,666	87,996,394
32	59,971,987	67,178,887	76,300,536	88,225,146
33	60,079,703	67,313,334	76,473,170	88,455,079
34	60,187,796	67,448,309	76,646,573	88,686,196
35	60,296,268	67,583,815	76,820,751	88,918,508
36	60,405,121	67,719,855	76,995,710	89,152,021
37	60,514,358	67,856,423	77,171,455	89,386,745
38	60,623,981	67,993,549	77,347,991	89,622,688
39	60,733,992	68,131,209	77,525,324	89,859,858
40	60,844,392	68,269,416	77,703,459	90,098,268
41	60,955,184	68,408,173	77,882,402	90,337,927
42	61,066,370	68,547,438	78,062,159	90,578,848
43	61,177,952	68,687,350	78,242,737	90,821,043
44	61,289,930	68,827,777	78,424,142	91,064,526
45	61,402,307	68,968,768	78,606,379	91,309,309
46	61,515,085	69,110,326	78,789,454	91,555,401
47	61,628,267	69,252,455	78,973,371	91,802,810
48	61,741,856	69,395,158	79,158,136	92,051,546
49	61,855,854	69,538,439	79,343,754	92,301,618
50	61,970,263	69,682,302	79,530,231	92,553,036
51	62,085,085	69,826,751	79,717,572	92,805,759
52	62,200,323	69,971,789	79,905,783	93,059,875
53	62,315,979	70,117,419	80,094,869	93,315,361
54	62,432,056	70,263,645	80,284,835	93,572,238
55	62,548,556	70,410,470	80,475,688	93,830,595
56	62,665,481	70,557,898	80,667,435	94,090,270
57	62,782,833	70,705,932	80,860,083	94,351,448
58	62,900,615	70,854,576	81,053,639	94,614,055
59	63,018,829	71,003,833	81,248,110	94,878,103
60	63,137,478	71,153,706	81,443,502	95,143,611

E

	84	85	86
0	95,143,611	114,300,579	143,006,601
1	95,410,585	114,684,819	143,606,943
2	95,679,034	115,071,619	144,212,307
3	95,948,971	115,461,005	144,822,757
4	96,220,411	115,853,017	145,438,358
5	96,493,467	116,247,668	146,059,175
6	96,767,939	116,644,985	146,685,275
7	97,044,063	117,044,995	147,316,726
8	97,321,646	117,447,864	147,953,611
9	97,600,890	117,853,346	148,595,987
10	97,881,716	118,261,757	149,244,148
11	98,164,135	118,672,834	149,897,753
12	98,448,162	119,086,890	150,557,233
13	98,733,810	119,503,669	151,222,301
14	99,021,104	119,923,488	151,893,462
15	99,310,047	120,346,233	152,570,581
16	99,600,655	120,771,937	153,253,487
17	99,893,042	121,200,643	153,942,729
18	100,187,022	121,632,370	154,638,158
19	100,482,822	122,067,151	155,339,855
20	100,780,346	122,505,017	156,047,923
21	101,079,507	122,946,003	156,762,433
22	101,380,525	123,390,142	157,483,474
23	101,683,314	123,837,634	158,211,136
24	101,987,889	124,288,195	158,945,509
25	102,294,266	124,742,169	159,686,753
26	102,602,473	125,199,280	160,434,770
27	102,912,514	125,659,878	161,189,849
28	103,224,405	126,123,842	161,952,305
29	103,538,166	126,591,211	162,721,698
30	103,853,919	127,062,036	163,498,660

	84	85	86
30	103,853,919	127,062,036	163,498,660
31	104,171,468	127,536,341	164,282,764
32	104,491,055	128,014,165	165,074,651
33	104,812,581	128,495,548	165,873,906
34	105,136,063	128,980,531	166,681,172
35	105,461,519	129,469,305	167,496,287
36	105,788,969	129,961,652	168,319,085
37	106,118,428	130,457,692	169,150,247
38	106,449,917	130,957,670	169,989,613
39	106,783,466	131,461,286	170,837,394
40	107,119,198	131,968,930	171,693,461
41	107,456,902	132,480,297	172,558,198
42	107,796,712	132,995,769	173,431,641
43	108,138,767	133,515,636	174,313,925
44	108,482,852	134,038,804	175,205,183
45	108,829,233	134,566,419	176,105,555
46	109,177,805	135,098,153	177,015,180
47	109,528,589	135,634,096	177,934,219
48	109,881,598	136,174,272	178,862,806
49	110,236,864	136,718,731	179,801,085
50	110,594,415	137,267,523	180,749,537
51	110,954,264	137,820,702	181,707,670
52	111,316,432	138,378,319	182,676,299
53	111,680,940	138,940,429	183,654,941
54	112,047,814	139,507,087	184,644,417
55	112,417,202	140,078,545	185,644,562
56	112,788,878	140,654,481	186,655,202
57	113,163,056	141,235,334	187,677,257
58	113,539,681	141,820,765	188,710,414
59	113,918,875	142,411,234	189,755,028
60	114,300,579	143,006,601	190,811,200

	87	88	89
0	190,811,200	286,362,498	572,899,830
1	191,879,163	288,770,746	582,610,421
2	192,959,095	291,219,764	592,655,713
3	194,051,200	293,710,598	603,057,015
4	195,155,685	296,244,357	613,825,994
5	196,273,146	298,823,024	624,990,311
6	197,403,054	301,445,987	636,564,040
7	198,545,993	304,115,322	648,578,536
8	199,702,191	306,833,212	661,050,728
9	200,871,878	309,599,077	674,016,435
10	202,055,705	312,416,191	687,500,739
11	203,253,093	315,283,945	701,531,474
12	204,464,726	318,204,757	716,149,676
13	205,691,260	321,181,137	731,385,593
14	206,932,111	324,212,583	747,289,264
15	208,188,402	327,302,782	763,899,813
16	209,459,545	330,451,272	781,259,259
17	210,746,693	333,661,982	799,432,199
18	212,049,271	336,934,467	818,463,792
19	21,3638,214	340,272,744	838,430,438
20	214,704,085	343,677,949	859,395,374
21	216,956,022	347,150,587	881,427,652
22	217,425,507	350,695,255	904,627,361
23	218,812,405	354,312,962	929,081,086
24	220,217,049	358,006,024	954,893,332
25	221,639,784	361,776,788	982.180,553
26	223,080,983	365,626,388	1,011,062,679
27	224,540,987	369,560,062	1,041,705,454
28	226,020,167	373,579,199	1,074,263,399
29	227,518,902	377,686,614	1,108,922,084
30	229,037,584	381,885,288	1,145,891,136

	87	88	89
30	229,037,584	381,885,288	1,145,891,136
31	230,576,614	386,178,258	1,185,395,877
32	232,132,427	390,568,737	1,227,736,470
33	233,717,425	395,060,088	1,273,213,435
34	235,320,041	399,655,828	1,322,188,681
35	236,945,285	404,359,642	1,375,082,163
36	238,592,501	409,175,388	1,432,363,027
37	240,262,714	414,107,152	1,494,645,462
38	241,957,021	419,159,137	1,562,590,046
39	243,674,732	424,335,793	1,637,005,697
40	245,417,543	429,641,796	1,718,863,124
41	247,184,785	435,082,056	1,809,337,410
42	248,978,216	440,661,780	1,909,864,971
43	250,797,165	446,386,310	2,022,219,818
44	252,643,455	452,261,453	2,148,619,711
45	254,517,088	458,293,185	2,291,873,854
46	256,417,991	464,487,853	2,455,533,838
47	258,348,100	470,852,152	2,644,433,955
48	260,307,416	477,393,195	2,864,819,229
49	262,296,605	484,118,351	3,125,276,745
50	264,316,358	491,038,024	3,437,829,002
51	266,366,704	498,155,754	3,819,696,333
52	268,449,755	505,482,730	4,297,181,900
53	270,565,570	513,030,946	4,911,245,459
54	272,714,927	520,805,157	5,729,633,839
55	274,898,633	528,821,258	6,875,680,006
56	277,117,516	537,085,003	8,594,012,547
57	279,372,435	545,610,968	11,458,686,834
58	281,664,304	554,414,914	17,188,033,688
59	283,994,009	563,504,309	34,376,070,815
60	286,362,498	572,899,830	Infinitum.

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	0	1	2	3
0	10,000,000	10,001,524	10,006,095	10,013,723
1	10,000,001	10,001,574	10,006,198	10,013,875
2	10,000,002	10,001,626	10,006,301	10,014,029
3	10,000,004	10,001,679	10,006,405	10,014,184
4	10,000,008	10,001,733	10,006,509	10,014,339
5	10,000,010	10,001,788	10,006,615	10,014,495
6	10,000,014	10,001,844	10,006,721	10,014,653
7	10,000,020	10,001,900	10,006,828	10,014,811
8	10,000,027	10,001,957	10,006,936	10,014,970
9	10,000,034	10,002,015	10,007,045	10,015,130
10	10,000,042	10,002,074	10,007,155	10,015,291
11	10,000,051	10,002,134	10,007,265	10,015,453
12	10,000,060	10,002,195	10,007,376	10,015,615
13	10,000,071	10,002,256	10,007,488	10,015,778
14	10,000,083	10,002,318	10,007,601	10,015,942
15	10,000,095	10,002,381	10,007,716	10,016,107
16	10,000,108	10,002,445	10,007,831	10,016,273
17	10,000,122	10,002,510	10,007,946	10,016,440
18	10,000,137	10,002,576	10,008,062	10,016,608
19	10,000,152	10,002,642	10,008,179	10,016,777
20	10,000,168	10,002,709	10,008,298	10,016,946
21	10,000,186	10,002,777	10,008,417	10,017,116
22	10,000,204	10,002,846	10,008,537	10,017,287
23	10,000,223	10,002,916	10,008,658	10,017,459
24	10,000,243	10,002,987	10,008,779	10,017,632
25	10,000,264	10,003,058	10,008,902	10,017,806
26	10,000,285	10,003,130	10,009,025	10,017,981
27	10,000,308	10,003,203	10,009,149	10,018,157
28	10,000,332	10,003,277	10,009,274	10,018,333
29	10,000,357	10,003,352	10,009,400	10,018,510
30	10,000,381	10,003,428	10,009,527	10,018,687

	0	1	2	3
30	10,000,381	10,003,428	10,009,527	10,018,687
31	10,000,407	10,003,505	10,009,655	10,018,865
32	10,000,433	10,003,582	10,009,783	10,019,044
33	10,000,461	10,003,660	10,009,912	10,019,224
34	10,000,489	10,003,739	10,010,043	10,019,405
35	10,000,518	10,003,819	10,010,174	10,019,587
36	10,000,548	10,003,900	10,010,306	10,019,770
37	10,000,579	10,003,982	10,010,439	10,019,954
38	10,000,611	10,004,065	10,010,572	10,020,138
39	10,000,643	10,004,148	10,010,706	10,020,324
40	10,000,677	10,004,232	10,010,841	10,020,510
41	10,000,711	10,004,317	10,010,977	10,020,698
42	10,000,746	10,004,403	10,011,114	10,020,886
43	10,000,782	10,004,490	10,011,252	10,021,086
44	10,000,819	10,004,578	10,011,390	10,021,266
45	10,000,857	10,004,666	10,011,529	10,021,456
46	10,000,895	10,004,755	10,011,670	10,021,649
47	10,000,934	10,004,845	10,011,811	10,021,842
48	10,000,975	10,004,936	10,011,952	10,022,035
49	10,001,016	10,005,028	10,012,098	10,022,239
50	10,001,058	10,005,122	10,012,238	10,022,424
51	10,001,100	10,005,216	10,012,383	10,022,620
52	10,001,144	10,005,310	10,012,528	10,022,817
53	10,001,188	10,005,405	10,012,674	10,023,015
54	10,001,233	10,005,501	10,012,822	10,023,213
55	10,001,280	10,005,598	10,012,970	10,023,412
56	10,001,327	10,005,696	10,013,119	10,023,612
57	10,001,375	10,005,795	10,013,269	10,023,813
58	10,001,423	10,005,894	10,013,419	10,024,014
59	10,001,473	10,005,994	10,013,570	10,024,217
60	10,001,524	10,006,095	10,013,723	10,024,420

F

	4	5	6	7
0	10,024,420	10,038,198	10,055,082	10,075,098
1	10,024,625	10,038,454	10,055,390	10,075,459
2	10,024,830	10,038,710	10,055,699	10,075,820
3	10,025,036	10,038,968	10,056,009	10,076,182
4	10,025,242	10,039,226	10,056,320	10,076,545
5	10,025,450	10,039,486	10,056,632	10,076,909
6	10,025,658	10,039,746	10,056,944	10,077,274
7	10,025,868	10,040,008	10,057,256	10,077,639
8	10,026,078	10,040,269	10,057,570	10,078,005
9	10,026,289	10,040,532	10,057,884	10,078,372
10	10,026,500	10,040,796	10,058,200	10,078,740
11	10,026,713	10,041,061	10,058,517	10,079,009
12	10,026,927	10,041,326	10,058,834	10,079,479
13	10,027,141	10,041,592	10,059,153	10,079,850
14	10,027,357	10,041,859	10,059,472	10,080,222
15	10,027,573	10,042,128	10,059,792	10,080595
16	10,027,790	10,042,397	10,060,113	10,080,968
17	10,028,009	10,042,667	10,060,435	10,081,332
18	10,028,227	10,042,936	10,060,757	10,081,717
19	10,028,447	10,043,207	10,061,080	10,082,093
20	10,028,667	10,043,479	10,061,405	10,082,470
21	10,028,889	10,043,752	10,061,730	10,082,848
22	10,029,111	10,044,025	10,062,056	10,083,226
23	10,029,334	10,044,300	10,062,383	10,083,606
24	10,029,559	10,044,576	10,062,711	10,083,987
25	10,029,784	10,044,853	10,063,039	10,084,368
26	10,030,009	10,045,130	10,063,369	10,084,750
27	10,030,236	10,045,409	10,063,700	10,085,134
28	10,030,463	10,045,689	10,064,031	10,085,518
29	10,030,692	10,045,969	10,064,364	10,085,903
30	10,030,920	10,046,250	10,064,696	10,086,289

	4	5	6	7
30	10,030,920	10,046,250	10,064,696	10,086,289
31	10,031,150	10,046,532	10,065,035	10,086,677
32	10,031,381	10,046,815	10,065,365	10,087,065
33	10,031,614	10,047,098	10,065,701	10,087,454
34	10,031,846	10,047,383	10,066,038	10,087,843
35	10,032,079	10,047,669	10,066,376	10,088,243
36	10,032,314	10,047,954	10,066,715	10,088,623
37	10,032,550	10,048,241	10,067,054	10,089,015
38	10,032,786	10,048,529	10,067,394	10,089,408
39	10,033,023	10,048,818	10,067,735	10,089,802
40	10,033,261	10,049,107	10,068,076	10,090,196
41	10,033,500	10,049,398	10,068,419	10,090,592
42	10,033,740	10,049,690	10,068,763	10,090,988
43	10,033,981	10,049,983	10,069,107	10,091,385
44	10,034,223	10,050,276	10,069,452	10,091,783
45	10,034,465	10,050,571	10,069,808	10,092,182
46	10,034,708	10,050,865	10,070,155	10,092,582
47	10,034,952	10,051,160	10,070,493	10,092,983
48	10,035,196	10,051,456	10,070,842	10,093,385
49	10,035,441	10,051,753	10,071,192	10,093,787
50	10,035,688	10,052,051	10,071,543	10,094,190
51	10,035,936	10,052,350	10,071,895	10,094,624
52	10,036,184	10,052,649	10,072,247	10,095,030
53	10,036,434	10,052,951	10,072,600	10,095,406
54	10,036,684	10,053,252	10,072,954	10,095,813
55	10,036,934	10,053,555	10,073,310	10,096,221
56	10,037,185	10,053,858	10,073,666	10,096,630
57	10,037,438	10,054,162	10,074,023	10,097,040
58	10,037,690	10,054,468	10,074,380	10,097,451
59	10,037,944	10,054,775	10,074,737	10,097,863
60	10,038,198	10,055,082	10,075,098	10,098,275

	8	9	10	11
0	10,098,275	10,124,650	10,154,264	10,187,166
1	10,098,698	10,125,117	10,154,786	10,187,743
2	10,099,103	10,125,585	10,155,308	10,188,320
3	10,099,518	10,126,054	10,155,831	10,188,899
4	10,099,934	10,126,524	10,156,356	10,189,478
5	10,100,351	10,126,994	10,156,881	10,190,058
6	10,100,769	10,127,465	10,157,407	10,190,639
7	10,101,188	10,127,947	10,157,934	10,191,221
8	10,101,607	10,128,410	10,158,462	10,191,804
9	10,102,028	10,128,684	10,158,991	10,192,387
10	10,102,450	10,129,358	10,159,520	10,192,972
11	10,102,872	10,129,634	10,160,051	10,193,557
12	10,103,295	10,130,311	10,160,582	10,194,144
13	10,103,720	10,130,788	10,161,114	10,194,732
14	10,104,144	10,131,266	10,161,648	10,195,320
15	10,104,570	10,131,746	10,162,182	10,195,910
16	10,104,996	10,132,226	10,162,707	10,196,500
17	10,105,423	10,132,707	10,163,252	10,197,092
18	10,105,851	10,133,189	10,163,789	10,197,684
19	10,106,286	10,133,672	10,164,327	10,198,277
20	10,106,710	10,134,156	10,165,865	10,198,872
21	10,107,140	10,134,641	10,165,495	10,199,467
22	10,107,572	10,135,127	10,165,944	10,200,063
23	10,108,005	10,135,614	10,166,485	10,200,660
24	10,108,438	10,136,102	10,167,028	10,201,258
25	10,108,873	10,136,591	10,167,571	10,201,857
26	10,109,309	10,137,080	10,168,116	10,202,457
27	10,109,755	10,137,571	10,168,661	10,203,058
28	10,110,182	10,138,163	10,169,207	10,203,659
29	10,110,620	10,138,555	10,169,765	10,204,262
30	10,111,059	10,139,048	10,170,303	10,204,867

	8	9	10	11
30	10,111,059	10,139,048	10,170,303	10,204,867
31	10,111,509	10,139,543	10,170,852	10,209,470
32	10,111,940	10,140,038	10,171,401	10,200,075
33	10,112,482	10,140,534	10,171,952	10,206,681
34	10,112,825	10,141,036	10,172,504	10,207,289
35	10,113,279	10,141,528	10,173,056	10,207,897
36	10,113,713	10,142,027	10,173,609	10,208,506
37	10,114,159	10,142,526	10,174,163	10,209,116
38	10,114,606	10,143,026	10,174,718	10,209,727
39	10,115,053	10,143,528	10,175,274	10,210,339
40	10,115,501	10,144,030	10,175,831	10,210,952
41	10,115,951	10,144,533	10,176,389	10,211,566
42	10,116,401	10,145,037	10,176,947	10,211,180
43	10,116,852	10,145,542	10,177,507	10,212,796
44	10,117,303	10,146,048	10,178,068	10,213,412
45	10,117,754	10,146,554	10,178,630	10,214,030
46	10,118,209	10,147,062	10,179,193	10,214,668
47	10,118,663	10,147,572	10,179,756	10,215,268
48	10,119,118	10,148,082	10,180,321	10,215,889
49	10,119,574	10,148,593	10,180,886	10,216,510
50	10,120,031	10,149,104	10,181,453	10,217,113
51	10,120,439	10,149,615	10,182,021	10,217,756
52	10,120,948	10,150,128	10,182,589	10,218,380
53	10,121,408	10,150,642	10,183,158	10,219,015
54	10,121,868	10,151,156	10,183,728	10,219,631
55	10,122,330	10,151,672	10,184,299	10,220,2[illegible]8
56	10,122,[illegible]92	10,152,188	10,184,870	10,220,885
57	10,123,256	10,152,705	10,185,443	10,221,514
58	10,123,720	10,153,224	10,186,017	10,222,143
59	10,124,275	10,153,744	10,186,591	10,222,7[illegible]4
60	10,124,650	10,154,264	10,187,166	10,223,405

	12	13	14	15
0	10,223,405	10,263,040	10,306,136	10,352,762
1	10,224,037	10,263,730	10,306,884	10,353,569
2	10,224,671	10,264,420	10,307,633	10,354,377
3	10,225,305	10,265,112	10,308,383	10,355,186
4	10,225,941	10,265,804	10,309,134	10,355,996
5	10,226,577	10,266,498	10,309,886	10,356,807
6	10,227,215	10,267,192	10,310,639	10,357,619
7	10,227,854	10,267,888	10,311,393	10,358,433
8	10,228,493	10,268,584	10,312,148	10,359,247
9	10,229,134	10,269,281	10,312,903	10,360,063
10	10,229,775	10,269,979	10,313,660	10,360,880
11	10,230,417	10,270,688	10,314,417	10,361,698
12	10,231,060	10,271,379	10,315,176	10,362,517
13	10,231,644	10,272,080	10,315,935	10,363,337
14	10,232,288	10,272,782	10,316,696	10,364,158
15	10,232,994	10,273,485	10,317,457	10,364,980
16	10,233,641	10,274,190	10,318,220	10,365,802
17	10,234,289	10,274,895	10,318,984	10,366,626
18	10,234,938	10,275,601	10,319,749	10,367,450
19	10,235,587	10,276,318	10,320,525	10,368,276
20	10,236,238	10,277,016	10,321,282	10,369,102
21	10,236,889	10,277,726	10,322,050	10,369,930
22	10,237,541	10,278,436	10,322,819	10,370,758
23	10,238,195	10,279,148	10,323,589	10,371,588
24	10,238,849	10,279,860	10,324,359	10,372,418
25	10,239,505	10,280,573	10,325,131	10,373,250
26	10,240,161	10,281,287	10,325,903	10,374,092
27	10,240,818	10,282,002	10,326,677	10,374,916
28	10,241,476	10,282,717	10,327,451	10,375,750
29	10,242,135	10,283,434	10,328,127	10,376,586
30	10,242,795	10,284,151	10,329,003	10,377,422

	12	13	14	15
30	10,242,795	10,284,151	10,329,003	10,377,422
31	10,243,456	10,284,870	10,329,381	10,378,260
32	10,244,118	10,285,589	10,330,559	10,379,098
33	10,245,782	10,286,310	10,331,339	10,379,938
34	10,245,445	10,287,032	10,332,119	10,380,778
35	10,246,110	10,287,754	10,332,902	10,381,620
36	10,246,776	10,288,478	10,333,684	10,382,463
37	10,247,442	10,289,202	10,334,467	10,383,307
38	10,248,110	10,289,928	10,335,252	10,384,153
39	10,248,778	10,290,654	10,336,037	10,384,999
40	10,249,448	10,291,381	10,336,824	10,385,846
41	10,250,119	10,292,119	10,337,612	10,386,694
42	10,250,790	10,292,838	10,338,400	10,387,543
43	10,251,461	10,293,569	10,339,189	10,388,393
44	10,252,136	10,294,300	10,339,980	10,389,244
45	10,252,811	10,295,043	10,340,771	10,390,096
46	10,253,482	10,295,766	10,341,564	10,390,949
47	10,254,162	10,296,501	10,342,347	10,391,803
48	10,254,839	10,297,237	10,343,152	10,392,657
49	10,255,517	10,297,973	10,343,947	10,393,513
50	10,256,196	10,298,710	10,344,743	10,394,370
51	10,256,876	10,299,449	10,345,541	10,395,228
52	10,257,557	10,300,188	10,346,340	10,396,087
53	10,258,239	10,300,928	10,347,139	10,396,947
54	10,258,922	10,301,669	10,347,940	10,397,808
55	10,259,606	10,302,411	10,348,741	10,398,670
56	10,260,291	10,303,154	10,349,544	10,399,533
57	10,260,977	10,303,898	10,350,347	10,400,397
58	10,261,661	10,304,643	10,351,151	10,401,262
59	10,262,351	10,305,390	10,351,956	10,402,128
60	10,263,040	10,306,136	10,352,762	10,402,994

	16	17	18	19
0	10,402,994	10,456,917	10,514,621	10,576,207
1	10,403,862	10,457,847	10,515,616	10,577,267
2	10,404,730	10,458,779	10,516,612	10,578,328
3	10,405,590	10,459,711	10,517,609	10,579,400
4	10,406,471	10,460,645	10,518,607	10,580,463
5	10,407,343	10,461,580	10,519,606	10,581,518
6	10,408,216	10,462,516	10,520,606	10,582,583
7	10,409,091	10,463,453	10,521,607	10,583,650
8	10,409,966	10,464,391	10,522,608	10,584,717
9	10,410,843	10,465,330	10,523,611	10,585,795
10	10,411,721	10,466,270	10,524,615	10,586,855
11	10,412,600	10,467,211	10,525,620	10,587,925
12	10,413,479	10,468,153	10,526,626	10,588,997
13	10,414,360	10,469,096	10,527,633	10,590,070
14	10,415,241	10,470,041	10,528,642	10,591,145
15	10,416,124	10,470,986	10,529,651	10,592,220
16	10,417,007	10,471,933	10,530,662	10,593,297
17	10,417,892	10,472,880	10,531,673	10,594,375
18	10,418,778	10,473,829	10,532,686	10,595,455
19	10,419,665	10,474,778	10,533,699	10,396,534
20	10,420,553	10,475,729	10,534,714	10,597,615
21	10,421,442	10,476,680	10,535,730	10,598,697
22	10,422,333	10,477,633	10,536,747	10,599,780
23	10,423,224	10,478,587	10,537,765	10,600,865
24	10,424.116	10,479,542	10,538,785	10,601,950
25	10,425,009	10,480,498	10,539,805	10,603,037
26	10,425,903	10,481,454	10,540,826	10,604,125
27	10,426,798	10,482,412	10,541,848	10,605,214
28	10,427,694	10,483,371	10,542,872	10,606,304
29	10,428,591	10,484,331	10,543,897	10,607,395
30	10,429,489	10,485,292	10,544,923	10,608,487

	16	17	18	19
30	10,429,489	10,485,292	10,544,923	10,608,487
31	10,430,388	10,486,254	10,545,950	10,609,580
32	10,431,288	10,487,217	10,546,977	10,610,675
33	10,432,189	10,488,181	10,548,006	10,611,770
34	10,433,091	10,489,146	10,549,036	10,612,867
35	10,433,995	10,490,113	10,550,067	10,613,964
36	10,434,899	10,491,080	10,551,099	10,615,063
37	10,435,805	10,492,049	10,552,133	10,616,163
38	10,436,711	10,493,018	10,553,168	10,617,264
39	10,437,619	10,493,989	10,554,204	10,618,366
40	10,438,528	10,494,961	10,555,241	10,619,469
41	10,439,436	10,495,934	10,556,279	10,620,574
42	10,440,346	10,496,908	10,557,318	10,621,680
43	10,441,257	10,497,883	10,558,359	10,622,787
44	10,442,170	10,498,059	10,559,400	10,623,895
45	10,443,083	10,499,836	10,560,443	10,625,004
46	10,443,998	10,500,814	10,561,496	10,626,114
47	10,444,913	10,501,793	10,562,531	10,627,226
48	10,445,830	10,502,773	10,563,577	10,628,338
49	10,446,749	10,503,754	10,564,623	10,629,451
50	10,447,668	10,504,736	10,565,670	10,630,566
51	10,448,588	10,505,719	10,566,719	10,631,682
52	10,449,509	10,506,704	10,567,769	10,632,799
53	10,450,431	10,507,689	10,568,820	10,633,917
54	10,451,354	10,508,676	10,569,872	10,635,037
55	10,452,279	10,509,664	10,570,925	10,636,157
56	10,453,204	10,510,653	10,571,980	10,637,279
57	10,454,131	10,511,643	10,573,034	10,638,402
58	10,455,058	10,512,635	10,574,091	10,639,526
59	10,455,987	10,513,627	10,575,149	10,640,651
60	10,456,917	10,514,621	10,576,207	10,641,777

	20	21	22	23
0	10,641,777	10,711,449	10,785,347	10,863,603
1	10,642,905	10,712,646	10,786,516	10,864,945
2	10,644,034	10,713,888	10,787,885	10,866,289
3	10,645,164	10,715,042	10,789,155	10,867,633
4	10,646,295	10,716,242	10,790,427	10,868,979
5	10,647,427	10,717,444	10,791,700	10,870,326
6	10,648,560	10,718,647	10,792,974	10,871,675
7	10,649,694	10,719,850	10,794,250	10,873,024
8	10,650,829	10,721,056	10,795,527	10,874,374
9	10,651,965	10,722,261	10,796,805	10,875,626
10	10,653,103	10,723,469	10,798,085	10,877,079
11	10,654,242	10,724,677	10,799,365	10,878,434
12	10,655,381	10,725,887	10,800,647	10,879,790
13	10,656,522	10,727,098	10,801,930	10,881,147
14	10,657,664	10,728,310	10,803,214	10,882,506
15	10,658,807	10,729,524	10,804,500	10,883,865
16	10,659,951	10,730,738	10,805,787	10,885,226
17	10,661,097	10,731,953	10,807,074	10,886,588
18	10,662,244	10,735,170	10,808,363	10,887,952
19	10,663,392	10,734,387	10,809,652	10,889,317
20	10,664,541	10,735,606	10,810,942	10,890,683
21	10,665,692	10,736,826	10,812,234	10,892,051
22	10,666,844	10,738,048	10,813,528	10,893,417
23	10,667,996	10,739,270	10,814,823	10,894,788
24	10,669,150	10,740,494	10,816,119	10,896,159
25	10,670,304	10,741,719	10,817,417	10,897,531
26	10,671,460	10,742,945	10,818,715	10,898,905
27	10,672,617	10,744,173	10,820,015	10,900,280
28	10,673,776	10,745,401	10,821,316	10,901,656
29	10,674,936	10,746,631	10,822,617	10,903,033
30	10,676,096	10,747,864	10,823,920	10,904,413

	20	21	22	23
30	10,676,096	10,747,864	10,823,920	10,904,413
31	10,677,258	10,749,094	10,825,225	10,905,790
32	10,678,420	10,750,327	10,826,531	10,907,171
33	10,679,584	10,751,561	10,827,838	10,908,553
34	10,680,749	10,752,797	10,829,146	10,909,936
35	10,681,915	10,754,034	10,830,455	10,911,322
36	10,683,082	10,755,273	10,831,766	10,912,709
37	10,684,250	10,756,513	10,833,078	10,914,096
38	10,685,420	10,757,753	10,834,391	10,915,484
39	10,686,591	10,758,995	10,835,706	10,916,874
40	10,687,763	10,760,237	10,837,023	10,918,265
41	10,688,936	10,761,481	10,838,341	10,919,657
42	10,690,111	10,762,726	10,839,660	10,921,051
43	10,691,287	10,763,972	10,840,980	10,922,436
44	10,692,464	10,765,220	10,842,301	10,923,833
45	10,693,642	10,766,469	10,843,623	10,925,241
46	10,694,821	10,767,720	10,844,947	10,926,641
47	10,696,001	10,768,971	10,846,272	10,928,041
48	10,697,182	10,770,224	10,847,597	10,929,442
49	10,698,364	10,771,477	10,848,924	10,930,846
50	10,699,548	10,772,732	10,850,252	10,932,249
51	10,700,732	10,773,988	10,851,583	10,933,654
52	10,701,918	10,775,244	10,852,914	10,935,061
53	10,703,105	10,776,502	10,854,246	10,936,469
54	10,704,294	10,777,761	10,855,578	10,937,879
55	10,705,483	10,779,022	10,856,912	10,939,290
56	10,706,674	10,780,284	10,858,247	10,940,702
57	10,707,866	10,781,547	10,859,584	10,942,115
58	10,709,059	10,782,802	10,860,922	10,943,527
59	10,710,254	10,784,078	10,862,262	19,944,945
60	10,711,449	10,785,347	10,863,603	10,946,362

	24	25	26	27
0	10,946,362	11,033,783	11,126,021	11,223,262
1	10,947,781	11,035,280	11,127,601	11,224,927
2	10,949,201	11,036,779	11,129,182	11,226,593
3	10,950,622	11,038,279	11,130,765	11,228,260
4	10,952,045	11,039,780	11,132,349	11,229,929
5	10,953,469	11,041,283	11,133,933	11,231,599
6	10,954,894	11,042,787	11,135,519	11,233,270
7	10,956,320	11,044,293	11,137,106	11,234,943
8	10,957,747	11,045,799	11,138,694	11,236,617
9	10,959,175	11,047,306	11,140,284	11,238,292
10	10,960,605	11,048,815	11,141,875	11,239,969
11	10,962,036	11,050,325	11,143,467	11,241,648
12	10,963,469	11,051,937	11,145,061	11,243,329
13	10,964,903	11,053,350	11,146,656	11,245,011
14	10,966,338	11,254,365	11,148,254	11,246,694
15	10,967,775	11,056,381	11,149,853	11,248,378
16	10,969,213	11,057,898	11,151,453	11,250,064
17	10,970,652	11,059,420	11,153,055	11,251,751
18	10,972,092	11,060,939	11,154,658	11,253,440
19	10,973,533	11,062,461	11,156,262	11,255,130
20	10,974,976	11,063,985	11,157,868	11,256,822
21	10,976,420	11,065,510	11,159,475	11,258,516
22	10,977,865	11,067,037	11,161,084	11,260,211
23	10,979,312	11,068,564	11,162,694	11,261,907
24	10,980,760	11,070,092	11,164,306	11,263,605
25	10,982,210	11,071,621	11,165,919	11,265,304
26	10,983,661	11,073,152	11,167,533	11,267,005
27	10,985,113	11,074,684	11,169,149	11,268,707
28	10,986,567	11,076,218	11,170,766	11,270,410
29	10,988,022	11,077,753	11,172,385	11,272,114
30	10,989,480	11,079,289	11,174,006	11,273,820

	24	25	26	27
30	10,989,480	11,079,289	11,174,006	11,273,820
31	10,990,938	11,080,827	11,175,627	11,275,528
32	10,992,398	11,082,366	11,177,249	11,277,238
33	10,993,859	11,083,906	11,178,873	11,278,949
34	10,995,321	11,085,448	11,180,499	11,280,661
35	10,996,783	11,086,990	11,182,125	11,282,374
36	10,998,247	11,088,536	11,183,753	11,284,089
37	10,999,712	11,090,082	11,185,383	11,285,805
38	11,001,179	11,091,629	11,187,014	11,287,524
39	11,002,647	11,093,178	11,188,647	11,289,244
40	11,004,116	11,094,729	11,190,281	11,290,965
41	11,005,587	11,096,280	11,191,916	11,292,688
42	11,007,059	11,097,833	11,193,553	11,294,412
43	11,008,533	11,099,387	11,195,191	11,296,132
44	11,010,008	11,100,943	11,196,831	11,297,864
45	11,011,484	11,102,500	11,198,472	11,299,593
46	11,012,962	11,104,058	11,200,114	11,301,324
47	11,014,441	11,105,618	11,203,758	11,303,056
48	11,015,921	11,107,179	11,203,404	11,304,789
49	11,017,402	11,108,741	11,205,051	11,306,523
50	11,018,884	11,110,306	11,206,700	11,308,259
51	11,020,367	11,111,871	11,208,350	11,309,996
52	11,021,852	11,113,438	11,210,001	11,311,735
53	11,023,338	11,115,006	11,211,654	11,313,476
54	11,024,826	11,116,575	11,213,308	11,315,218
55	11,026,315	11,118,145	11,214,963	11,316,961
56	11,027,806	11,119,717	11,216,620	11,318,706
57	11,029,298	11,121,290	11,218,278	11,319,452
58	11,030,791	11,122,865	11,219,938	11,322,199
59	11,032,287	11,124,442	11,221,599	11,323,949
60	11,033,783	11,126,021	11,223,262	11,325,700

	28	29	30	31
0	11,325,700	11,433,540	11,547,004	11,666,331
1	11,327,452	11,435,384	11,548,944	11,668,371
2	11,329,206	11,437,230	11,550,886	11,670,413
3	11,330,961	11,439,078	11,552,829	11,672,457
4	11,332,718	11,440,927	11,554,774	11,674,502
5	11,334,479	11,442,777	11,556,720	11,676,548
6	11,336,237	11,444,629	11,558,669	11,678,597
7	11,337,999	11,446,483	11,560,619	11,680,647
8	11,339,762	11,448,339	11,562,570	11,682,698
9	11,341,526	11,450,196	11,564,523	11,684,752
10	11,343,292	11,452,054	11,566,480	11,686,807
11	11,345,060	11,453,915	11,568,434	11,688,864
12	11,346,830	11,455,776	11,570,393	11,690,923
13	11,348,601	11,457,639	11,572,353	11,692,984
14	11,350,373	11,459,503	11,574,314	11,695,046
15	11,352,149	11,461,370	11,576,277	11,697,110
16	11,353,923	11,463,238	11,578,242	11,699,176
17	11,355,698	11,465,107	11,580,208	11,701,243
18	11,357,475	11,466,978	11,582,175	11,703,312
19	11,359,255	11,468,850	11,584,145	11,705,383
20	11,361,036	11,470,723	11,586,116	11,707,455
21	11,362,819	11,472,599	11,588,089	11,709,530
22	11,364,603	11,474,483	11,590,064	11,711,606
23	11,366,389	11,476,354	11,592,040	11,713,684
24	11,368,177	11,478,235	11,594,018	11,715,764
25	11,369,966	11,480,117	11,595,998	11,717,845
26	11,371,756	11,482,001	11,597,979	11,719,928
27	11,373,548	11,483,887	11,599,961	11,722,012
28	11,375,341	11,485,774	11,601,946	11,724,099
29	11,377,136	11,487,662	11,603,932	11,726,187
30	11,378,933	11,489,353	11,605,919	11,728,276

	28	29	30	31
30	11,378,933	11,489,353	11,605,919	11,728,276
31	11,380,731	11,491,445	11,607,909	11,730,367
32	11,382,530	11,493,338	11,609,900	11,732,460
33	11,384,331	11,495,233	11,611,893	11,734,555
34	11,386,134	11,497,140	11,613,888	11,736,652
35	11,387,938	11,499,028	11,615,876	11,738,751
36	11,389,744	11,500,928	11,617,882	11,740,851
37	11,391,551	11,502,829	11,619,881	11,742,953
38	11,393,359	11,504,731	11,621,882	11,745,057
39	11,395,169	11,506,626	11,623,885	11,747,162
40	11,396,981	11,508,532	11,625,889	11,749,269
41	11,398,794	11,510,450	11,627,996	11,751,378
42	11,400,609	11,512,360	11,629,904	11,753,489
43	11,402,425	11,514,271	11,631,913	11,755,603
44	11,404,243	11,516,183	11,633,924	11,757,718
45	11,406,063	11,518,097	11,635,937	11,759,834
46	11,407,884	11,520,013	11,637,952	11,761,951
47	11,409,706	11,521,930	11,639,968	11,764,069
48	11,411,530	11,523,849	11,641,986	11,766,190
49	11,413,356	11,525,770	11,644,005	11,768,312
50	11,415,183	11,527,692	11,646,026	11,770,437
51	11,417,012	11,529,616	11,648,049	11,772,564
52	11,418,842	11,531,542	11,650,075	11,774,696
53	11,420,673	11,533,469	11,652,099	11,776,822
54	11,422,507	11,535,398	11,654,127	11,778,954
55	11,424,342	11,537,328	11,656,156	11,781,088
56	11,426,178	11,539,260	11,658,188	11,783,223
57	11,428,016	11,541,193	11,660,221	11,785,361
58	11,429,856	11,543,128	11,662,256	11,787,500
59	11,431,689	11,545,065	11,664,292	11,789,640
60	11,433,540	11,547,004	11,666,331	11,791,783

	32	33	34	35
0	11,791,783	11,923,633	12,062,179	12,207,745
1	11,793,927	11,925,886	12,064,546	12,210,233
2	11,796,073	11,928,141	12,066,916	12,212,723
3	11,798,221	11,930,397	12,069,286	12,215,214
4	11,800,371	11,932,656	12,071,660	12,217,708
5	11,802,522	11,934,917	12,074,036	12,220,204
6	11,804,675	11,937,180	12,076,413	12,222,702
7	11,806,830	11,939,445	12,078,792	12,225,201
8	11,808,987	11,941,701	12,081,174	12,227,703
9	11,811,145	11,943,979	12,083,558	12,230,207
10	11,813,306	11,946,250	12,085,943	12,232,713
11	11,815,468	11,948,522	12,088,330	12,235,221
12	11,817,632	11,950,796	12,090,720	12,237,732
13	11,819,797	11,953,071	12,093,111	12,240,245
14	11,821,965	11,955,349	12,095,504	12,242,759
15	11,824,134	11,957,629	12,097,899	12,245,275
16	11,826,306	11,959,910	12,100,296	12,247,794
17	11,828,479	11,962,194	12,102,696	12,250,315
18	11,830,654	11,964,479	12,105,097	12,252,837
19	11,832,830	11,966,766	12,107,500	12,255,361
20	11,835,008	11,969,055	12,109,905	12,257,888
21	11,837,188	11,971,346	12,112,312	12,260,417
22	11,839,369	11,973,638	12,114,722	12,262,948
23	11,841,552	11,975,932	12,117,133	12,265,481
24	11,841,737	11,978,229	12,119,546	12,268,016
25	11,845,924	11,980,527	12,121,960	12,270,553
26	11,848,114	11,982,828	12,124,377	12,273,093
27	11,850,305	11,985,131	12,126,796	12,275,634
28	11,852,498	11,987,435	12,129,216	12,278,187
29	11,854,693	11,989,741	12,131,638	12,280,722
30	11,856,890	11,992,050	12,134,063	12,283,270

	32	33	34	35
30	11,856,890	11,992,050	12,134,063	12,283,270
31	11,859,088	11,994,360	12,136,490	12,285,820
32	11,861,288	11,996,672	12,138,919	12,288,372
33	11,863,489	11,998,986	12,141,350	12,290,925
34	11,865,693	12,001,303	12,143,783	12,293,481
35	11,867,899	12,003,619	12,146,218	12,296,039
36	11,870,107	12,005,938	12,148,656	12,298,599
37	11,872,316	12,008,259	12,150,095	12,301,161
38	11,874,527	12,010,582	12,153,536	12,303,725
39	11,876,739	12,012,907	12,155,978	12,306,291
40	11,878,954	12,015,233	12,158,423	12,308,859
41	11,881,171	12,017,562	12,160,870	12,311,430
42	11,883,389	12,019,893	12,163,319	12,314,003
43	11,885,609	12,022,226	12,165,770	12,316,578
44	11,887,831	12,024,560	12,168,223	12,319,156
45	11,890,054	12,026,897	12,170,677	12,321,736
46	11,892,280	12,029,236	12,173,135	12,324,317
47	11,894,508	12,031,576	12,175,594	12,326,900
48	11,896,737	12,033,919	12,178,055	12,329,486
49	11,898,968	12,036,264	12,180,518	12,332,074
50	11,901,202	12,038,610	12,182,983	12,334,664
51	11,903,437	12,040,958	12,185,450	12,337,256
52	11,905,674	12,043,309	12,187,919	12,339,851
53	11,907,912	12,045,661	12,190,390	12,342,448
54	11,910,153	12,048,016	12,192,864	12,345,046
55	11,912,395	12,050,372	12,195,340	12,347,646
56	11,914,640	12,052,730	12,197,817	12,350,249
57	11,916,886	12,055,089	12,200,296	12,352,854
58	11,919,133	12,057,451	12,202,777	12,355,460
59	11,921,382	12,059,814	12,205,260	12,358,068
60	11,923,633	12,062,179	12,207,745	12,360,678

	36	37	38	39
0	12,360,678	12,521,357	12,690,184	12,867,599
1	12,363,290	12,524,103	12,693,070	12,870,632
2	12,365,906	12,526,851	12,695,957	12,873,667
3	12,368,524	12,529,601	12,698,847	12,876,704
4	12,371,144	12,532,354	12,701,739	12,879,744
5	12,373,766	12,535,110	12,704,634	12,882,787
6	12,376,391	12,537,867	12,707,531	12,885,832
7	12,379,018	12,540,627	12,710,430	12,888,879
8	12,381,647	12,543,389	12,713,332	12,891,929
9	12,384,278	12,546,152	12,716,236	12,894,982
10	12,386,911	12,548,918	12,719,143	12,898,037
11	12,389,546	12,551,686	12,722,052	12,901,094
12	12,392,183	12,554,456	12,724,964	12,904,155
13	12,394,822	12,557,229	12,727,878	12,907,218
14	12,397,464	12,560,005	12,730,794	12,910,283
15	12,400,108	12,562,783	12,733,713	12,913,351
16	12,402,754	12,565,563	12,736,635	12,916,422
17	12,405,402	12,568,345	12,739,559	12,919,494
18	12,408,053	12,571,130	12,742,485	12,922,569
19	12,410,705	12,573,917	12,745,413	12,925,647
20	12,413,359	12,576,706	12,748,344	12,028,727
21	12,416,015	12,579,597	12,751,277	12,931,809
22	12,418,674	12,582,912	12,754,213	12,934,895
23	12,421,335	12,585,087	12,757,151	12,937,983
24	12,423,998	12,587,885	12,760,092	12,941,073
25	12,426,663	12,590,685	12,763,035	12,944,166
26	12,429,331	12,593,488	12,765,981	12,947,262
27	12,432,001	12,596,293	12,768,929	12,950,360
28	12,434,673	12,599,101	12,771,880	12,953,461
29	12,437,348	12,601,911	12,774,833	12,956,565
30	12,440,024	12,604,724	12,777,788	12,959,671

	36	37	38	39
30	12,440,024	12,604,724	12,777,788	12,959,671
31	12,442,702	12,607,539	12,780,746	12,962,780
32	12,445,383	12,610,356	12,783,707	12,965,892
33	12,448,066	12,613,175	12,786,670	12,969,007
34	12,450,751	12,615,997	12,789,635	12,972,124
35	12,453,438	12,618,821	12,792,602	12,975,243
36	12,456,128	12,621,648	12,795,573	12,978,366
37	12,458,821	12,624.477	12,798,546	12,981,491
38	12,461,516	12,627,308	12,801,521	12,984,618
39	12,464,213	12,630,141	12,804,498	12,987,747
40	12,466,913	12,632,977	12,807,478	12,990,880
41	12,469,614	12,635,815	12,810,460	12,994,015
42	12,472,317	12,638,655	12,813,445	12,997,153
43	12,475,022	12,641,597	12,816,432	12,000,293
44	12,477,730	12,644,343	12,819,422	13,003,436
45	12,480,440	12,646,191	12,822,415	13,006,582
46	12,483,152	12,650,041	12,825,410	13,009,730
47	12,485,866	12,652,893	12,828,407	13,012,881
48	12,488,583	12,655,748	12,831,407	13,016,034
49	12,491,302	12,658,605	12,834,409	13,019,189
50	12,494,022	12,661,464	12,837,414	13,022,348
51	12,496,744	12,664,325	12,840,421	13,025,509
52	12,499,469	12,667,189	12,843,431	13,028,673
53	12,502,197	12,670,055	12,846,443	13,031,839
54	12,504,927	12,672,924	12,849,458	13,035,008
55	12,507,659	12,675,795	12,852,475	13,038,180
56	12,510,394	12,678,668	12,855,495	13,041,354
57	12,513,132	12,681,543	12,858,517	13,044,530
58	12,515,871	12,684421	12,861,542	13,047,710
59	12,518,613	12,687,301	12,864,569	13,050,892
60	12,521,357	12,690,184	12,867.599	13,054,077

	40	41	42	43
0	13,054,077	13,250,131	13,456,326	13,673,275
1	13,057,264	13,253,482	13,459,851	13,676,986
2	13,060,455	13,256,835	13,463,380	13,680,700
3	13,063,646	13,260,192	13,466,912	13,684,417
4	13,066,843	13,263,582	13,470,447	13,688,138
5	13,070,041	13,266,915	13,473,985	13,691,861
6	13,073,242	13,270,282	13,477,527	13,695,587
7	13,076,445	13,273,651	13,481,071	13,699,316
8	13,079,651	13,277,023	13,484,618	13,703,048
9	13,082,859	13,280,397	13,488,168	13,706,783
10	13,086,071	13,283,775	13,491,721	13,710,523
11	13,089,285	13,287,155	13,495,276	13,714,266
12	13,092,502	13,290,538	13,498,835	13,718,012
13	13,095,721	13,293,924	13,502,397	13,721,761
14	13,098,944	13,297,313	13,505,962	13,725,514
15	13,102,169	13,300,704	13,509,530	13,729,270
16	13,105,397	13,304,098	13,513,101	13,733,029
17	13,108,627	13,307,495	13,516,675	13,736,790
18	13,111,861	13,310,896	13,520,252	13,740,555
19	13,114,098	13,314,299	13,523,832	13,744,322
20	13,118,337	13,317,705	13,527,416	13,748,092
21	13,121,578	13,321,114	13,531,003	13,751,867
22	13,124,823	13,324,526	13,534,593	13,755,644
23	13,128,070	13,327,941	13,538,185	13,759,424
24	13,131,320	13,331,359	13,541,781	13,763,209
25	13,134,572	13,334,779	13,545,380	13,766,997
26	13,137,828	13,338,203	13,548,981	13,770,788
27	13,141,085	13,341,629	13,552,585	13,774,582
28	13,144,346	13,345,058	13,556,193	13,778,380
29	13,147,509	13,348,490	13,559,803	13,782,181
30	13,150,874	13,351,924	13,563,417	13,785,985

	40	41	42	43
30	13,150,874	13,351,924	13,563,417	13,785,985
31	13,154,142	13,355,361	13,567,034	13,789,792
32	13,157,413	13,358,802	13,570,654	13,793,603
33	13,160,687	13,362,245	13,574,277	13,797,416
34	13,163,964	13,365,691	13,577,903	13,801,233
35	13,167,243	13,369,140	13,581,532	13,805,053
36	13,170,526	13,372,592	13,585,164	13,808,876
37	13,173,811	13,376,057	13,588,799	13,812,703
38	13,177,099	13,379,505	13,592,438	13,816,534
39	13,180,389	13,382,966	13,596,079	13,820,368
40	13,183,682	13,386,430	13,599,723	13,824,205
41	13,186,978	13,389,897	13,603,370	13,828,045
42	13,190,276	13,393,367	13,607,021	13,831,889
43	13,193,577	13,396,839	13,610,675	13,835,736
44	13,196,882	13,400,315	13,614,332	13,839,586
45	13,200,189	13,403,794	13,617,992	13,843,439
46	13,203,499	13,407,275	13,621,656	13,847,296
47	13,206,812	13,410,759	13,625,323	13,851,156
48	13,210,128	13,414,247	13,628,993	13,855,019
49	13,213,447	13,417,738	13,632,666	13,858,885
50	13,216,769	13,421,232	13,636,342	13,862,755
51	13,220,093	13,424,728	13,640,021	13,866,628
52	13,223,421	13,428,227	13,643,704	13,870,505
53	13,226,750	13,431,729	13,647,390	13,874,385
54	13,230,082	13,435,234	13,651,078	13,878,268
55	13,233,417	13,438,742	13,654,769	13,882,154
56	13,236,754	13,442,253	13,658,464	13,886,044
57	13,240,094	13,445,767	13,662,162	13,889,936
58	13,243,437	13,449,284	13,665,863	13,893,833
59	13,246,783	13,452,804	13,669,567	13,897,733
60	13,250,131	13,456,326	13,673,275	13,901,636

	44	45	46	47
0	13,901,636	14,142,135	14,395,564	14,662,790
1	13,905,542	14,146,251	14,399,901	14,667,366
2	13,909,452	14,150,371	14,404,242	14,671,946
3	13,913,365	14,154,494	14,408,587	14,676,530
4	13,917,281	14,158,621	14,412,937	14,681,119
5	13,921,201	14,162,751	14,417,290	14,685,712
6	13,925,126	14,166,884	14,421,647	14,690,309
7	13,929,052	14,171,021	14,426,008	14,694,910
8	13,922,982	14,175,162	14,430,374	14,699,514
9	13,936,916	14,179,306	14,434,743	14,704,122
10	13,940,854	14,183,454	14,439,116	14,708,735
11	13,944,795	14,187,606	14,443,493	14,713,352
12	13,948,739	14,191,761	14,447,874	14,717,973
13	13,952,686	14,195,919	14,452,259	14,722,598
14	13,956,638	14,200,082	14,456,648	14,727,228
15	13,960,592	14,204,248	14,461,040	14,731,862
16	13,964,550	14,208,418	14,465,437	14,736,500
17	13,968,511	14,212,591	14,469,838	14,741,142
18	13,972,476	14,216,769	14,474,242	14,745,788
19	13,976,444	14,220,950	14,478,650	14,750,438
20	13,980,416	14,225,135	14,483,062	14,755,094
21	13,984,391	14,229,324	14,487,478	14,759,753
22	13,988,370	14,233,517	14,491,898	14,764,416
23	13,992,352	14,237,713	14,496,322	14,769,083
24	13,996,338	14,241,912	14,500,750	14,773,755
25	14,000,327	14,246,115	14,505,182	14,778,430
26	14,004,319	14,250,321	14,509,617	14,783,110
27	14,008,315	14,254,531	14,514,056	14,787,794
28	14,012,314	14,258,745	14,518,500	14,792,482
29	14,016,316	14,262,961	14,522,946	14,797,174
30	14,020,322	14,267,182	14,527,397	14,801,871

	44	45	46	47
30	14,020,322	14,267,182	14,527,397	14,801,871
31	14,024,332	14,271,407	14,531,852	14,806,571
32	14,028,345	14,275,635	14,536,311	14,811,276
33	14,032,361	14,279,867	14,540,773	14,815,985
34	14,036,381	14,284,103	14,545,240	14,820,698
35	14,040,404	14,288,343	14,549,711	14,825,416
36	14,044,431	14,292,587	14,554,186	14,830,139
37	14,048,461	14,296,834	14,558,665	14,834,866
38	14,052,494	14,301,086	14,563,148	14,839,597
39	14,056,531	14,305,331	14,567,635	14,844,332
40	14,060,572	14,309,599	14,572,126	14,849,072
41	14,064,616	14,313,861	14,576,621	14,853,815
42	14,068,664	14,318,127	14,581,120	14,858,563
43	14,072,715	14,322,396	14,585,624	14,863,315
44	14,076,770	14,326,670	14,590,131	14,868,071
45	14,080,829	14,330,947	14,594,642	14,872,831
46	14,084,891	14,335,228	14,599,157	14,877,597
47	14,088,956	14,339,513	14,603,676	14,882,377
48	14,093,026	14,343,802	14,608,199	14,887,141
49	14,097,099	14,348,095	14,612,725	14,891,919
50	14,101,175	14,352,391	14,617,256	14,896,701
51	14,105,255	14,356,691	14,621,791	14,901,487
52	14,109,339	14,360,995	14,626,330	14,906,278
53	14,113,427	14,365,303	14,630,873	14,911,073
54	14,117,518	14,369,615	14,635,421	14,915,873
55	14,121,612	14,373,930	14,639,973	14,920,677
56	14,125,709	14,378,350	14,644,528	14,925,486
57	14,129,810	14,382,573	14,649,087	14,930,299
58	14,133,915	14,386,900	14,653,651	14,935,116
59	14,138,023	14,391,230	14,658,218	14,939,938
60	14,142,135	14,395,564	14,662,790	14,944,764

	48	49	50	51
0	14,944,764	15,242,532	15,557,239	15,890,158
1	14,949,594	15,247,634	15,562,635	15,895,869
2	14,954,429	15,252,741	15,568,036	15,901,586
3	14,959,268	15,257,852	15,573,441	15,907,307
4	14,964,112	15,262,969	15,578,852	15,913,034
5	14,968,960	15,268,990	15,584,267	15,918,766
6	14,973,812	15,273,216	15,589,688	15,924,504
7	14,978,668	15,278,347	15,595,114	15,930,247
8	14,983,530	15,283,484	15,600,545	15,936,095
9	14,988,396	15,288,626	15,605,981	15,941,748
10	14,993,266	15,293,773	15,611,422	15,947,508
11	14,998,104	15,298,924	15,616,868	15,953,273
12	15,003,020	15,304,080	15,622,319	15,959,044
13	15,007,903	15,309,240	15,627,775	15,964,820
14	15,012,791	15,314,405	15,633,237	15,970,603
15	15,017,683	15,319,574	15,639,704	15,976,390
16	15,022,580	15,324,748	15,644,177	15,982,184
17	15,027,481	15,329,926	15,649,655	15,987,983
18	15,032,387	15,335,109	15,655,138	15,993,788
19	15,037,297	15,340,297	15,660,626	15,999,599
20	15,042,212	15,345,491	15,666,119	16,005,416
21	15,047,131	15,350,689	15,671,617	16,011,237
22	15,052,054	15,355,892	15,677,121	16,017,065
23	15,056,982	15,361,100	15,682,630	16,022,898
24	15,061,915	15,366,313	15,688,144	16,028,736
25	15,066,852	15,371,530	15,693,663	16,034,579
26	15,071,791	15,375,753	15,699,188	16,040,429
27	15,076,739	15,381,980	15,704,717	16,046,283
28	15,081,690	15,387,212	15,710,252	16,052,143
29	15,086,645	15,392,449	15,715,792	16,058,008
30	15,091,605	15,397,692	15,721,337	16,063,878

	48	49	50	51
30	15,091,605	15,397,692	15,721,337	16,063,878
31	15,096,569	15,402,939	15,726,887	16,069,754
32	15,101,538	15,408,191	15,732,443	16,075,637
33	15,106,571	15,413,447	15,738,003	16,081,524
34	15,111,490	15,418,708	15,743,569	16,087,418
35	15,116,472	15,423,974	15,749,141	16,093,318
36	15,121,459	15,429,246	15,754,718	16,099,224
37	15,126,451	15,434,522	15,760,300	16,105,135
38	15,131,447	15,439,803	15,765,887	16,111,053
39	15,136,447	15,445,089	15,771,479	16,116,976
40	15,141,453	15,450,380	15,777,077	16,122,905
41	15,146,463	15,455,675	15,782,680	16,128,839
42	15,151,478	15,460,976	15,788,289	16,134,779
43	15,156,497	15,466,282	15,793,903	16,140,724
44	15,161,520	15,471,593	15,799,523	16,146,676
45	15,166,548	15,476,908	15,805,147	16,152,634
46	15,171,581	15,482,229	15,810,777	16,158,598
47	15,176,619	15,487,554	15,816,412	16,164,567
48	15,181,661	15,492,885	15,822,052	16,170,542
49	15,186,708	15,498,220	15,827,697	16,176,522
50	15,191,760	15,503,560	15,833,349	16,182,509
51	15,196,816	15,508,905	15,839,005	16,188,501
52	15,201,877	15,514,256	15,844,667	16,194,499
53	15,206,943	15,519,611	15,850,335	16,200,503
54	15,212,013	15,524,972	15,856,008	16,206,513
55	15,217,088	15,530,338	15,861,676	16,212,528
56	15,222,168	15,535,710	15,867,370	16,218,550
57	15,227,253	15,541,083	15,873,058	16,224,577
58	15,232,342	15,546,463	15,878,753	16,230,610
59	15,237,435	15,551,848	15,884,453	16,236,648
60	15,242,532	15,557,239	15,890,158	16,242,692

	52	53	54	55
0	16,242,692	16,616,401	17,013,017	17,434,469
1	16,248,742	16,622,819	17,019,832	17,441,715
2	16,254,799	16,629,243	17,026,654	17,448,968
3	16,260,861	16,635,673	17,033,482	17,456,229
4	16,266,929	16,642,109	17,040,318	17,463,499
5	16,273,003	16,648,551	17,047,160	17,470,775
6	16,279,083	16,655,001	17,054,010	17,478,059
7	16,285,169	16,661,457	17,060,866	17,485,351
8	16,291,261	16,667,919	17,067,729	17,492,650
9	16,297,358	16,674,408	17,074,599	17,499,957
10	16,303,461	16,680,864	17,081,476	17,507,272
11	16,309,570	16,687,345	17,088,359	17,514,594
12	16,315,685	16,693,834	17,095,250	17,521,924
13	16,321,806	16,700,328	17,102,148	17,529,262
14	16,327,934	16,706,829	17,109,053	17,536,607
15	16,334,067	16,713,336	17,115,965	17,543,959
16	16,340,197	16,719,850	17,122,885	17,551,319
17	16,346,353	16,726,362	17,129,812	17,558,687
18	16,352,505	16,732,877	17,136,747	17,566,063
19	16,358,663	16,739,430	17,143,689	17,573,446
20	16,364,827	16,745,970	17,150,638	17,580,837
21	16,370,996	16,752,517	17,157,593	17,588,236
22	16,377,172	16,759,070	17,164,556	17,595,643
23	16,383,359	16,765,629	17,171,525	17,603,057
24	16,389,542	16,772,195	17,178,502	17,610,480
25	16,395,736	16,778,767	17,185,485	17,617,909
26	16,401,936	16,785,347	17,192,476	17,625,347
27	16,408,152	16,791,933	17,199,472	17,632,793
28	16,414,365	16,798,525	17,206,477	17,640,246
29	16,420,573	16,805,124	17,213,488	17,647,707
30	16,426,798	16,811,729	17,220,507	17,655,175

	52	53	54	55
30	16,426,798	16,811,729	17,220,507	17,655,175
31	16,433,027	16,818,341	17,227,532	17,662,651
32	16,439,263	16,824,960	17,234,565	17,670,136
33	16,445,505	16,831,585	17,241,605	17,677,627
34	16,451,754	16,838,217	17,248,653	17,685,127
35	16,458,008	16,844,856	17,255,708	17,692,635
36	16,464,269	16,851,502	17,262,770	17,700,151
37	16,470,536	16,858,154	17,269,839	17,707,674
38	16,456,809	16,864,813	17,276,917	17,715,206
39	16,483,089	16,871,479	17,284,002	17,722,744
40	16,489,385	16,878,151	17,291,095	17,730,290
41	16,495,668	16,884,830	17,298,194	17,737,844
42	16,501,967	16,891,515	17,305,300	17,745,407
43	16,508,272	16,898,207	17,302,413	17,752,978
44	16,514,582	16,904,907	17,319,514	17,760,555
45	16,520,898	16,911,613	17,326,662	17,768,142
46	16,527,220	16,918,326	17,333,798	17,775,740
47	16,533,548	16,925,046	17,340,941	17,783,343
48	15,539,883	16,931,772	17,348,091	17,790,955
49	16,546,224	16,948,504	17,355,249	17,798,575
50	16,552,571	16,945,244	17,362,415	17,806,203
51	16,558,925	16,951,990	17,369,587	17,813,838
52	16,565,286	16,958,743	17,376,767	17,821.481
53	16,571,642	16,965,495	17,383,954	17,829,132
54	16,578,026	16,972,270	17,391,148	17,836,792
55	16,584,406	16,979,044	17,398,350	17,844.460
56	16,590,792	16,985,824	17,405,560	17,852,135
57	16,597,184	16,992,611	17,412,776	17,859,818
58	16,603,584	16,999,406	17,420,000	17,867,509
59	16,609,989	17,006,208	17,427.231	17,875,209
60	16,616,401	17,013,017	17,434.469	17.882,917

	56	57	58	59
0	17,882,917	18,360,816	18,870,800	19,416,039
1	17,890,632	18,369,014	18,879,589	19,425,445
2	17,898,356	18,377,251	18,888,389	19,434,862
3	17,906,089	18,385,497	18,897,196	19,444,290
4	17,913,830	18,393,753	18,906,018	19,453,727
5	17,921,579	18,402,017	18,914,846	19,463,175
6	17,929,337	18,410,291	18,923,685	19,472,635
7	17,937,102	18,418,574	18,932,534	19,482,114
8	17,944,876	18,426,865	18,941,393	19,491,595
9	17,952,658	18,435,165	18,950,261	19,501,076
10	17,960,448	18,443,454	18,959,139	19,510,578
11	17,968,247	18,451,792	18,968,027	19,520,091
12	17,976,054	18,460,120	18,976,926	19,529,615
13	17,983,869	18,468,456	18,985,834	19,539,150
14	17,991,693	18,476,802	18,994,752	19,548,697
15	17,999,525	18,485,157	19,003,680	19,558,254
16	18,007,365	18,493,521	19,012,618	19,567,822
17	18,015,214	18,501,895	19,021,516	19,577,401
18	18,023,071	18,510,278	19,030,523	19,586,991
19	18,030,936	18,518,670	19,039,491	19,596,592
20	18,038,811	18,527,072	19,048,468	19,606,204
21	18,046,693	18,535,483	19,057,455	19,615,827
22	18,054,584	18,543,903	19,066,453	19,625,462
23	18,062,482	18,552,332	19,075,461	19,635,107
24	18,070,389	18,560,770	19,084,480	19,644,765
25	18,078,305	18,569,217	19,093,509	19,654,434
26	18,086,229	18,577,674	19,102,549	19,664,114
27	18,094,161	18,586,139	19,111,598	19,673,805
28	18,102,102	18,594,614	19,120,658	19,683,507
29	18,110,051	18,603,098	19,129,727	19,693,220
30	18,118,009	18,611,591	19,138,807	19,702,945

	56	57	58	59
30	18,118,009	18,611,591	19,138,807	19,702,945
31	18,125,975	18,620,094	19,147,897	19,712,680
32	18,133,950	18,628,606	19,156,998	19,722,428
33	18,141,934	18,637,127	19,166,109	19,732,186
34	18,149,926	18,645,658	19,175,231	19,741,956
35	18,157,927	18,654,198	19,184,362	19,751,738
36	18,165,937	18,662,748	19,193,504	19,761,531
37	18,173,956	18,671,307	19,202,656	19,771,335
38	18,181,984	18,679,875	19,211,818	19,781,141
39	18,190,021	18,688,452	19,220,990	19,790,968
40	18,198,065	18,697,038	19,230,172	19,800,808
41	18,206,118	18,705,634	19,239,365	19,810,658
42	18,214,179	18,714,239	19,248,569	19,820,520
43	18,222,249	18,722,854	19,257,783	19,830,393
44	18,230,328	18,731,480	19,267,008	19,840,277
45	18,238,416	18,740,115	19,276,242	19,850,172
46	18,246,513	18,748,760	19,285,488	19,860,079
47	18,254,618	18,757,414	19,294,744	19,869,997
48	18,262,732	18,766,078	19,304,010	19,879,927
49	18,270,854	18,774,752	19,313,287	19,889,868
50	18,278,986	18,783,436	19,322,574	19,899,820
51	18,287,126	18,792,130	19,331,872	19,909,784
52	18,295,276	18,800,833	19,341,181	19,919,760
53	18,303,434	18,809,546	19,350,501	19,929,748
54	18,311,601	18,818,268	19,359,831	19,939,749
55	18,319,776	18,826,999	19,369,172	19,949,760
56	18,327,961	18,835,741	19,378,524	19,959,784
57	18,336,154	18,844,492	19,387,886	19,969,820
58	18,344,356	18,853,252	19,397,260	19,979,868
59	18,352,567	18,862,021	19,406,644	19,989,928
60	18,360,816	18,870,800	19,416,039	20,000,000

	60	61	62	63
0	20,000,000	20,626,654	21,300,545	22,026,892
1	20,010,083	20,637,484	21,312,206	22,039,475
2	20,020,179	20,648,338	21,323,882	22,052,074
3	20,030,285	20,659,184	21,335,570	22,064,690
4	20,040,404	20,670,054	21,347,275	22,077,322
5	20,050,534	20,680,937	21,358,993	22,089,970
6	20,060,676	20,691,834	21,370,727	22,102,635
7	20,070,832	20,702,744	21,382,475	22,115,316
8	20,080,995	20,713,667	21,394,238	22,128,014
9	20,091,172	20,724,603	21,407,016	22,140,727
10	20,101,361	20,735,554	21,417,808	22,153,459
11	20,111,562	20,746,517	21,429,615	22,166,204
12	20,121,776	20,757,494	21,441,438	22,178,971
13	20,132,001	20,768,484	21,453,275	22,191,751
14	20,142,239	20,779,488	21,465,128	22,204,548
15	20,152,489	20,790,505	21,476,995	22,217,361
16	20,162,751	20,801,535	21,488,877	22,230,191
17	20,173,035	20,812,579	21,500.774	22,243,038
18	20,183,321	20,823,636	21,512,686	22,255,902
19	20,193,619	20,834,706	21,524.612	22,268,782
20	20,203,930	20,845,791	21,536,553	22,281,680
21	20,214,252	20,856,888	21,548,509	22,294,595
22	20,224,588	20,868,000	21,560,481	22,307,526
23	20,234,936	20,879,125	21,572,467	22,320,474
24	20,245,296	20,890,264	21,584,469	22,333,439
25	20,255,669	20,901.416	21,596,487	22,346,420
26	20,266,054	20,912,582	21,608,520	22,359,419
27	20,276,452	20,923,761	21,620,568	22,372,434
28	20,286,863	20,934,955	21,632,631	22,385,466
29	20,297,286	20,946,162	21,644,710	22,398,418
30	20,307,721	20,957,383	21,656,804	22,411,584

	60	61	62	63
30	20,307,721	20,957,383	21,656,804	22,411,584
31	20,318,170	20,968,618	21,668,913	22,424,667
32	20,328,630	20,979,867	21,681,038	22,437,768
33	20,339,102	20,991,130	21,693,178	22,450,886
34	20,349,587	21,002,406	21,705,334	22,464,022
35	20,360,084	21,013,696	21,717,505	22,477,175
36	20,370,594	21,025,001	21,729,691	22,490,346
37	20,381,116	21,036,319	21,741,893	22,503,543
38	20,391,751	21,047,651	21,754,111	22,516,748
39	20,402,198	21,058,997	21,766,344	22,529,965
40	20,412,758	21,070,357	21,778,593	22,543,201
41	20,423,331	21,081,731	21,770,858	22,556,358
42	20,433,916	21,093,119	21,803,138	22,569,723
43	20,444,514	21,104,522	21,815,434	22,583,025
44	20,455,126	21,115,938	21,827,745	22,596,336
45	20,465,750	21,127,368	21,840,072	22,609,663
46	20,476,387	21,138,814	21,852,415	22,623,009
47	20,487,037	21,150,273	21,864,774	22,636,372
48	20,497,700	21,161,747	21,877,149	22,659,753
49	20,508,376	21,173,235	21,889,539	22,663,152
50	20,519,064	21,184,737	21,901,946	22,676,569
51	20,529,765	21,196,253	21,914,369	22,690,004
52	20,540,479	21,207,783	21,926,808	22,703.456
53	20,551,205	21,219,328	21,939,263	22,716,924
54	20,561,945	21,230,887	21,951,734	22,730,414
55	20,572,697	21,242,460	21,964,220	22,743,919
56	20,583,463	21,254,048	21,976,722	22,757,443
57	20,594.242	21,265,650	21,989,240	22,770,984
58	20,605,033	21,277,267	22,001,775	22,784,543
59	20,615,837	21,288,899	22,014.325	22,798,120
60	20,626,654	21,300,545	22,026 892	22,811,726

	64	65	66	67
0	22,811,726	23,662,013	24,585,936	25,593,051
1	22,825,329	23,676,784	24,602,010	25,610,602
2	22,838,962	23,691,575	24,618,107	25,628,180
3	22,852,612	23,706,387	24,634,227	25,645,783
4	22,866,281	23,721,220	24,650,370	25,663,414
5	22,879,968	23,736,073	24,666,536	25,681,071
6	22,893,674	23,750,947	24,682,727	25,698,754
7	22,807,387	23,765,842	24,698,940	25,716,464
8	22,921,140	23,780,757	24,715,178	25,734,201
9	22,934,901	23,795,692	24,731,439	25,751,965
10	22,948,680	23,810,648	24,747,724	25,769,755
11	22,962,478	23,825,625	24,764,033	25,787,582
12	22,976,294	23,840,623	24,780,365	25,805,417
13	22,990,129	23,855,642	24,796,721	25,823,287
14	23,003,983	23,870,683	24,813,101	25,841,185
15	23,017,855	23,885,844	24,829,504	25,859,104
16	23,031,747	23,900,827	24,845,932	25,877,061
17	23,045,657	23,915,931	24,862,383	25,895,040
18	23,059,586	23,931,055	24,878,858	25,913,046
19	23,073,534	23,946,200	24,895,356	25,931,080
20	23,087,501	23,961,366	24,911,878	25,949,142
21	23,101,486	23,976,553	24,928,423	25,967,230
22	23,115,490	23,991,762	24,944,993	25,985,345
23	23,129,513	24,006,992	24,961,587	26,003,487
24	23,143,556	24,022,245	24,978,205	26,021,658
25	23,157,616	24,037,518	24,994,847	26,039,855
26	23,171,696	24,052,814	25,011,514	26,058,081
27	23,185,795	24,068,130	25,028,205	26,076,333
28	23,199,913	24,083,469	25,044,920	26,094,614
29	23,214,050	24,098,830	25,061,660	26,112,923
30	23,228,205	24,114,213	25,078,426	26,131,259

	64	65	66	67
30	23,228,205	24,114,213	25,078,426	26,131,259
31	23,242,380	24,129,616	25,095,216	26,149,623
32	23,256,574	24,145,041	25,112,030	26,168,015
33	23,270,797	24,160,487	25,128,869	26,186,436
34	23,285,021	24,175,956	25,145,732	26,204,884
35	23,299,273	24,191,445	25,162,620	26,223,361
36	23,313,546	24,206,956	25,179,532	26,241,867
37	23,327,838	24,222,488	25,196,469	26,260,400
38	23,342,150	24,238,043	25,213,432	26,278,963
39	23,356,481	24,253,619	25,230,418	26,297,555
40	23,370,832	24,269,217	25,247,431	26,316,176
41	23,385,203	24,284,838	25,264,468	26,334,825
42	23,399,593	24,300,481	25,281,531	26,353,503
43	23,414,003	24,316,147	25,298,620	26,372,209
44	23,428,433	24,331,835	25,315,734	26,390,945
45	23,442,882	24,347,546	25,332,874	26,409,709
46	23,457,351	24,363,281	25,350,039	26,428,502
47	23,471,840	24,379,038	25,367,229	26,447,323
48	23,486,348	24,394,818	25,384,445	26,466,174
49	23,500,876	24,410,620	25,401,687	26,485,053
50	23,515,424	24,426,446	25,418,956	26,503,962
51	23,529,992	24,442,294	25,436,250	26,522,890
52	23,544,580	24,458,164	25,453,570	26,541,867
53	23,559,188	24,474,056	25,470,915	26,560,863
54	23,573,817	24,489,973	25,488,286	26,579,889
55	23,588,565	24,505,908	25,505,683	26,598,945
56	23,603,134	24,521,869	25,523,005	26,618,030
57	23,617,822	24,537,851	25,540,553	26,637,145
58	23,632,532	24,553,857	25,558,027	26,656,291
59	23,647,262	24,569,885	25,575,526	26,675,466
60	23,662,013	24,585,936	25,593,051	26,694,672

	68	69	70	71
0	26,694,672	27,904,284	29,238,045	30,715,531
1	26,713,907	27,925,445	29,261,433	30,741,500
2	26,733,172	27,946,642	29,284,861	30,767,516
3	26,752,467	27,967,873	29,308,328	30,793,579
4	26,771,791	27,989,139	29,331,835	30,819,689
5	26,791,145	28,010,440	29,355,382	30,845,846
6	26,810,529	28,031,776	29,378,970	30,872,051
7	26,829,942	28,053,147	29,402,599	30,898,304
8	26,849,390	28,074,553	29,426,268	30,924,605
9	26,868,867	28,095,994	29,449,978	30,950,953
10	26,888,373	28,117,469	29,473,728	30,977,350
11	26,907,910	28,138,980	29,497,519	31,003,793
12	26,927,479	28,160,527	29,521,350	31,030,285
13	26,947,078	28,182,108	29,545,222	31,056,824
14	26,966,709	28,203,725	29,569,136	31,083,412
15	26,986,370	28,225,378	29,593,090	31,110,047
16	27,006,062	28,247,067	29,617,087	31,136,731
17	27,025,785	28,268,793	29,641,124	31,163,462
18	27,045,539	28,290,553	29,665,204	31,190,241
19	27,065,323	28,312,349	29,689,326	31,217,019
20	27,085,138	28,334,181	29,713,488	31,243,945
21	27,104,985	28,356,049	29,737,692	31,270,871
22	27,124,864	28,377,954	29,761,938	31,297,848
23	27,144,774	28,399,894	29,786,227	31,324,873
24	27,164,717	28,421,871	29,810,558	31,351,948
25	27,184,690	28,443,884	29,834,931	31,379,072
26	27,204,686	28,465,934	29,859,347	31,406,247
27	27,224,734	28,488,021	29,883,705	31,433,472
28	27,244,804	28,510,144	29,908,306	31,460,747
29	27,264,906	28,532,304	29,932,850	31,488,072
30	27,285,040	28,554,501	29,957,438	31,515,448

	68	69	70	71
30	27,285,040	28,554,501	29,957,438	31,515,448
31	27,305,205	28,576,735	29,982,069	31,542,873
32	27,325,402	28,599,007	30,006,743	31,570,349
33	27,345,631	28,521,316	30,031,460	31,597,875
34	27,365,893	28,643,662	30,056,220	31,625,453
35	27,386,286	28,666,045	30,081,023	31,653,080
36	27,406,513	28,688,467	30,105,870	31,680,758
37	27,426,872	28,710,925	30,130,760	31,708,486
38	27,447,264	28,733,422	30,155,714	31,736,265
39	27,497,688	28,755,956	30,180,672	31,764,094
40	27,488,145	28,778,549	30,205,694	31,791,974
41	27,508,635	28,801,139	30,230,760	31,819,906
42	27,529,157	28,823,787	30,255,871	31,847,891
43	27,549,722	28,846,473	30,281,026	31,875,929
44	27,570,301	28,869,196	30,306,226	31,904,019
45	27,590,922	28,891,957	30,331,460	31,932,164
46	27,611,578	28,914,756	30,356,759	31,960,358
47	27,632,266	28,937,594	30,382,092	31,988,606
48	27,652,989	28,960,471	30,407,470	32,016,909
49	27,673,745	28,983,386	30,432,893	32,045,263
50	27,694,535	29,006,340	30,458,361	32,073,672
51	27,715,358	29,029,332	30,483,873	32,102,132
52	27,736,215	29,052,363	30,509,430	32,130,646
53	27,757,105	29,075,435	30,535,033	32,159,212
54	27,778,029	29,098,546	30,560,682	32,187,832
55	27,798,987	29,121,697	30,586,375	32,216,504
56	27,819,978	29,144,888	30,617,115	32,245,231
57	27,841,003	29,168,118	30,637,890	32,274,012
58	27,862,060	29,191,388	30,663,732	32,302,846
59	27,883,156	29,214,697	30,689,608	32,331,735
60	27,904,284	29,238,045	30,715,531	32,360,678

	72	73	74	75
0	32,360,678	34,203,038	36,279,559	38,637,042
1	32,389,676	34,235,609	36,316,402	38,679,033
2	32,418,726	34,268,245	36,353,333	38,721,117
3	32,447,837	34,300,947	36,390,323	38,763,296
4	32,477,001	34,333,716	36,427,401	38,805,571
5	32,506,219	34,366,553	36,464,558	38,847,941
6	32,535,494	34,399,452	36,561,793	38,890,408
7	32,564,823	34,432,420	36,539,107	38,932,971
8	32,594,209	34,465,456	36,576,511	38,975,632
9	32,623,651	34,498,557	36,613,973	38,018,390
10	32,653,148	34,531,726	36,651,525	39,061,246
11	32,682,701	34,564,959	36,689,156	39,104,200
12	32,712,311	34,598,259	36,726,868	39,147,252
13	32,741,977	34,631,626	36,764,660	39,190,423
14	32,771,699	34,665,061	36,702,533	39,233,653
15	32,801,478	34,698,564	36,840,488	39,277,002
16	32,831,314	34,732,135	36,878,524	39,320,449
17	32,861,207	34,765,775	36,916,641	39,363,994
18	32,891,157	34,799,483	36,954,842	39,407,640
19	32,921,165	34,833,259	36,993,127	39,451,384
20	32,951,231	34,867,105	37,031,496	39,495,228
21	32,981,355	34,901,024	37,069,947	39,539,172
22	33,011,537	34,935,005	37,108,482	39,583,218
23	33,041,776	34,969,052	37,147,101	39,627,364
24	33,072,074	35,003,172	37,185,803	39,671,613
25	33,102,431	35,037,361	37,224,589	39,715,965
26	33,132,846	35,071,621	37,263,459	39,760,420
27	33,163,320	35,105,952	37,302,413	39,704,979
28	33,193,853	35,140,354	37,341,453	39,849,642
29	33,224,444	35,174,826	37,380,577	39,894,411
30	33,255,094	35,209,369	37,419,788	39,939,286

	72	73	74	75
30	33,255,094	35,209,369	37,419,788	39,939,286
31	33,285,803	35,243,981	37,459,081	39,984,263
32	33,316,571	35,278,664	37,498,460	40,029,344
33	33,347,398	35,313,418	37,537,923	40,074,528
34	33,378,286	35,348,244	37,577,471	40,119,816
35	33,409,232	35,383,140	37,617,104	40,165,209
36	33,440,240	35,418,110	37,656,824	40,210,709
37	33,471,307	35,453,152	37,696,632	40,256,316
38	33,502,436	35,488,268	37,736,518	40,302,033
39	33,533,625	35,423,456	37,776,513	40,347,858
40	33,564,875	35,558,718	37,816,588	40,393,792
41	33,596,187	35,594,052	37,856,751	40,439,834
42	33,627,561	35,629,460	37,897,004	40,485,985
43	33,658,998	35,664,940	37,937,146	40,532,245
44	33,690,497	35,700,494	37,977,779	40,578,613
45	33,722,059	35,736,121	38,018,300	40,625,091
46	33,753,683	35,771,822	38,058,912	40,671,678
47	33,785,370	35,807,597	38,099,614	40,718,374
48	33,817,120	35,843,447	38,140,406	40,765,180
49	33,848,934	35,879,373	38,181,288	40,812,093
50	33,880,813	35,915,374	38,222,261	40,859,121
51	33,912,753	35,951,451	38,263,324	40,906,259
52	33,944,756	35,987,602	38,304479	40,953,510
53	33,976,821	36,023,829	38,345,725	41,000,876
54	34,008,950	36,060,132	38,387,064	41,048,358
55	34,041,141	36,096,510	38,428,495	41,095,957
56	34,073,395	36,132,966	38,470,019	41,143,668
57	34,105,712	36,169,497	38,511,635	41,191,492
58	34,138,091	36,206,107	38,553,344	41,239,431
59	34,170,523	36,242,794	38,595,146	41,287,425
60	34,203,038	36,279,559	38,637,042	41,335,654

	76	77	78	79
0	41,335,654	44,454,097	48,097,335	52,408,433
1	41,383,937	44,510,183	48,163,251	52,486,983
2	41,432,338	44,566,415	48,229,350	52,565,774
3	41,480,856	44,622,793	48,295,633	52,644,807
4	41,529,492	44,679,318	48,362,102	52,724,084
5	41,578,245	44,735,990	48,428,756	52,803,604
6	41,627,117	44,792,810	48,495,599	52,883,368
7	41,676,108	44,849,777	48,562,631	52,963,377
8	41,725,219	44,906,892	48,629,854	53,043,632
9	41,774,450	44,964,155	48,697,269	53,124,134
10	41,823,802	45,021,567	48,764,877	53,204,885
11	41,873,273	45,079,129	48,832,678	53,285,884
12	41,922,863	45,136,843	48,900,673	53,367,134
13	41,972,573	45,194,707	48,968,853	53,448,635
14	42,022,405	45,252,726	49,037,249	53,530,390
15	42,072,357	45,310,898	49,105,830	53,612,399
16	42,122,431	45,369,224	49,174,607	53,694,666
17	42,172,625	45,427,703	49,243,590	53,777,191
18	42,222,942	45,486,338	49,312,751	53,859,976
19	42,273,380	45,545,127	49,382,118	53,943,022
20	42,323,942	45,604,073	49,451,684	54,026,331
21	42,374,627	45,663,175	49,521,449	54,109,903
22	42,425,439	45,722,435	49,591,416	54,193,739
23	42,476,377	45,781,853	49,661,584	54,277,840
24	42,527,442	45,841,429	49,731,956	54,362,207
25	42,578,635	45,901,164	49,802,532	54,446,842
26	42,629,957	45,961,059	49,873,313	54,531,744
27	42,681,409	46,021,115	49,944,301	54,616,915
28	42,732,991	46,081,333	50,015,[illegible]7	54,702,356
29	42,784,705	46,141,715	50,086,901	54,788,068
30	42,836,551	46,202,261	50,158,514	54,874,053

	76	77	78	79
30	42,836,551	46,202,261	50,158,514	54,874,053
31	42,888,527	46,262,969	50,230,335	54,960,312
32	42,940,631	46,323,841	50,302,367	55,046,847
33	42,992,865	46,384,877	50,374,610	55,133,659
34	43,045,229	46,446,076	50,447,065	55,220,751
35	43,097,722	46,507,440	50,519,732	55,308,122
36	43,150,347	46,568,970	50,592,614	55,395,775
37	43,203,103	46,630,665	50,665,711	55,483,710
38	43,255,992	46,692,527	50,739,024	55,571,930
39	43,309,012	46,754,555	50,812,553	55,660.434
40	43,362,166	46,816,752	50,886,299	55,749,226
41	43,415,454	46,879,117	50,960,263	55,838,300
42	43,468,877	46,941,653	51,034,447	55,927,677
43	43,522,435	47,004,361	51,108,850	56,017,340
44	43,576,129	47,067,242	51,183,475	56,107,297
45	43,629,959	47,130,297	51,258,321	56,197,549
46	43,683,925	47,193,526	51,333,391	56,288,099
47	43,738,028	47,256,930	51,408,684	56,378,948
48	43,792,268	47,320,509	51,484,204	56,470,097
49	43,846,646	47,384,264	51,559,951	56,561,548
50	43,901,162	47,448,195	51,635,936	56,653,302
51	43,955,817	47,512,302	51,712,129	56,745,360
52	44,000,612	47,576,586	51,788,563	56,837,723
53	44,065,548	47,641,048	51,865,227	56,930,392
54	44,120.625	47,705,689	51,942,124	57,023,369
55	44,175,844	47,770,510	52,019,254	57,116,653
56	44,231,207	47,835,511	52,096,618	57,210,246
57	44,286,712	47,900,693	52,174,216	57,304,150
58	44,342,362	47,966,058	52,252,051	57,398,367
59	44 398,156	48,031,605	52,330,123	57,492,896
60	44,454,097	48,097,335	52,408,433	57,587,740

	80	81	82	83
0	57,587,740	63,924,495	71,852,975	82,055,127
1	57,682,901	64,042,118	72,002,006	82,249,986
2	57,778,381	64,160,180	72,151,659	82,445,779
3	57,874,180	64,278,683	72,301,942	82,642,513
4	57,970,302	64,397,632	72,452,863	82,840,196
5	58,066,748	64,517,028	72,604,421	83,038,833
6	58,163,520	64,636,873	72,756,618	83,238,436
7	58,260,619	64,757,168	72,909,461	83,439,009
8	58,358,049	64,877,918	73,062,954	83,640,561
9	58,455,810	64,999,124	73,217,100	83,843,097
10	58,553,904	65,120,789	73,371,903	84,046,626
11	58,652,333	65,242,916	73,527,367	84,251,153
12	58,751,099	65,365,508	73,683,499	84,456,680
13	58,850,205	65,488,566	73,840,302	84,663,213
14	58,949,653	65,612,095	73,997,782	84,870,760
15	59,049,444	65,736,097	74,155,942	85,079,327
16	59,149,581	65,859,675	74,314,786	85,288,957
17	59,250,065	65,985,531	74,474,318	85,499,628
18	59,350,898	66,110,967	74,634,544	85,711,347
19	59,452,082	66,246,886	74,795,468	85,924,121
20	59,553,618	66,363,291	74,957,095	86,137,958
21	59,655,506	66,490,185	75,119,429	86,352,864
22	59,757,728	66,617,572	75,282,475	86,568,849
23	59,860,346	66,745,453	75,446,238	86,785,921
24	59,963,291	66,873,831	75,610,721	87,004,089
25	60,066,612	67,002,708	75,775,928	87,223,362
26	60,170,285	67,132,088	75,941,864	87,443,750
27	60,274,319	67,261,972	76,108,533	87,665,261
28	60,378,718	67,392,365	76,275,941	87,887,909
29	60,483,482	67,523,270	76,444,091	88,111,704
30	60,588,615	67,654,691	76,612,989	88,336,657

	80	81	82	83
30	60,588,615	67,654,691	76,612,989	88,336,657
31	60,694,118	67,786,629	76,782,641	88,562,776
32	60,799,995	67,919,089	76,953,050	88,790,069
33	60,906,246	68,052,073	77,124,223	89,018,543
34	61,012,875	68,185,585	77,296,165	89,248,201
35	61,119,882	68,319,630	77,468,882	89,479,054
36	61,227,271	68,454,208	77,642,381	89,711,108
37	61,335,043	68,589,313	77,816,665	89,944,373
38	61,443,202	68,724,977	77,991,740	90,178,856
39	61,551,749	68,861,175	78,167,612	90,414,568
40	61,660,686	68,997,920	78,344,287	90,651,519
41	61,770,013	69,135,315	78,521,769	90,889,717
42	61,879,735	69,273,018	78,700,066	91,129,181
43	61,989,853	69,411,469	78,879,183	91,369,917
44	62,100,367	69,550,434	79,059,128	91,611,941
45	62,211,280	69,689,963	79,239,905	91,855,265
46	62,322,594	69,830,059	79,421,520	92,099,899
47	62,434,312	69,970,726	79,603,976	92,345,849
48	62,546,437	70,111,967	79,787,381	92,593,126
49	62,658,971	70,253,786	79,971,439	92,841,739
50	62,771,918	70,396,188	80,156,456	93,091,699
51	62,885,274	70,539,174	80,342,336	93,342,963
52	62,999,049	70,682,751	80,529,087	93,595,620
53	63,113,241	70,826,919	80,716,713	93,849,647
54	63,227,855	70,971,684	80,905,219	94,105,066
55	63,342,890	71,117,047	81,094,612	94,361,964
56	63,458,352	71,263,014	81,284,899	94,620,181
57	63,574,240	71,409,586	81,476,087	94,879,901
58	63,690,559	71,556,760	81,668,183	95,141,050
59	63,807,309	71,704,564	81,861,195	95,403,639
60	63,924,495	71,852,975	82,055,127	95,667,689

	84	85	86
0	95,667,689	114,737,188	143,355,808
1	95,933,204	115,119,970	143,954,694
2	96,200,195	115,505,313	144,558,602
3	96,468,673	115,893,242	145,167,595
4	96,738,655	116,283,797	145,781,740
5	97,010,253	116,676,991	146,401,101
6	97,283,267	117,072,851	147,025,745
7	97,557,932	117,471,403	147,655,740
8	97,834,057	117,872,815	148,291,169
9	98,111,843	118,276,840	148,932,108
10	98,391,211	118,683,794	149,578,791
11	98,672,171	119,093,414	150,230,942
12	98,954,738	119,506,013	150,888,966
13	99,238,930	119,921,335	151,552,578
14	99,524,766	120,339,695	152,222,283
15	99,812,250	120,760,985	152,897,946
16	100,101,400	121,185,232	153,579,394
17	100,392,329	121,612,482	154,267,179
18	100,684,851	122,042,752	154,961,155
19	100,979,193	122,476,076	155,661,396
20	101,275,259	122,912,485	156,368,008
21	101,572,962	123,352,014	157,081,063
22	101,872,522	123,794,696	157,800,648
23	102,173,854	124,240,732	158,526,854
24	102,476,971	124,689,836	159,259,771
25	102,781,890	125,142,353	159,999,560
26	103,088,639	125,598,007	160,746,121
27	103,397,202	126,057,149	161,499,724
28	103,707,656	126,519,656	162,260,744
29	104,019,959	126,985,568	163,028,671
30	104,334,254	127,454,936	163,804,188

	84	85	86
30	104,334,254	127,454,936	163,804,188
31	104,650,345	127,927,785	164,586,836
32	104,968,474	128,404,152	165,377,268
33	105,288,542	128,884,078	166,175,067
34	105,610,566	129,367,604	166,980,877
35	105,934,564	129,854,921	167,794,536
36	106,260,557	130,345,812	168,615,879
37	106,588,558	130,840,395	169,445,585
38	106,918,589	131,338,917	170,283,495
39	107,250,680	131,841,076	171,129,820
40	107,584,955	132,347,264	171,984,431
41	107,921,201	132,857,174	172,847,712
32	108,259,554	133,371,390	173,719,700
43	108,600,151	133,889,600	174,600,528
44	108,942,779	134,411,312	175,490,331
45	109,287,702	134,937,471	176,389,247
46	109,634,817	135,467,749	177,297,417
47	109,984,143	136,002,235	178,215,000
48	110,335,695	136,540,955	179,142,131
49	110,689,503	137,083,887	180,078,954
50	111,045,597	137,631,223	181,025,951
51	111,403,988	138,183,016	181,982,628
52	111,764,699	138,739,177	182,949,802
53	112,127,750	139,299,830	183,926,988
54	112,493,167	139,865,032	184,915,009
55	112,861,097	140,435,034	185,913,698
56	113,231,316	141,009,514	186,922,883
57	113,604,036	141,588,910	187,943,432
58	113,979,204	142,172,885	188,975,184
59	114,356,941	142,761,897	190,018,342
60	114,737,188	143,355,808	191,073,059

	87	88	89
0	191,073,059	286,537,048	572,987,098
1	192,139,567	288,943,841	582,696,234
2	193,218,044	291,391,404	592,740,072
3	194,308,693	293,880,683	603,139,919
4	195,411,723	296,413,087	613,907,444
5	196,527,729	298,990,299	625,070,305
6	197,656,182	301,611,807	636,642,580
7	198,797,665	304,279,687	648,655,621
8	199,952,408	306,996,123	661,126,359
9	201,120,639	309,760,533	674,090,521
10	202,303,011	312,576,192	687,573,461
11	203,498,943	315,442,491	701,612,741
12	204,709,121	318,361,849	716,229,489
13	205,934,200	321,336,774	731,453,951
14	207,173,596	324,366,765	747,356,168
15	208,428,431	327,455,509	763,965,262
16	209,698,119	330,602,545	781,323,254
17	210,983,811	333,811,800	799,494,739
18	212,284,914	337,082,830	818,524,878
19	213,602,421	340,419,652	838,490,069
20	214,936,837	343,823,403	859,453,551
21	216,287,319	347,294,586	881,484,374
22	217,655,350	350,837,799	904,682,629
23	219,040,792	354,454,051	929,134,899
24	220,443,981	358,145,679	954,945,691
25	221,865,261	361,914,968	982,231,457
26	223,305,005	365,763,113	1,011,112,129
27	224,763,453	369,695,332	1,041,753,449
28	226,241,278	373,713,015	1,074,309,940
29	227,738,558	377,818,975	1,108,967,170
30	229,255,785	382,016,194	1,145,934,768

	87	88	89
30	229,255,785	382,016,194	1,145,934,768
31	230,793,360	386,307,709	1,185,438,054
32	232,351,718	390,696,734	1,227,777,193
33	233,931,261	395,186,630	1,273,252,703
34	235,532,422	399,780,916	1,322,226,495
35	237,156,211	404,483,275	1,375,118,522
36	238,801,972	409,297,566	1,432,397,932
37	240,470,730	414,227,875	1,494,678,912
38	242,163,582	419,278,406	1,562,622,042
39	243,879,838	424,453,607	1,637,036,239
40	245,621,193	429,758,156	1,718,892,212
41	247,386,980	435,196,961	1,809,365,043
42	249,178,956	440,775,230	1,909,891,150
43	250,996,450	446,498,305	2,022,234,532
44	252,841,285	452,371,994	2,148,642,981
45	254,713,463	458,402,271	2,291,895,669
46	256,612,911	464,595,485	2,455,554,199
47	258,541,565	470,958,329	2,644,450,861
48	260,499,426	477,497,828	2,864,834,681
49	262,487,160	484,221,619	3,125,282,743
50	264,505,458	491,139,838	3,437,843,546
51	266,554,348	498,256,113	3,819,709,423
52	268,635,944	505,581,634	4,297,193,536
53	270,750,304	513,128,395	4,911,255,640
54	272,898,206	520,901,152	5,729,642,566
55	275,080,457	528,915,798	6,875,687,278
56	277,297,985	537,178,089	8,594,018,365
57	279,551,349	545,702,599	11,458,691,197
58	281,841,763	554,505,091	17,188,036,597
59	284,170,013	563,593,031	34,376,072,269
60	286,537,048	572,987,098	Infinitum.

TH. FINKII GEOMETRIAE ROTVNDI,

LIBER DECIMVS.

De calculo triangulorum.

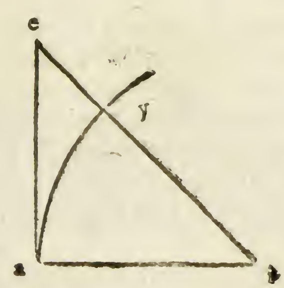
GEometria circularis canonem triangulorum adhuc nobis communicavit: sed jam etiam usum ejus in calculo triangulorum docebit: & quidem planorum hoc in loco. De sphæricis enim inferius erit suo loco.

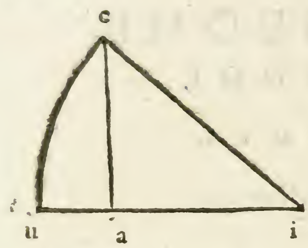
1. *Amplitudo anguli trianguli mensuratur arcu, circuli radii è dicto angulo in terminum cruris, angulum dictum subtendente.*

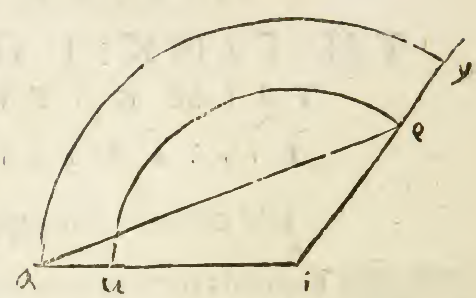
Id hic in calculo triangulorum sic sumimus. Est enim arcus ita descriptus basis anguli sectoris in centro: Et ut basis ea ad integram peripheriam sic angulus sectoris ad 4 rectos per 5.e.3.& compositionem proportionum. Sed peripheria & 4 recti æquatur sive eadem mensura mensurantur per 2.c.8.e.5.R. Ergo angulus & ei subtensus arcus eandem habebunt mensuram.

Sic in triangulis a e i. Si centro i radio i a describatur circulus: tum arcus a y subtensus scilicet angulo a i e, eundem metietur.

Eodem modo si radio i e describatur arcus e u. is angulum dictum mensurabit. Sunt enim arcus y a & e u similes.



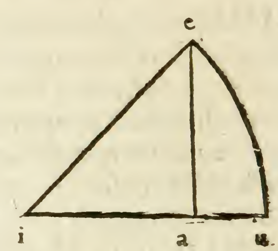




unde & hoc conſequitur arcum, circuli ex angulo tanquam centro deſcripti, quemlibet, modo baſis fiat anguli ſectoris in centro, eſſe menſuram anguli oppoſiti. At nos radium definivimus ob inſequentem triangulorum calculum.

Itaq;

2. *Si baſis recti fiat radius circuli: crura erunt ſinus ſibi oppoſitorum angulorum.* 20.p.1.Reg.triang.



Sit enim triangulum rectangulum a e i & baſis e i fiat radius circuli u e.

Dico e a eſſe ſinum anguli oppoſiti e i a. Nam cum e a ex theſi ſit perpendicularis è termino arcus e in diametrum i a ductam per terminum u. erit e a ſinus arcus e u hoc eſt ex i e anguli e i a.

Eodem argumento ſi centro e deſcribas arcum patebit i a eſſe ſinum anguli a e i. Vel hoc modo probatur. Angulum a metitur peripheriæ totius pars quarta:& complementum u e hoc eſt anguli e i a eſt reſiduum ad quadrantem, hoc eſt reliquum anguli i de recto nempe angulus a e i per 9.e.6.R. jam vero a i eſt ſinus complementi arcus u e hoc eſt complem. anguli i. id eſt anguli a e i. crus ſcilicet anguli oppoſiti.

Et

3. *Alterutrum crurum recti eſt ſinus complementi anguli ſui & baſis.*

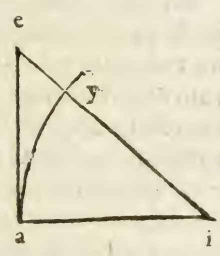
Id enim jam eſt demonſtratum ut e a eſt ſinus complementi anguli a e i nempe anguli e i a. Eſt enim angulus i. compl. anguli e ut patuit.

Hinc jam datis angulis trianguli rectanguli, laterum ratio dabitur. Qua de re eſt 30.p.1.Regiom.de triangulis.ut ſi detur angulus e.gr.30.erit angulus a i e gr. 60. Horum angulorum ſinus ex canone jam dantur:nẽpe a i 5,000,000.e a vero 8,660,254. in ea videlicet menſura in qua radius e i aſſumitur 10,000,000.Quare ratio e i ad a i eſt ratio,10,000,000 ad 5,000,000. ad e a vero ut 10,000,000 ad 8,660,254. & crurum inter ſe a i ad a e ut 5,000,000 ad 8,660,254. Copernicus hanc ad rem,ut & in reliquo hoc calculo diametro & inſcriptis integris utitur 13.c.1.

Et

4. Si crus alterum recti fiat radius ex angulo obliquo:reliquum eſt tangens anguli dicti,baſis vero ejuſdem ſecans.

Ea de re eſt apud Rheinholdum 8 præc. tabul. direct.& apud Rheticum in dialogo de canone triangulorum. Eſto rurſus triangulum a e i. & centro i radio i a deſcribatur circulus a y. dico a e nempe crus recti reliquum eſſe tangentẽ anguli oppoſiti nempe i ex quo circulus deſcriptus eſt : & baſin e i eſſe ejuſdem anguli ſecantem. Nam ut ex definitionibus tangentis & ſecantis patet a e & e i ſunt arcus a y hoc eſt per i e anguli e i a tangens & ſecans.



Hinc jam rurſus datis angulis dabitur ratio laterum:ſi crus alterutrum ſit radius.Sit enim angulus a e i gr.30. erit angulus e i a gr.60.Hujus tangens a e datur & ſecans ex ſuo qualibet canone illic 17,320,508 hic 20,000,000. Quare ratio a i ad a e eſt ut 10,000,000 ad 17,320,508 ad e i ut 10,000,000 ad 20,000,000.

itaque ſiue baſis recti ſumatur pro radio ſive crus recti : datis angulis laterum dabitur ratio in aſſumpta menſura. Quod ſi vero & latus dati trianguli unum in aliqua menſura detur : per re-

gulam auream & assumptam mensuram, in data mensura latera innotescent. Atq; is calculus triangulorum planorum vulgo dicitur. Qui ut magis reddatur perspicuus: sequentia consectaria è generali hac, per regulam auream & assumptam mensuram, reductione proponam. Notabis autem deinceps radium, sinum, tangentem & secantem anguli esse mensuræ assumptæ: crus vero latus & basin datæ. Et in rectangulus quidem basin & crus esse anguli recti.

Ac in triangulo rectangulo.

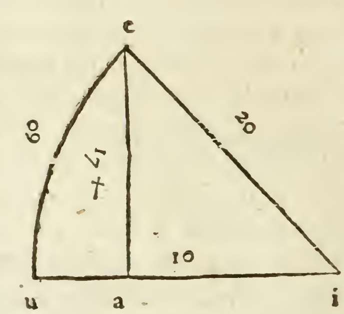
Et

5 *Radius est ad sinum anguli, ut basis ad crus dicto angulo oppositum.*

Per hoc elementum & data basi cum angulis, crurum erit inventio: & dato crure cum basi magnitudine aut ratione angulorum inquisitio. Hac de re est 29, 27, 28. p. 1. Regiom. de triangulis.

Primò itaq; data basi cum angulis crura inveniuntur.

Sit data trianguli a e i basis e i partium 20. & angulus i e a. gr. 30. & proinde angulus e i a grad. 60. tanquam prioris complementum ut patuit. jam quia e a & a i sunt sinus oppositorum angulorum: dabūtur ex Canone sinuum e a quidem 8,660,254. at a i 5,000,000. in ea mensura in qua radius e i est 10,000,000. Ergo per regulam auream si cum e i est 10,000,000. tū a i est 5,000,000. cum e i datur 20 tum a i erit 10.



e i	e i	a i	a i
10,000,000	20	5,000,000	10.

Rursus cum e i est 10,000,000. tum e a est 8,660,254. Ergo cum e i est 20 tum e a erit 17 & paulo ultra: nempe si ad sexagenariam analogiam reduxeris 19'.14" ferè.

e i

e i	e i	e a	e a
10,00,0000	20	8,660,254	

Factus per ſecundum & tertium in primum diviſus quotum invenit 17 & remanet 3,205,080. Quæ ſi per doctrinam reductionis Arithmeticæ per 60 multiplices & in radium dividas continuè: invenies minuta 19 & 14 ſecunda ferè. ut videre hic licet.

1,000,000.	60.	320,508	
		60	
		19|230,480	prima
		60	
		13|828,800	ſecunda
		60	
		49|728,000	tertia

Et ſic deinceps ad ſcrupula poſſes continuare ſexta antequam reliquum nullum invenires. Verum ſufficit huc uſque eſſe progreſſum: imo ad prima tranſiſſe ſatis eſt: abjectis ſecundis quæ 30 minora ſunt: alias ſi majora ſint pro iis aſſumitur 1 primum cum voce ferè.

Secundò data baſi cum crure anguli invenientur.

Detur in eadem figura e i baſis 20. at crus recti a i 10. ſive hæ horum laterum magnitudines ſint ſive laterum rationis termini perinde eſt: nec alia erit calculi ratio. His igitur datis quæruntur anguli. Cum itaq; a i per 2.e. ſit ſinus anguli a e i: is vero detur in ea menſura in qua datur baſis ei 20. Erit ergo ut 20 ad 10. ſic 10,000,000 radius aſſumptus ad 5,000,000 ſinum anguli ad e in menſura aſſumpta

e i	e i	a i	a i
2	10,000,000	1	5,000,000.

Sinus igitur iſte in canone ſinuum in ſua parte nempe interiore quæſitus in marginibus ſupero quidem gr. 30 ſiniſtro mi-

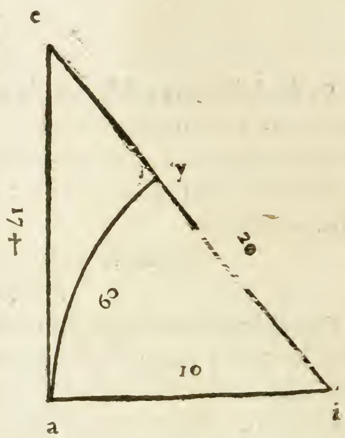
nutum nullum exhibet. Quare angulus a e i erit gr.30. Sed & hinc angulus ad i dabitur nempe gr. 60. tanquam prioris cōplementum, aut recti & gr.30 de duobus rectis reliquus per 9.e.6.R. Et

6. Radius est ad tangentem anguli, ut Crus recti & dicti ad crus reliquum.

Hoc elementum docebit: dato angulo cum crure, reliqui cruris inventionem: & datis cruribus, angulorum investigationem monstrabit. Ea de re est apud Rheticū in dialogo suo: & Rheinholdum præc.9.tab.direct.

Primò ergo dato crure cum angulo reliquum crus invenitur.

Sit igitur trianguli rectanguli e a i datum crus a i 10. & angulus a i e grad.60. Quæritur crus reliquum a e. in ea mensura in qua a i datur.



Fiat itaque i a radius: erit per 4.e a e tangens anguli e i a.

Quare ex canone tangentium dabitur a e 17,320,508 in mensura assumpta. Ergo si cum a i est 10,000,000. tū a e est 17,320,508. cum a i datur 10 erit a e 17. & paulo plus videlicet 19′,14″ plane ut ante.

a i	a i	a e	a e
1,000,000.	1.	17,320,508	

Et sic dato crure ac angulo, crus reliquum invenimus.

Secundò datis cruribus anguli, in assumpta mensura aut in qua determinatur rectus, dabuntur.

Dentur enim in præmisso schemate crura a i quidē 2,500,000. at a e 4,330,127.

Ex his cruribus anguli dati trianguli obliqui exquiruntur, fiat enim a i radius: erit a e tangens anguli i quę cū detur in ea menſura in qua datur etiam i a radius: erit per regulam auream ut 2,500,000 ad 4,330,127 ſic 10,000,000 ad 17,320,508.

a i	a i	a e
25	100	4,330,127.

atq; ea eſt tangens anguli i in aſſumpta menſura. quæ quæſita in canone tangentium monſtrat gr. 60 pro angulo i. Quare angulus e erit gr. 30. in ea menſura quæ peripheriam ſtatuit definitam partibus 360. Quod ſi alia detur peripheriæ menſura ſimul cum cruribus: ex ſuperiori libro reduces angulos inventos ad datam menſuram. ut ſi detur anguli recti amplitudo 11. erit ut 90 ad 60 ſic 11 ad $7\frac{1}{3}$ pro angulo i. & ut 90 ad 30 ſic 11 ad $3\frac{2}{3}$ pro angulo e. Quod hic ſemel repetiviſſe ſat eſt.

Hujus vero calculi exempla multa deſumi poſſunt ex 2. lib. M.C. Ptolemæi: ubi de ratione umbrarum agit: multa etiam paſſim apud Geographos & Phyſicos alios leguntur. Nos ſumemus illud Plinii quod Rheinholdus præc. 49. tab. direct. habet.

Plinius itaq; lib. 2. c. 74. vel ut alii legunt 73. ſic de gnomone & umbra æquinoctiali loquitur: Meridiano tempore æquinoctii die in urbe Roma nona pars gnomonis deeſt umbrę. hoc eſt ſi in præmiſſa figura a e ſtatuas gnomonem partium 9. erit a i umbra talium partium 8. Hinc inquirendus eſt angulus a e i. angulus ſcilicet inclinationis axis Æquatoris ad Horizontem: qui vulgò altitudo poli dicitur: à quo ſi nō ratione ſubjecto tamen differt. Si itaq; fiat a e radius: erit ut 9 ad 8 ſic 10,000,000 ad 8,888,889 ferè.

a e	a e	a i	a i
9.	10,000,000	8.	8,888,889

Atq; hæc a i eſt tangens anguli a e i menſuræ aſſumptæ: quare ex canone tangentium dabitur angulus ipſe e gr. 41,38′.

Simili modo erunt tractanda reliqua quæ dicto loco Plinius habet: & quæ apud Ptolemæum extant.

Tale eſt illud quod Vitruvius Alexandriæ umbram meridianam dicit æquinoctii die eſſe 3, quarum gnomon habeat 5. hoc

eſt

est esse a e 5, a i 3. Ex quib. angulus a e i invenietur gr. 30. 58′ ferè.

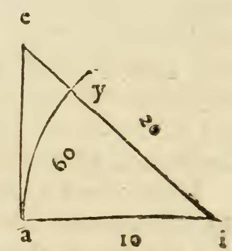
Et

7. *Radius est ad secantem anguli, ut Crus adjacens ad basin.*

Hoc elemento: Dato crure cum angulo invenitur basis: Et rursus data basi cum crure exquiruntur anguli.

Primò: Dato crure cum angulo invenitur basis.

Sit trianguli rectanguli e a i. datum crus a i 10. cum angulo e i a grad. 60. Quæritur hinc basis e i. fiat itaq; crus datum radius circuli a y. constat jam e i esse secantem anguli e i a. Ergo cum angulus i detur gr. 60. ipsa secans ex canone secantium dabitur 20,000,000. in ea mensura in qua radius a i est 10,000,000. Ergo in qua a i est 10 in ea e i erit 20.



a i	a i	e i	e i
1.	10.	2.	20.

Eadem res erit si detur crus reliquum e a. Nam cum ut radius ad secantem anguli e sic e a ad e i.

Secundò: Data basi cum crure anguli exquiruntur.

Detur enim basis e i 20 crus a i 10. & crus a i fiat radius, erit ergo e i secans anguli e i a. jam ut 10 ad 20 sic 10,000,000. ad 20,000,000. pro secante anguli i. Quare secantium canon monstrabit angulum i. grad. 60. & proinde reliquus erit gr. 30. nempe angulus ad e. itaque & per sinus & secantes data basi cum crure anguli exquiruntur: ut patuit supra 5 e, & hic apparet.

Atq; sic calculus triangulorum rectangulorum fuit: qui è datis duobus præter angulum rectum aut lateribus aut latere & angulo reliqua invenire docuit. Nam datis angulis tantum ratio laterum datur ut ad 3 & 4 e patuit. Sequitur hinc postulatum: tum ipse obliquangulorum calculus.

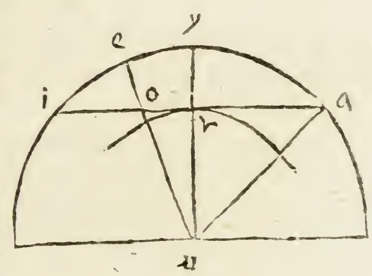
Et

Et licet

8. Data ſumma aut differentia duarum peripheriarum vel angulorum & ratione ſinuum, ſigillatim utrumq; ſecernere.

Sunt de hoc poſtulato ſecundum & quartum Theonis lemma cyclicum 11. c. 1. & 21,22,23,24.p.4. Regiom. de triangulis. Quorum inventionis rationem illic perlegere potes. Nobis hæc erit propoſita methodus.

1. ut ſemiſsis ſummæ terminorum rationis ſinuum ad differentiam ſemiſsis ac termini alterius, ſic tangens ſemiſsis ſummæ datæ ad tangentem peripheriæ peripheriarum qua minor quæſitarum ſemiſſe ſummæ minor, aut major, major eſt.



Eſto in ſequenti ſchemate data ſumma arcuum i e, e a gr. 40. Sitq; data ratio ſinuum harum peripheriarũ, hoc eſt ratio a o ad i o per 13.e.5. ut 7 ad 4. Quæruntur jam ſingulæ peripheriæ a e, e i. Ducantur radii u a in terminum inſcriptæ. u y biſecans ut inſcriptam ſic ſummam peripheriarum erit ergo angulus r u a grad. 20. cujus tangens eſt a r. illa datur ex canone 3,639,702. ex theſi $5\frac{1}{2}$. Nam i a eſt 11. ergo r a eſt $5\frac{1}{2}$. jam ergo ut 3,639,702 ad 10,000,000. ſic $5\frac{1}{2}$ ad 15 & paulo ultra. nempe remanent 808,940. jam r o eſt tangens anguli o u r: eaq; datur in menſura ea in qua u r radius eſt paulo ultra 15. datur inquam nempe $1\frac{1}{2}$ differentia a r $5\frac{1}{2}$ & a o 7. vel differentia i r $5\frac{1}{2}$ & i o 4. Quare ex 6 e rurſus ut u r datæ menſuræ ad radium, ſic o r datæ menſuræ ad tangentem anguli u, hoc eſt arcus e y. Hinc jam ſequitur calculus ſeu calculi ratio quam ante propoſuimus. Nam ut a r in data menſura $5\frac{1}{2}$ ad 3,639,702 in aſſumpta, tangentem

ſcilicet anguli r u a. ſic 15 +— nẽpe u r in data mẽſura ad 10,000,000.

Et ut 15 +— ad 10,000,000. ſic o r datæ menſuræ $1\frac{1}{2}$ ad 992,646 tangentem anguli o u r. Ex æquatione itaq; rationum ut ſemiſſis terminorum rationis ſinuum ſummæ ad differentiam ſemiſſis alteriuſq; termini, ſic tangens ſemiſsis ſummæ datæ ad tangentem peripheriæ proſthaphæreſeos. ut hic

a r	a r	o r	o r
$5\frac{1}{2}$	3,639,702	$1\frac{1}{2}$	992,646.

Tangens itaq; peripheriæ e y aut ſimilis eſt 992,646. cui competit ex canone tangentium gr. 5,40′ proſthaphæreſis nempe: quæ ablata ſemiſsi i y relinquit 14, 20′ addita ſemiſsi a y cõponit 25,40′. & ita peripheriarũ quæſitarũ major eſt 25,40′ minor 14,20′.

Detur jam peripheriarum differentia cum ratione ſinuum.

Erit itaq;

2. *ut ſemiſsis differentiæ terminorum rationis ad terminum minorem, ſic tangens ſemiſsis differentiæ peripheriarum datæ ad tangentem, peripheriæ minoris, ſemiſſe differentiæ auctæ, minorem tangente ſemiſsis differentiæ peripheriarum.*

Eſto in ſubjecta figura differentia peripheriarum o r, e r data gr. 40. Et ſit ratio data ſinuum o r, ad e r ut 20 ad 13.

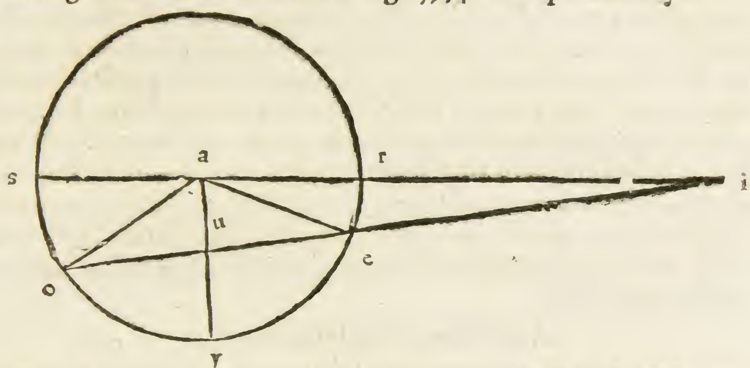
jam per puncta o & e inſcripta recta continuetur donec concurrat cum diametro in i. erit per 14. e. 5. ratio o i ad i e ut 20 ad 13. erit ergo o e 7. biſecetur o e erit u e $3\frac{1}{2}$. ſed & a y biſecat arcum o e. erit ergo y e gr. 20. ejusque aut ſimilis radio a u deſcripti tangens u e. 3,639,702. in ea menſura in qua a u eſt 10,000,000. Ergo per antecedentes rationes ut u e in data menſura $3\frac{1}{2}$ ad u e 3,639,702 ſic e i datę menſurę 13 ad 13,518,893. aſſumptę menſuræ.

u e	u e	e i	e i
$3\frac{1}{2}$	3,639,702	13	13,518,893.

jam ſi u e & e i componas habebis tangentem anguli u a r, hoc eſt arcus y r.

13,518,893
3,639,702
17,158,595

Tangens hæc arcum monstrat gr.59,46 nempe arcum y r.



Peripheria hæc est minor quæsitarum aucta semisse differentiæ. Ergo inde ablata semissis differentiæ grad.20. relinquit e r quæsitam minorem 39,46.

y r	59,	46.
y e	20	
r e	79,	46.

Quod si cũ semisse addas: totus erit arcus quæsitus major 76,46.

y r	59,	46.
y o	20	
r o	79,	46.

Quod si sinus major sit peripheriæ minoris ut si sit data ratio sinuum peripheriarum s o, 5 e ut 20ad 13. Hic majori sinui minor competit arcus; manebit eadem operatio &, invenietur r e ut ante 39,46′. Hujus autem reliquum cum ad semiperipheriam nempe 140,14′ est arcus major quæsitus. Et ex majore sublata differentia e o relinquit 100,14′ pro s o arcu minori. Sed hæc non sunt difficilia ei qui supra didicit sinum eundem esse peripheriarum duarum.

Sequetur deinceps calculus obliquangulorum: ubi non repe-

temus ea quorum noticia ante innotuit:ut exempli gratia: datis duobus angulis tertius invenitur 25.p.1. Reg. id docuit 9. e.6.R. nempe si tres sunt 2 recti & trium duo dantur: hi de duobus rectis relinquent: sic nec repetemus inventionem segmentorum basis sectæ perpendiculari: quæ est 43,44,45.p.1.R.Est enim supra dicta ad 11.e.2.Nec hic multum de datis declamabimus quot ea esse debeant:cum alias plura alias pauciora ad quęsitum deducere posśint pro ratione æqualitatis aut inæqualitatis laterum oblati trianguli.Nec est hic alia ratio quam in rectangulo : in rectangulo dantur tria angulus rectus & præter hunc alia duo : in obliquangulo dabuntur totidem aut latera : aut latera duo cum angulo comprehenso aut opposito : aut anguli cum latere adjacente aut opposito.prout hæc varietas ex sequentibus consectariis apparebit.

Ac in triangulo obliquangulo.

9. Crus alterum est ad segmentum basis in perpendicularem conterminum, ut radius ad sinum complementi anguli cruris & segmenti.

Hoc elementum docebit:datis tribus lateribus trianguli angulos invenire. de qua re est 47.54.p.1.Regiom.& 9.p.2.

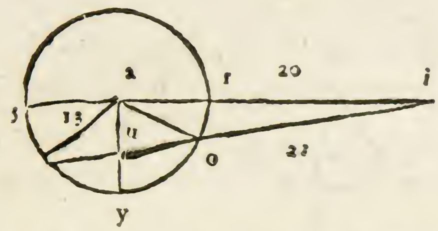
Esto triangulum a e i ab ejusq; angulo a. perpendicularis a u secet basin. Cadet autem intra triangulum aut extra prout ad 11.e.2.est monstratum. Verum sive intra sive extra cadat calculus perinde procedet.

Detur autem in primo triangulo.

crus	a i	20.
crus	a e	13
basis	e i	21.

Hinc 11.e.2. eliciet segmentum u e 5. u i vero 16. & simul concludet perpendicularem cecidisse intra triangulum.

Quia igitur per perpendicularem facta sunt duo triangula rectangula e u a, i u a. erit per 5 e.



ut e a ad e u sic radius ad ang. e a u
13 5 10,000,000 3,846,154

Sinus inventus in canone sinuum exhibet gr. 22,37′ & parum ultra. Quare angulus e a u erit gr. 22,37′. proindeq; ejus complementum a e u gr. 67,23′.

Rursus cum i u a triangulum sit rectangulum erit ut i a ad i u sic rad. ad sin. ang. u a i
20 16 10,000,000. 8,000,000.

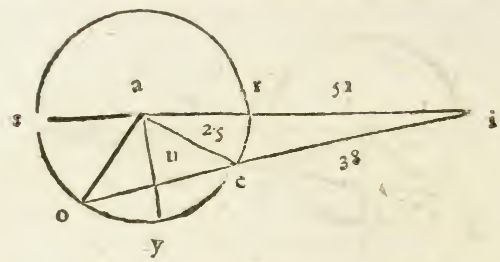
Sinus hic in canone sinuum quæsitus monstrat arcum grad. 53,7′,48″ pro angulo u a i. Ergo angulus a i u erit gr. 36,52′,12″. & sic habes trianguli a e i duos angulos: quos si conjunctim de 180 nempe duobus rectis auferas per 9. e. 6. R. relinquetur tibi angulus e a i gr. 75,45′.

e.	67,	22,	49″	180	
i.	36	52	12	104	15′
	104	15′		75	45

Atq; sic inquires angulos datorum laterum: si perpendicularis cadat intra triangulum: quod quando fiat in inventione segmenti u e docet 11. e. 2.

Sit jam datum crus	a i	51
crus	a e	25
basis	e i	38

Ex his datis colligetur u e 7, & i u 45 per 11. e. 2. unde & illud constabit perpendicularem cadere extra.



jam rursus quia fiunt duo triangula a u e, a u i rectangula: erit per 5 e.

ut a e ad e u sic radius ad sinum ang. u a e.

25 7 10,000,000. 2,800,000.

Hinc ex canone sinuum dabitur angulus u a e gr. 16,15′,37″. & proinde angulus u e a tanquam prioris complementum grad. 73,44′,23″ qui cum sit extra triangulum: angulus a u i per 1. c. 8.e. 5.R. dabitur gr. 106,15′,37″.

180
73, 44, 23″
106, 15, 37.

Rursus per 5 e in triang. rect. a u i.

ut a i ad i u sic radius ad sinum ang. u a i

51 45 10,000,000 8,823,529.

Hinc dabitur angulus a u i ex canone gr. 61.55′,39″, & proinde angulus a i e gr. 28.4′,21″.

jam igitur datis duobus angulis trianguli a e i dabitur angulus e a i gr. 45,40′.

e.	106, 15′, 37″	180
i.	28, 4, 21	134 20′
	134 20	45 40

Quod si oblatum triangulum æquicrurum sit: tum ut perpendiculari bisecatur basis: sic unicus angulus ad basin reliquos invenit. Nam inventus unus est etiam alter. Quare duplicatus de duobus rectis relinquit tertium. Sic si triangulum æquilaterum sit:

fit: sine hac inventione dantur anguli: nempe quilibet est bes recti per 3.c.10.e.6.R. Quare in assumpta mensura erit gr.60. Et simi lia compendia discipulum diligentem superioris Geometriæ latere non possunt.

Et

10. ut radius ad tangentem anguli cruris ac basis, sic segmentum basis, in perpendicularem, conterminum ad perpendicularem.

De hac perpendicularis inventione factæ sunt apud Regiom. propositiones 34,36,46, lib.1. de triangulis.

Consequitur ex 6 e. ut in præmissis triangulis: si in primo triangulo cupias inquirere perpendicularem a u: & datus sit angulus a e u grad. 67,23. & segmentum e u 5. si cogites e u esse radium erit per 6 e.

ut radius	ad tang. ang. e	sic u e	ad a u
10,000,000	24,003,779.	5	12

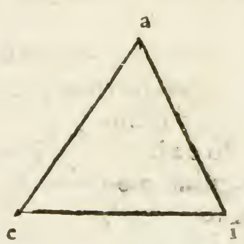
Itaq; perpendicularis erit 12. Eadem methodo alterius trianguli perpendicularem invenies.

Et

11. Latera sinibus oppositorum angulorum sunt proportionalia.

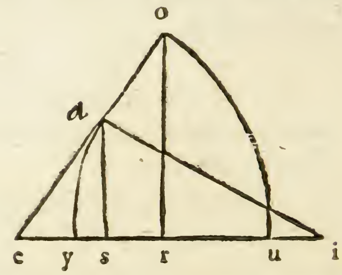
Hoc elementum sæpissimè & crebro ab Regiomontano & inculcatur & proponitur maxime 41,48. p. 1. & 1, 6. p. 2. de triangulis.

Esto triangulū obliquangulum (nam de rectangulo ad 3.e. fuit, a e i: & quidem varium sive lateribus inæqualibus. Nam si latera sint æqualia anguli oppositi quoq; æquantur per 10. e. 6.R. proindeq; sinus angulorum æqualium sunt æquales per 15 e 5. Sit itaq; varium dico esse a e ad sinum anguli a i e, sic a i ad sinum anguli a e i.



Conti-

Continuetur eñ crus minus e a in o, ut e o æquet̃ majori a i. jam centris e & i deſcribantur radiis æqualium crurum arcus o u, a y determinantes per i e angulorum amplitudines: & ſint arcuum ſinus a s, o r qui etiam ideo erunt ſinus angulorum a s quidem anguli i o r vero anguli e. Quia itaq; rectę e r. e o parallelis a s, o r per 12. e. 5. R. (ſunt enim anguli alterni recti) interſecantur: erit per 14. e. 5. R. ut e o hoc eſt a i ad o r ſinum anguli e oppoſiti: ſic e a ad a s ſinum anguli oppoſiti i.



Dentur enim in triangulo a e i.

	a e i	gr.	67,	22′,	49″
anguli	a i e	gr.	36,	52,	12
	e a i	gr.	75,	45.	

Eorumq; ex canone ſinus

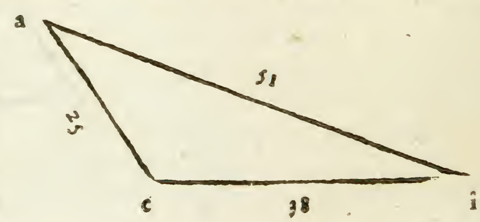
I.	9,230,780.
II.	6,000,016.
III.	9,692,309.

Ergo ratio a i ad a e eſt ut 9,230,780 ad 6,000,016. Et a i ad e i eſt ut 9,230,780 ad 9,692,309. Et e a ad e i ut 6,000,016 ad 9,692,309. Et

13. *Dati anguli data baſi ac crure anguli reliqui ac latus trianguli tertium invenitur.* *50. 51. p. 1. 5. p. 2. Regiom.*

Eſto primo in triangulo adjuncto e a i.

Datus angulus a e i obtuſus gr. 106, 15′, 37″. ejus baſis a i ſit 51. at crus a e 25. Quęrunt̃ reliqui anguli & crus e i.



per

per ii e,itaq; ut a i eſt ad ſinum anguli e,ſic a e eſt ad ſinũ anguli i. Datur autem a i,datur a e.canon etiam ſinuum ſuppeditat ſinum anguli gr.106,15,37″,hoc eſt per 8 e 5 ſinum gr.73,44′,23″ nempe 9,599,997. Ergo per regulam auream dabitur ſinus anguli a i e 4,705,881.

a i	a e	e	i
51	25	9,599,997	4,705,881

Quare ex canone ſinuum dabitur ipſe angulus i gr.28,4′ & pa rum ultra,nempe 20″.

jam datis his duobus angulis ex theſi & calculo : reliquus ad a erit gr.45,40′,3″.

e.	106, 15′ 37″		180
i.	28, 44 20		134, 20
	134, 19, 57		45, 40

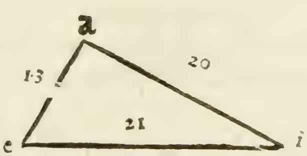
Quod ſi jam velis inquirere reliquum dati anguli crus:nempe e i.prædictum elementum inveniet 38. Nam ut ſinus anguli i ad a e ſic ſinus anguli a ad e i.

i.	a.	a e	e i
4,705,881.	7,152,964.	25.	38.

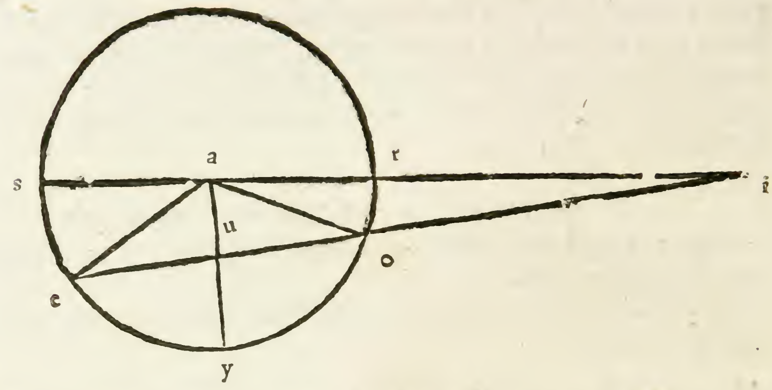
Exemplum aliud.

Sit in triangulo appoſito angulus e acutus grad.67, 22′, 49″: detur etiam a e 13. a i 20. Quæruntur reliqui anguli & latus e i.

Verum data non ſufficiunt niſi enim dederis acutus ne ſit aut obtuſus angulus à crure dati anguli ſubtenſus : ejus menſuram non definies.Nam ſi in hoc ſchemate ita ex elemento præmiſſo ratiocinatus fueris :



ut ſe habet a o ſeu a e(ſunt enim radii ejuſdẽ circuli) ad ſinum anguli i.ſic a i ad ſinum anguli oppoſiti. Verum dixiſti: at quem arcum competentem ſinui deſumes ? competentem ne angu-



lo a o i hoc est majorem peripheria quadrantis: an ne competen tem angulo a o e hoc est per 2. e.5. R. angulo a e o acuto. cum idem sinus his competat: latus etiam æquale & acuto & obtuso adjaceat. Quare cum triangulum his datis tam anceps sit & dubium: rectè Regiom. 51. p. 1. de triangulis monet datum angulum acutum cum crure & basi ad reliqua invenienda non sufficere. Quare angulum oppositum cruri dati anguli definiamus necesse est: sit'ne acutus an obtusus. itaq; ut ad triangulum datum descendamus: angulum pręter dicta data, a i e damus acutum. Hinc per pręmissas ratiocinationes invenietur angulus a i e gr. 36, 52', 12"

a i	a e	e	i
20.	13.	9,230,780.	6,000,006.

jam cum noti sint duo anguli e & i ille ex thesi hic ex calculo: erit angulus ad a. gr. 75, 45'.

jam pro latere e i inveniendo ratiocinatio præcedens valet & inveniet latus ipsum 21.

i	a	a e	e i
6,000,006.	9,692,309.	13.	21.

Et sic data basi datoq; crure dati anguli obtusi reliqua inveniuntur: data vero basi datoq; crure anguli acuti notaque specie anguli cruri dato oppositi reliqua inveniuntur. Sed plura nobis idem elementum suppeditat.

Et

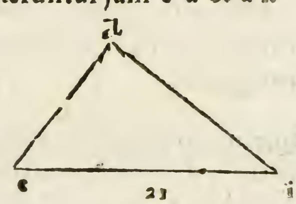
Et

14. Datis angulis cum uno trianguli latere, reliqua latera vestigantur.

Ea de re est apud Regiomontanum 52,53.p.1.ubi quæsita prout in antecedentibus fecit in deductis ad rectangula obliquangulis invenit. idem & alii præter Copernicum cum Ptolemæo fecerunt. Sed nos per plura quærere nolumus cum possimus per pauciora: rideretur sane qui secundum peripheriam vellet circumire cum posset secundum subtensam ingredi: nec perpendicularem aut segmenta lateris secta perpendiculari inquirere est necesse: non enim hæc quærenda proponuntur: Et si proponerentur: ex superioribus inventionis methodus esset. Et Regiom. 4.p.2.hanc methodum quæ hic erit retinet. Sed ad elementum. sint in triangulo a e i data.

Angulus	a e i	67,	22′,	49″
Angulus	a i e	36,	52,	12
Erit Angulus	i a e	75,	45.	

Datum sit etiam latus e i 21. Quæruntur jam e a & a i. inveniuntur, illud quidem 13 hoc vero 20. ut ex sequenti terminorum ex elemento proposito dispositione patet.



I.

a	i	e i	a e
9,692,309.	6,000,006.	21.	13.

II.

a	e	e i	a i
9,692,309.	9,230,780.	21.	20.

Vel

i	e	a e	a i
6,000,006.	9,230,780.	13.	20.

Quod ſi duo dentur latera & anguli: jam unica tantum inventione lateris eſt opus: quod nemo non intelligere poteſt. Nos ſumpſimus ea quæ dari minimum debent.

jam itaq; & datis tribus lateribus angulos invenimus: angulis datis laterum proportiones cognovimus: dati anguli baſi ac crure datis ad angulorum reliquorum ac tertii lateris cognitionem pervenimus: datis angulis cũ latere reliqua veſtigavimus. Supereſt jam adhuc alius caſus: nempe datis dati anguli cruribus angulos ac tertium latus eruamus. inventio hæc apud Regiomont. eſt 49.p.1. ubi perpendicularem in alterutrum crurum inquirit & ſegmenta cruris ſecti: Eſt etiam 2.p.2. at nimis laborioſa & tædioſa. Quare ex ſuperiori doctrina deducto calculo brevi & facili eoq; novo fruere.

Et

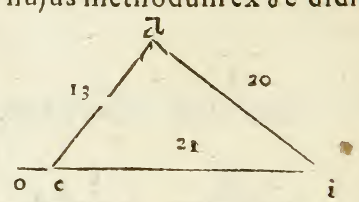
15. *ut ſemiſsis ſummæ crurum ad differentiam ſummæ ſemiſsis alteriusq; cruris, ſic tangens ſemiſsis anguli crurum exterioris ad tangentem anguli quo minor interiorum ſemiſſe dicti reliqui minor eſt, aut major, major.*

Eſto trianguli a e i datus angulus ad e. gr. 67,22′,49″.

Ejusq; crura a e 13.
e i 21.

Quæruntur reliqui anguli. Nam de latere inventis angulis ex ſuperioribus cõſtat. inventionis hujus methodum ex 8 e didici.

Primo latus i e continuetur in o. Erit angulus o e a. æqualis angulis ad a & i. per 1.c.9.e.6.R. At angulus o e a datur nempe reliquus ad duos rectos anguli a e i per 1.c.8.e.5.R. datur itaque inquam grad. 112.37′,11″. Quare ſumma angulorum a & i datur.



Secundo datur etiam ratio ſinuum angulorum a & i. Nam ut e i ad e a ſic ſinus anguli a ad ſinum anguli i per 11.e.

Itaq; data ſumma angulorum a & i. data etiam ratione ſinuum ſepara-

ſeparatorum angulorum:ſigillatim etiam dabuntur i & a.ea methodo quam ex 8 e jam propoſuimus.

Terminos itaq; aureæ regulæ colligamus. Ratio ſinuum anguli a ad ſinum anguli i eſt ut 21 ad 13.adde terminos: ſemiſsis 17. erit pro primo.jam aufert 17 de 21, aut de 17 aufer 13. differentia 4 erit pro ſecundo.

Summa angulorum datur 112,37′,11″.

Ergo ſemiſsis eſt 56,18′,35″. Hujus tangens ex canone datur 149,999.ne ſcrupuloſius omnia perſequamur.

Ex hiſce tribus aurea regula inveniet 35,294.

17. 149,999. 4. 35,294.

Tangentem ſcilicet anguli quo angulus i minor eſt quam ſemiſsis anguli a e o.

Dabitur itaq; ex canone ſinuum gr.19, 26′, 24″. in deſumendo autem duas primas dextras tangentium notas prætermitte ea conditione qua ſupra monui. Et in præmiſsis elementis obſervare potuiſſem : ſed volui plenos ſumere numeros ne turbarentur tyrones. Qui plenis uti ſcit : in uſu facilè pro ratione quoſdam præcidere poteſt.

Jam cum notus ſit dimidius angulus a e o, notus etiam ſit angulus quo dimidius angulum i ſuperat.ulterius angulus i non latebit:ablato igitur angulo gr.19,26′,24″ de angulo gr.56,18′,35″. reliquus erit angulus a i e gr.36,52′,11″.

Rurſus ſi addas proſthaphæreſin ad ſemiſſem anguli exterioris:ſumma eſt pro angulo e a i gr.75,45′. De magnitudine anguli ex 11.e.6.R.judicium fiet:cum dentur latera iis oppoſita.

56,	18′,	35″
19,	26,	24
36,	52,	11
75,	44,	59

O 3 Exemplum

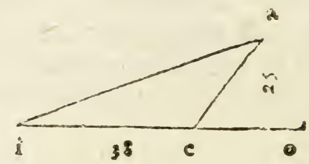
Exemplum aliud.

Detur angulus a e i gr. 106, 15′, 37″.

Crus a e 25
Crus e i 38.

Quæruntur anguli reliqui.

Hic rursus quia datur angulus a e i. dabit angulus a e o grad. 73, 44′, 23″. Summa angulorū a & i.



Sed & ratio sinuum anguli i ad a est ut a e 25 ad 38. Quare per 8 e dabitur ipse angulus i & a. illic grad. 28, 4′, 21″. hic grad 45, 40′.

Colligamus terminos aureæ regulæ.

Erit ergo primus $31\frac{1}{2}$.

a e	25
e i	38
	63

Secundus erit differentia $31\frac{1}{2}$ & 25 nempe $6\frac{1}{2}$.

Tertius erit tangens semissis anguli a e o nempe 74,999.

Hinc regula aurea invenit quartum 15,476.

$31\frac{1}{2}$ 74,999. $6\frac{1}{2}$ 15,476.

tangentem nempe anguli in ea mensura in qua radius concipitur 100,000. anguli, inquam, quo anguli a e o semissis major est angulo i. Datur autem hic ex canone grad. 8, 47′, 50″. Ergo de 36, 52′, 11″, relinquit angulum a i e grad. 28, 4′, 21″. At cum grad. 36, 52′, colligit grad. 45, 40′ pro angulo a. latus a i ex superioribus datur.

Atque

Atque ſic expoſitus nobis ſit inſignis ille & aureus triangulorum calculus: cujus uſus in Aſtronomia, Geodæſia imò artibus omnibus liberalibus & vita tota conſpicuus eſt. In eo itaq; ſedulo & ſerio, philomathes, te exerceas oportet ſi fructum hunc uberrimum conſequi volueris. Habes hic expoſita ea: quæ generalia videbantur & ſunt. in ſpecie enim de æquilateris aliquid, rurſus nonnihil de æquicruro proponere noluimus: propterea quod calculus communis ſit. Si vero quæ occurrunt in datis compendia prout ad 9 e monſtravi: ea non niſi à cæcis in Geometria non videbuntur. Omiſimus etiam ea in quibus tanquam ſpeciebus calculus hic generalis exerceri poſſet: quod eſſe id artis non videbatur. Regiomontanus aliquot caſus in ſecundo libro de triangulis collegit: quos illic videre potes. Cujus certe libri à ſtudioſis avidè legi debent: & cum fructu legi poſſunt. Habui id quidem in animo ut calculum Regiomontani itemque Ptolemæi & Copernici cum hoc præſenti conferrem: Verum ego hinc tyrones turbari exiſtimavi: nec ab horum artificum immortalitati conſecratorum monumentis abducere ſtudioſos volui. ubi hic perceptus fuerit: facile collatio inſtitui poterit. Nos breviſsimam, uti ſpero, viam ingreſsi ſumus: ſi deviavimus certè reducem æquo animo, & animo grato ferre poſſumus: & feremus.

Ne tamen nullus indicetur præſtantiſsimi hujus
calculi uſus: fructum Geometricum
qui hinc promanat ſequenti
libro exponemus.

TH. FIN.

TH. FINKII GEOMETRIAE ROTVNDI,

LIBER VNDECIMVS.

De Geodæsia rectarum.

CAlculus triangulorum planorum expositus facilem suppeditat rectarũ Geodæsiam per radium.

Hic fructus calculi Geometrici Geometricus est: per omnia magnitudinum genera diffusus. Lineas enim hæc Geodæsia metitur: ergo & superficies & corpora metietur: atq; hac arte areas triangulorum exquirit, sed & triangulata è triangulis composita sunt: quare metiendis triangulorum areis, metietur & areas triangulatorum. Sed & diametros circulorum tanquam rectas metitur, metietur itaq; & areas circulorum, omniumq; ejus segmentorum. Sic eadem hæc geodæsia bases & altitudines corporum metitur: metitur itaque & ipsas soliditates. usum hunc nobis potissimum præstat trianguli rectanguli calculus. Vulgò & à præstantibus mathematicis triangulum rectangulum magister matheseos appellatur. Et certè vires in Geometria præcipuas habet. Magisterium & hic verum deprehenditur in geodæsia rectarum.

Sed instrumento geodætico opus est. Eam ad rem alii quadratum Geometricum adsciscunt: nonnulli gnomonẽ seu simplicem normam in consilium vocant: aliis per umbras mensura placuit: aliis per speculum: quam utramque in opticis Euclides retinuit: alii quadrantem adhibent, & eum vel simplicem vel cum inscripto quadrato: alii alia arripuerunt instrumenta. Arrisit etiam nonnullis radius: qui, quasi à sancto illo patriarcha jam olim inventus sit, baculus jacobi vulgò dicitur. Nos ex iis duo selegimus radium & quadrantem simplicem: propterea ut collatione facta geodæsia & certior & facilior esset. Vulgò hæc duo instru-

inſtrumenta duobus quoque inſigniuntur nominibus. Nos eodem nomine Radium vocamus. Et certè ſi rem rectè perpendamus, inſtrumentum idem eſt. inſtrumentum ſane perantiquum & verè Geometricum: hoc eſt magnitudinibus metiendis cujuſcunq; materiæ aptum. Sic & metiendis terris uſurpatur, ut eſt apud Virgil. 3. Eclog.

———Ecquis fuit alter,
Deſcripſit Radio totum qui gentibus orbem?

Et aſtris loco & ordine definiendis, cunctisque ſuperlunaris iſtius civitatis regionibus ac viis deſcribendis eidem illi poëtæ aptum inſtrumentum viſum eſt.

———Cœliq; meatus,
Deſcribent Radio: & ſurgentia ſidera dicent.

Sed ſatis de uſu & fructu hujus geodæſiæ: ſatisq; de inſtrumenti nomine: ejus fabricam & uſum doceamus.

2 *Radius eſt norma crurum inæqualium: alterius recti metiendæ magnitudini reliqui paralleli.*

Quid, inquis, facis idem inſtrumentum baculum & quadrantem? an non baculus eſt ut ipſe definis norma crurum inæqualium? quid ſimile in quadrante? an non crura recti hic ſunt radii & proinde te ipſo teſte 2.e.1. æquales? quam cupis differentiam & diſſenſionem horum inſtrumentorum majorem? ubi peripheria quadrantis in baculo conſpicitur? Sed heus quid dicam: audi crurum definitiones: & ſitum eorum tam in quadrante quam baculo deprehendes. Fateor quidem actu ipſo exiſtere in baculo: non item in quadrante: ubi tamen concipi commodè & aptè videbis, & tam commode: quam in uſu baculi tu peripheriã quadrantis radio, ſegmento alterius cruris delineatam concipis: aut ſaltem concipere debes: ſi calculi veritatem commodè tueri vis & uſum inſtrumenti docere: etenim an non terminus cruris applicandus ad os genæ eſt? imò. Sed cur? quia ſcilicet viſus ex centro circuli exire debeat. An non hic circulus conceptus? cur non & in quadrante crura baculi mihi concipere liceat? At dices cur

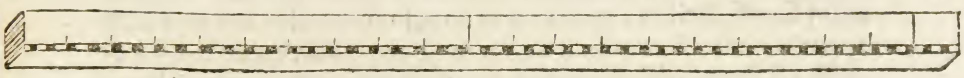
non concipere liceret? sed quemadmodum nec radius circulus dicitur, quia circulus illic concipiatur: sic nec quadrans baculus vocabitur aut radius quia in eo radius concipi posſit. Sed heus: si de nomine lis eſt, voces quadrantem, voces radium: & ego his nominibus uſitatis utor. Si de re litigas: audi quid quæram: an non illic circuli centrum concipitur ubi viſus exit? imo dices: neque enim negare potes: Ergo ſi viſus exeat à termino peripheriæ quadrantis, quod ſæpè fit, ubi jam eo centro deſcripta peripheria in quadrante apparet. At inquis angulus qui fit à reliquo latere quadrantis & perpendiculo è centro, angulus eſt quæſitus: bene ſe jam res habet: id eſt idem quod ajo. Si viſus exeat à termino peripheriæ illic crus eſt radii unum: alterum eſt ex centro perpendiculum: angulus à radio optico & crure è dicto centro eſt angulus quæſitus: hoc eſt angulus qui fit à latere reliquo & perpendiculo. Ergo ut tandem tibi ſatisfaciam: Quemadmodũ in quadrante ſi non pro centro uſurpetur quod actu centrum eſt, peripheria quadrantis, ex altero centro, actu non exiſtit: ſic radii hoc eſt quadrantis crura actu adeſſe ſemper non eſt neceſſum. Sed nimis forſan hac de re: crura doceantur.

3. Crura radii ſunt index & tranſverſarium. quorum terminis dioptræ affixæ ſunt.

Reperiuntur hodie multi: qui egregiè ſe & terrena menſos eſſe & cœleſtes deſcripſiſſe diſtantias putant adminiculo radii. interim tamen ſine omnibus dioptris viſum expaciari ſinunt: cum tamen tanta viſus ſit hallucinatio: ut vix etiam dioptris conſtringi poſſit. Eſt enim ut omnium ſenſuum celerrimus ſic maxime diſsipalis. nec enim tantum à centro oculi aut vertice coni optici (cum Euclide enim loquamur) radii excurrunt: ſed ut in Sole ab univerſo corpore. Quare conſultum arbitramur viſum dioptris conſtringi: ut in rem tantum metiendam deferri poſſit.
Sed ecce figuram radii.

4. *Index*

a



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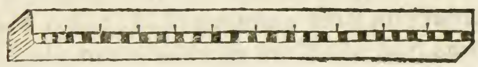

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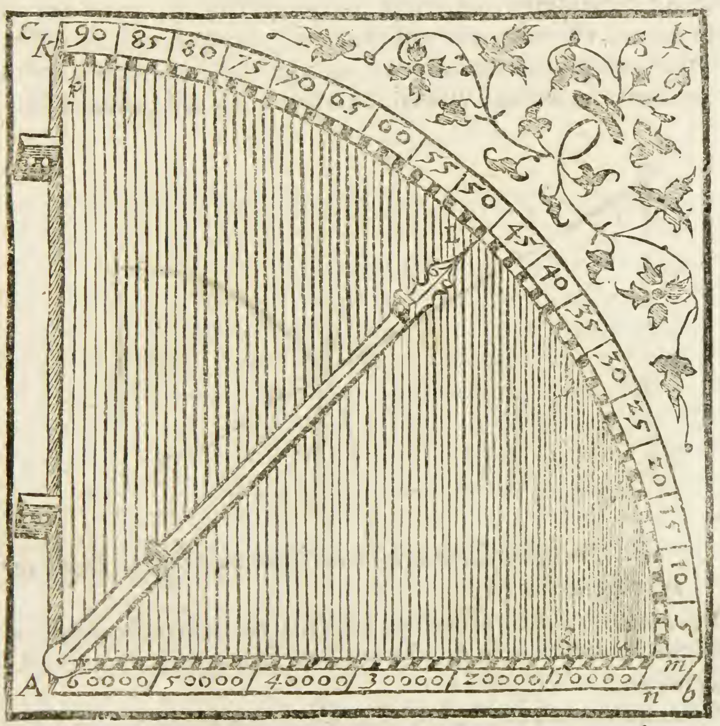
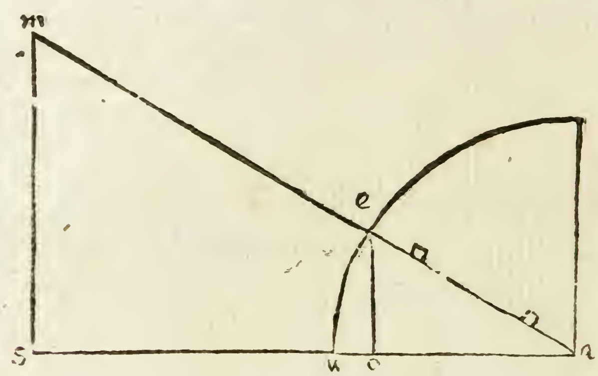

C
K
90 85 80 75 70 65 60 55 50 45 40 35 30 25 20 15 10 5
k
A
60000 50000 40000 30000 20000 10000
m
n
b

4. Index est radii crus majus, à cujus termino visus est.

Id sic sumimus ut semper à termino indicis visus sit: Videtur enim sic esse facilitas quædam. in baculo sive radio index facile apparet. nam actu datur ut a e, in quadrante indicem concipi necesse est. Et ut facilius concipias: post indicis definitionem hanc quadrantis fabricam accipe. E' centro, quod actu datur, amussis sive regula est. Ea dioptras habet aut non habet: si habet per quas visus incidat: jam basis quadrantis est index. Si non habet amussis dioptras, hoc est si filum ex centro cum plumbea sphærula dimittatur: erunt in latere quadrantis dioptræ: itaq; recta è termino peripheriæ, ubi visus incipit perpendicularis regulæ dependenti, est index: aut recta è centro perpendicularis rectæ è termino peripheriæ est index: si nempe visus à centro per dioptras lateris sit. ut hic si visus sit ex a per e. a s erit index radii.

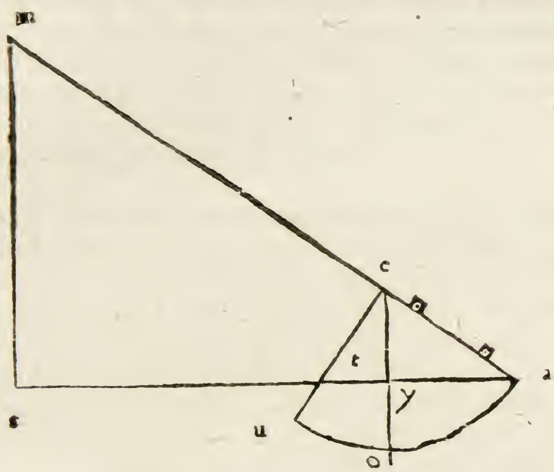


4. Transversarium est radii crus minus, per cujus terminum visus est.

Vocabulum hic transversarii est Vitruvii. in baculo transversarium actu datur i o. in quadrante definitio monstrabit transversarium in additis figuris e o.

Cæterum in usu radii utile est multa habere transversaria: par-

tim



tīm quę poſsint imponi ita indici ut ab ea biſecentur partim quę īmponantur ſine biſectione.

jam vero illud non eſt artis docere materiam inſtrumenti. parum enim intereſt modo ſolida ſit, firma, ſtabilis, lignea vel metallaris. Sed ut radium ad uſum melius accommodes: utrumq; ejus crus certis partibus definias neceſſe eſt. Sunt qui indici radium hypotheticum aſcribant: ſunt qui majorem ſunt qui minorem aſſumunt. Et id parum refert: modo ſecundum eandem menſuram & index & tranſverſaria particularia dividantur.

jam in uſu diſtantia juſta poſtulatur: nec enim viſus eſt infinitus. Vis optica requiritur in unum oculum diducta: cum unita fortius collimet. tranquillitas & immobilitas inſtrumenti neceſſaria eſt: manus itaq; quietæ ſint oportet: ſecus enim, ſi trepident, geodæſiæ turbabitur proportio. Sed & coincidentiam centri oculi cum centro circuli abſoluti uſus poſtulat. Secus enim radius opticus cum indice non angulum in centro, ſed ad aliud diametri punctum concludet. unde nova exiſtit geodæſię hallucinatio. Summopere itaq; hanc ἐκκεντρότητα & viſus παράλλαξιν

vitabimus. Quadrantis quidem usus ab hac est hallucinatione liber: quod in eadem recta è centro per dioptras visus feratur. At radius huic aberrationi obnoxius existit. Aberratio tamē corrigi poterit inventa ekkentrotete. Ejus inventionem accuratiorem, quam forsan Astrologia requirit, eò reservabimus: contenti ea quæ hic sufficit. illa igitur ex Thomæ Digesei alis mathematicis sic est.

Si visus à termino indicis per terminos transversariorū, quorum majus minoris duplum sit incidat in metam magnitudinis: differentia semissis indicis & segmenti in minus transversarium est dimidia ekkentrotes visus. Hac itaq; inventa potes numerationis in indice initium facere, aut ab ipso termino & visum secundum quantitatem eccentricitatis ab eo amovere: aut à termino in indice eam numerare: tumque ibi initium facere ubi ἐκκεντρότης finit. Verum si dioptris visus constrictus sit modica aberratio in hac quidem geodæsia merito non curatur.

5. *Geodæsia distantiæ est unius aut duplicis: Geodæsia unius distantiæ est ubi mensor in metienda magnitudine locum non mutat.*

Hic

6. *Si visus est in metam magnitudinis metiendæ: magnitudo erit tangens anguli indicis atq; radii optici: tumq; index magnitudini rectus est.*

Magnitudinem intelligo lineam aut magnitudinem lineatam in qua lineæ existunt. Verbi gratia si sit metienda latitudo fluvii alicujus: nos lineam consideramus quæ latitudinem mensurat. Quod si jam visus è termino indicis per terminum transversarii cadat in metam ac finem magnitudinis metiendę: linea quę metitur magnitudinem erit tangens anguli indicis atq; radii optici. atq; tum index est rectus magnitudini metiendæ: & proinde per 2. e. transversarium erit eidem parallelū. Et in hoc casu suaserim: ut tot sint in promptu transversaria particularia ut segmentum indicis semper radius hypotheticus sit, aliquot tamen circulis dextris minutus.

7. Sin

7. Sin visus est in metam magnitudinis ad metiendam rectæ: metienda est tangens anguli transversarii & radii optici: tumq; metiendæ transversarium est rectum.

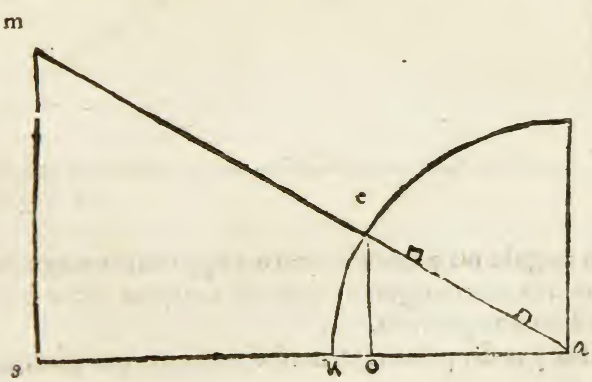
Et hinc rursus per 2. e. concludes indicem tum esse parallelum metiendæ magnitudini. Hic opera erit danda: ut transversarium habeat mensuram radii.

Cæterum quod ad perpendiculum & parallelismum attinet sæpe uti poteris plumbo suspenso in longitudine parallela horizonti: & norma etiam sæpè sufficiet: ut usus ipse docebit.

Jam vero antequam ad calculum hinc deductum descendam: anguli hujus inventionem docebo.

Est autem inventio in radio difficilior quam in quadrante: nisi quod supra monui in radio feceris: nempe ut aut segmentum indicis aut transversarium sit radius. Quod si est: jam transversarium illic exhibet tangentem anguli indicis atq; radii optici hic segmentum indicis tangentem anguli transversarii atque radii optici: & quidem in mensura assumpta. Ergo si usus postulet angulus dabitur. Et sic cognosces esse faciliorem usum radii quam quadrantis.

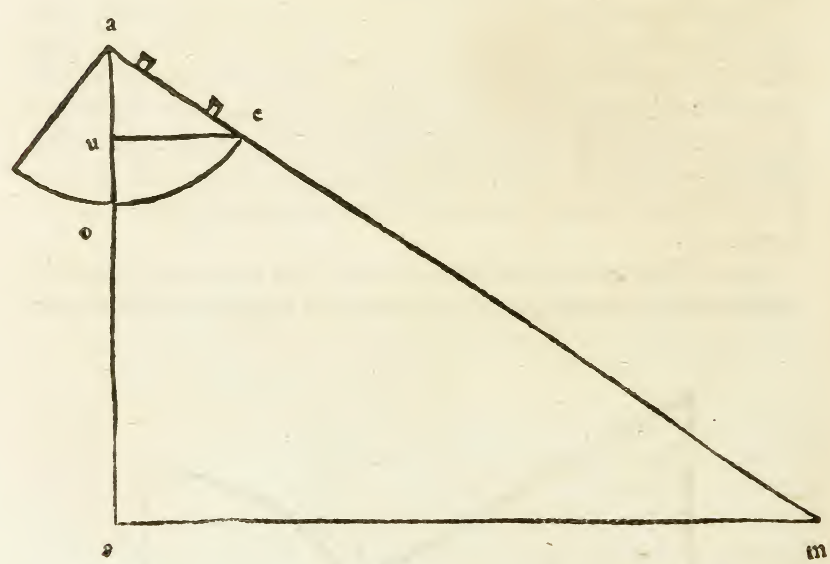
Quod si segmentum indicis non sit radius 6. e. 10. angulos inveniet vel eorum tangentes. in quadrante angulus statim ex pe-



ripheria

ripheria innotescit. Sed & id variè. Nam si visus est per regulæ dioptras è centro: peripheria inter indicem & amussim est anguli oppositæ magnitudinis ut hic cum visus fertur per e à puncto a in metam m angulus m a s est is cujus tangens est magnitudo m s hoc est arcus u e.

Sin autem visus est per dioptras lateris, erit è centro quod actu est, aut è termino peripheriæ quadrantis: si est è centro: angulus magnitudinis in cujus metam visus cadit, est peripheria inter latus dioptrarum & filum è centro pendulum. ut in sequenti schemate apparebit.



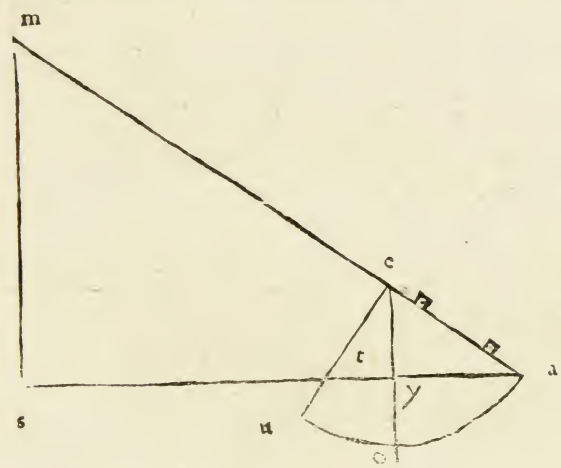
Nam angulo o a e hoc est arcui o e opponitur magnitudo s m tangens nimirum anguli a. quia est tangens arcus è radio a s subtendentis angulum a.

Si visus per dioptras lateris est è termino peripheriæ in metam

tam magnitudinis oppositæ: angulus oppositæ magnitudinis est peripheria inter latus quadrantis reliquum & filum è centro pendulum.

Esto enim visus ab a per e in terminum oppositæ magnitudinis m s. Erit per s e angulus e a s 15 cujus m s est tangens. Eum determinari dico in quadrante arcu u o.

Nam peripheria u o determinat angulum u e y.



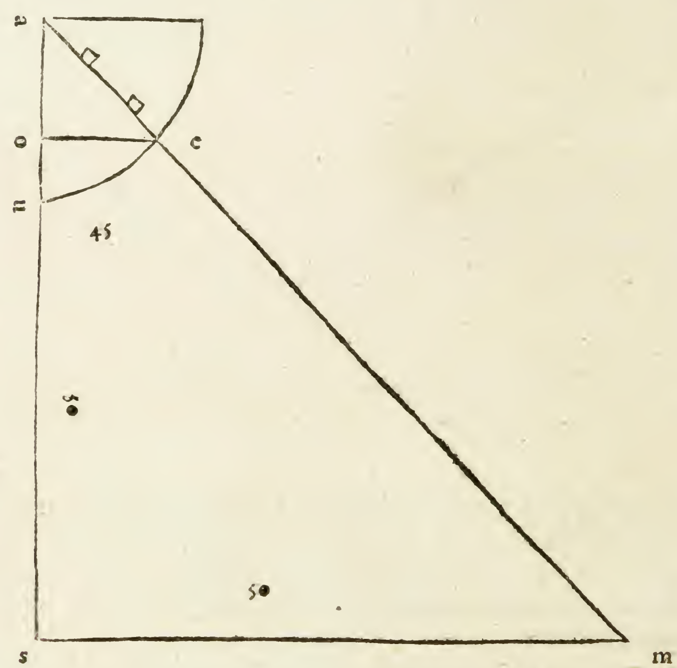
At angulus u e y æquatur angulo e a t. Nam angulus u e a ex fabrica rectus est: est etiam angulus e y t rectus ex thesi. præterea triangula e a t, e y t communem habent ad t. Quare per c.3.e.7.R. angulus u e y æquatur angulo e a t. Ergo angulus e a t mensuratur peripheria u o.

Hinc tandem sequetur ipsa rectarum geodæsia. ubi angulus magnitudinis is erit qui subtenditur à metienda magnitudine.

Itaq;

8. Radius est ad tangentem anguli longitudinis ut nota altitudo ad longitudinem.

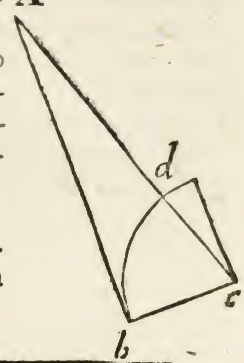
Longitudinis inventio hoc elemento docetur: sive jam visus sit in metam magnitudinis metiendæ sive magnitudinis ad metiendam rectæ ac notæ: methodus tamen eadem fuerit. Esto primum visus in metam longitudinis. Sit ergo metienda longitudo s m è loco superiore a. & detur altitudo a s pedum 50. Quæritur s m. Capiatur itaq; angulus s a m longitudinis. Sitq; gr. 45. arcus nempe u e aut ei similis è radio a s. Hujus tangens erit 10,000,000.



Ergo per 6. e. 10. ut radius ad 10,000,000. sic 50 altitudo ad s m longitudinem etiam 50.

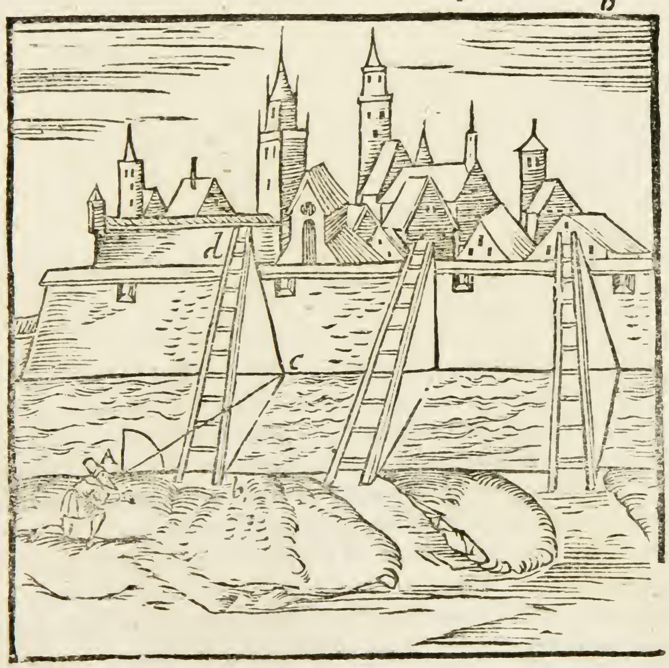
Hinc patet cur facilitas calculi sit si semper segmentum indicis recti sit radius assumptus. Nam per 9. e. 7. R. ut a o ad o e sic a s ad

ad s m, hoc eſt ut radius ad tangentem anguli a. ſic a s ad s m. Quod hic ſemel repetiviſſe ſatis ſit.



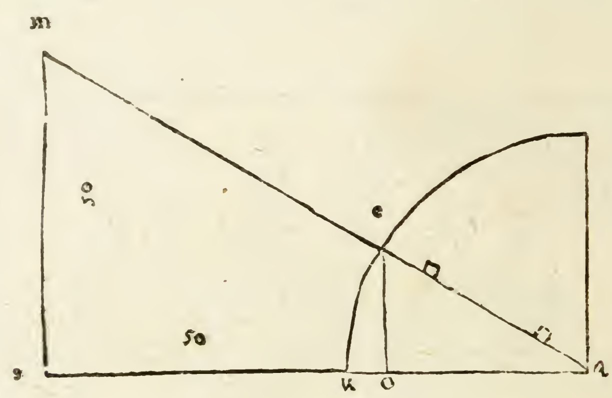
Nec quicquam intereſt: utrum longitudo ſit in ſubjecto plano, an in aſcenſu deſcenſu've montis. modo index rectus ſit longitudini metiendæ. ut in ſequenti figura patet: ſi b a ſit ſuperficies montis metiendi.

Et hoc modo latitudines fluuiorum, vallium, foſſarum metiri potes: ut in appoſita hic figura apparet.



Et sane dimensio latitudinis fossarum magni est momenti in oppugnationibus arcis, urbis: ut ex cognita latitudine & profunditate scalæ pro irruptione satis longæ & aptæ fabrefieri possint. Sic apud Polybium Philippus ad Meliteorum urbem capiendam scalas non satis longas attulit. itaq; gravi clade affectus discessit. Et Gallis ad Mediolanũ eadẽ res perinde nocuit.

Sit deinceps visus in metam altitudinis nempe in metam magnitudinis ad metiendam rectæ. & quærenda longitudo a s.

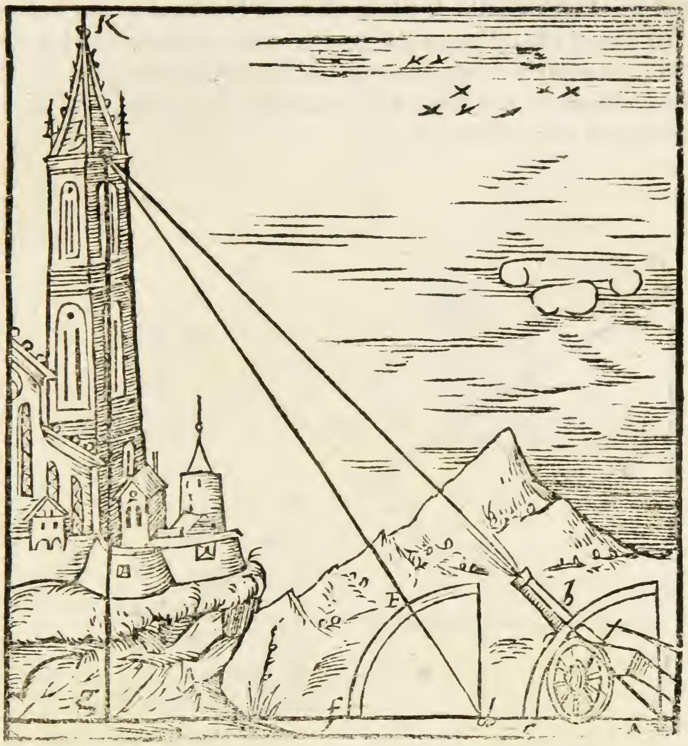


Sit data altitudo m s pedum 50. jam opticus radius incidat in m. sitque angulus a gr. 45. erit etiam o e a hoc est per 12. e. 5. R. angulus s m a nempe complementum anguli prioris. Et quia hic continetur à transversario ac optico radio erit a s ejus tangens.

Si itaq; sit ut radius m s ad tangentem a s 10,000,000. erit ut m s 50, ad s a etiam 50. pro longitudine quæsita.

Hinc licebit oblato acumine turris notæ distantiam à nobis ad eam reperire: quæ res in bellis maximi est momenti, ut sciamus distantiam ex quanta machinæ sphærula in hostium urbem sit emittenda: ut ex sequenti figura intelligi potest.

Quod



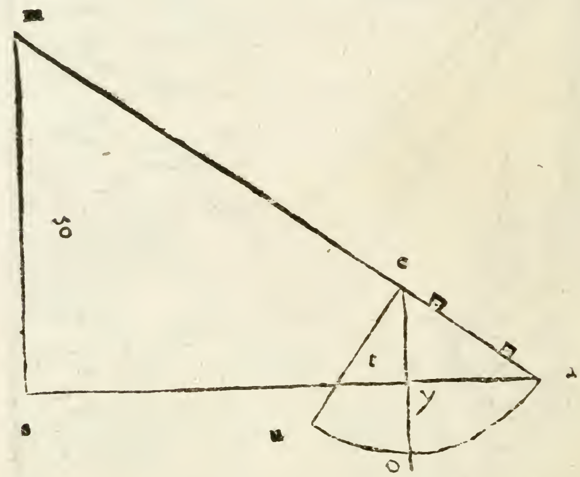
Quod ſi hic,ut ante indicis ſegmentum,ſic tranſverſarium rectum aſſumas totidem partium quot radius eſt definitus: multum inventionis compendium habebis. Nam per 9.e.7.R.ut e o ad o a.ſic s m ad s a,hoc eſt ut radius ad tangentem anguli longitudinis ſic altitudo ad longitudinem. Quod etiã hic de tranſverſario ad magnitudinem metiendam recto ſemel monuiſſe ſufficiat. Et

9. *Radius eſt ad tangentem anguli altitudinis, ut data longitudo ad altitudinem.*

Longitudinis geodæſia ex eodem quidem loco fuit: ſequi-

tur ſimilis altitudinis dimenſio.ubi index metiendæ rectus eſt.

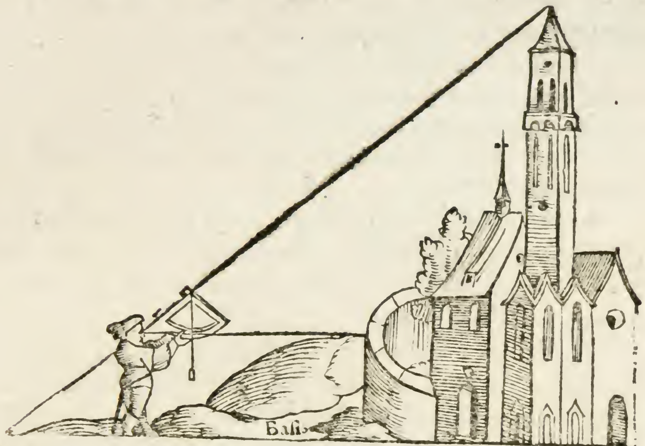
Sit data a s longitudo pedum 50. & angulus indicis, radiique optici ſit 45.erit tum in triangulo rectangulo a s m, ut radius a s aſſumptæ menſuræ ad tangentem anguli 10,000,000 ſic a s 50 pedum ad s m pedum 50.

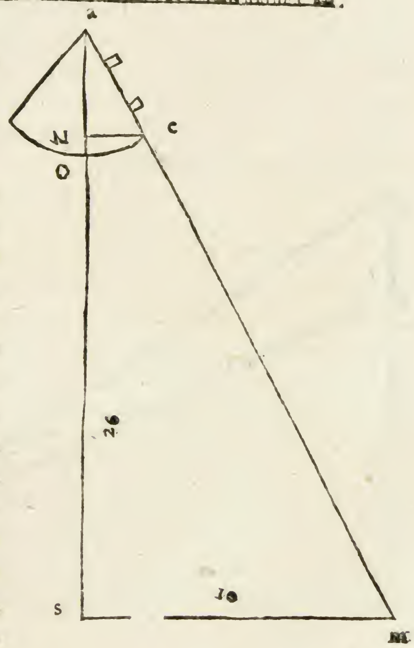


Et hoc modo turrium,ædificiorum aliorumque rerum metiri altitudines poteris.

Sed hoc tamen notandum altitudinem eam tantum inveniri quæ eſt à continuato indice ad verticem altitudinis.Quare menſoris altitudo ad inventam altitudinem addenda eſt:

ut ſi ſit data longitudo pedum 250.& ſegmentum indicis 10,000 tanquam radius,tranſverſariam vero partium totidem erit altitudo 254.addita ſcilicet menſoris altitudine pedum 4.





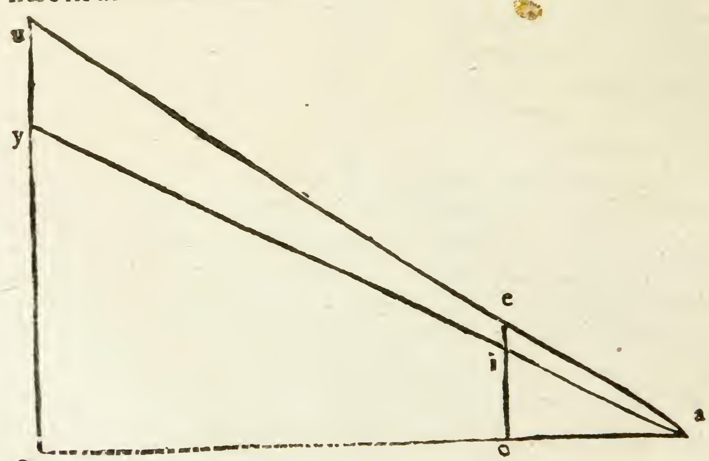
Sit deinceps index parallelus metiendæ altitudini. ut accidit in altitudine everſa puteorum & ſimiliũ rerum rupis, turris.

Eſto enim metienda altitudo a s, & data ſit longitudo verbi gratia diameter putei s m 10. Quæritur a s profunditas putei. ſitq; angulus o a e grad. 21 & 2′ ferè. erit proinde angulus altitudinis a e u, hoc eſt a m s grad. 68, 58′. Ergo in triangulo rectangulo a m s, ut radius ad tangentem anguli m 26,005,663 ſic s m longitudo 10 pedum ad a s profunditatem 26 idem fiet ſi radio uſus fueris. Nam ut u e ad u a, hoc

hoc est ex fabrica quam supra præscripsi radius ad tangentem anguli m u ut m s ad s a. Sed tamen radiū quadrantis aut segmentum indicis supra orā metiendę magnitudinis ex inventa quantitate tollitur ut vera relinquatur profunditas. ut si a o esset pedum 2. & a s pedum 26. profunditas o s esset pedum 24. Schemata dimensionis puteorum vide in Geometria Rami & practica Geometria Orontii. Itaq;

10. Si visus sit in terminos notæ partis metiendæ magnitudinis: erit ut differentia transversariorum ejusdem indicis segmenti ad transversarium minus sic nota pars remotior ad reliquam.

Id primum in radio videbimus: deinde in quadrante. in radio eam ad rem nonnulli unicum saltem transversarium adhibent, & per ejus pinnas duas transmittūt visum: pinnarum altera immobilis sistitur ad terminum transversarii: altera quæ cursor dicitur hinc inde obvolvitur: donec per utramq; pinnam visus incidat. Verum ego consultum arbitror: ut ad idem quidem indicis segmentum duo sint transversaria: & quidem propter calculi brevitatem & facilitatem paucarum admodum partium. ut hæc in usu facile notabis.



Sic

Sit exempli gratia ædificii alicujus u s altitudo fenestræ in edita parte u y pedum 6. Et cupias hinc investigare reliquam altitudinem y s.

Admove radium & mitte visum primò in u erit tum a o segmentum indicis: e o transversarium seu tangens anguli e a o si segmentum indicis intelligatur radius.

Rursus eodem segmento indicis per transversarium 10 mitte visum in y. erit 10 tangens anguli i a o.

Sit autem transversarium e o partium 12, i o partium 10. Erit ergo ut 2 ad 10, sic 6 ad 30 reliquam partem.

Nam quia e i & u y, itẽ i o & y s sunt parallelę erit per 14.e.5.R.

ut e i ad u y sic i a ad a y
ut i a ad a y sic i o ad y s.

Ergo per æquationem rationum & alterne.

ut e i ad i o sic u y ad y s.

Eadem ratio in quadrante est: Nam differentia transversariorum, ut patuit, est differentia tangentium e a o, i a o. Ergo ut hæc differentia ad tangentem minorem sic nota pars ad reliquam. Quod monuisse satis sit.

Et sic geodæsia unius distantiæ fuit longitudinis & altitudinis: cur etiam non latitudinis? an non fieri poterit ut linea transversa unica statione mensuretur? si nempe index rectus sit transversæ ad terminum latitudinis: & regula visum in reliquum terminum deducat: an non jam angulus latitudinis notus est: & proinde ejus tangens? verum est: si sit instrumentum, latitudini rectum, parallelum horizonti: & tum si detur tertius aliquis terminus datur latitudo: quæ tamen tum aliud nihil est quam longitudo vel altitudo. Quod si latitudo non sit in eadem altitudine cum mensore: certe invento angulo latitudinis, & non dato termino tertio nempe distantia à loco observatoris in latitudinem unica distantia non sufficit. Quod si ergo inventa sit distantia illa in latitudinem à loco observationis unica statio sufficiet. inventio autem erit ex calculo triangulorum. Nam

ut radius ad tangentem anguli longitudinis aut altitudi-

nis, ſic altitudo aut longitudo ad diſtantiam à puncto in quod viſus incidit ad viſum.

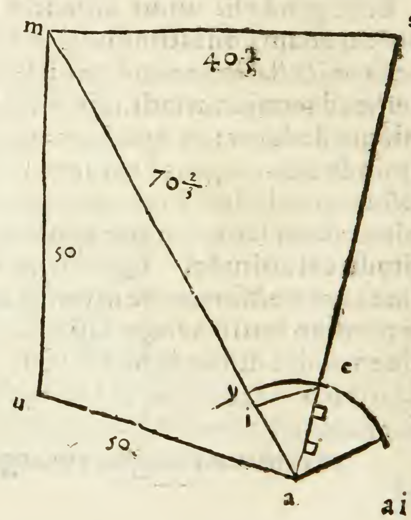
ut in figura 9 e angulus m a s altitudinis erat grad. 45. ejus ſecans, ſi radius fiat a s, eſt 14,142, ne reliqua jam curemus. & data fuit menſura radii a s pedum 50. ergo per 7 e 10 ut radius 10,000 ad 14,142, ſic a s pedum 50, ad a m pedum $70\frac{2}{3}$ ac paulo ultra ſed tamen non ultra dodrantem partis. Hac inventa diſtantia geodæſia latitudinis etiam fiet ex una diſtantia hoc modo.

Et

11. *Si viſus eſt in metam latitudinis: radius eſt ad tangentem anguli latitudinis, ut diſtantia à viſu ad indicem in latitudinem continuatum ad latitudinem.*

Hæc eſt geodæſia latitudinis unius diſtantiæ. Et tamen dimenſio ex una diſtantia eſſe duplex poteſt. Nam index perpendicularis latitudini metiendæ aut eſt perpendicularis ad ejus terminum aut aliud punctum.

Eſto primum ad terminum ejus. ut ſit metienda linea tranſverſa m s exempli gratia latitudo arcis ſuperioris.



Sitq; a u longitudo pedum 50. & angulus altitudinis m u grad. 45. erit m a pedum $70\frac{2}{3}$.

Et ſic jam angulus latitudinis y a e grad. 30. erit tangens ejus 5,774. jam cum in triangulo a m s rectangulo ad m ex theſi detur angulus a. erit ut radius ad 5,774. Sic a m diſtantia $70\frac{2}{3}$ ad m s latitudinẽ pedum 40 & $\frac{4}{5}$ ferè.

10,000. 5,774. $70\frac{2}{3}$ $40\frac{4}{5}$.

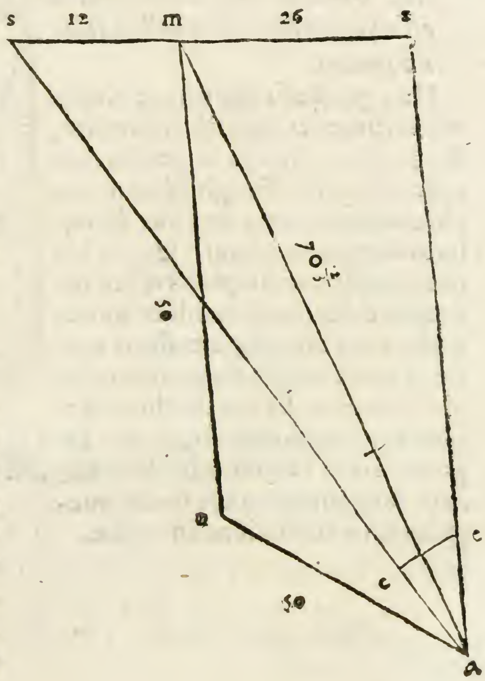
Idem fuerit in radii uſu:

ai ſit

a i ſit 10,000. i e 5,774. Ergo ut 10,000. ad 5,774 ſic $70\frac{2}{3}$ ad $40\frac{4}{5}$. Quod ante monui.

Sit jam index perpendicularis latitudini ad aliud punctum. tam particulares latitudines reſtituent totam. Erit autem perpendicularis ad medium aut extra. Si ad medium jam via pateſacta eſt. Nam latitudo dicto modo inventa ſi duplicet, eſt quęſita.

Experimentum autem facies: ſi imponas tranſverſariũ quod ab indice biſecetur: & tum viſus immoto radio ſit in utramque metam latitudinis. Tum enim angulus indicis radiique optici unius, eſt angulus ſemiſsis latitudinis. Sed quia laborioſa eſt & moleſta inventio iſtius puncti biſectionis imo nec ſemper detur acceſſus ejuſmodi: ut index in medium punctum latitudini ſit perpendicularis: utile eſt à quocunque puncto latitudinis ſegmenta menſurari: primò primũ ſecundò ſecundum. At quomodo idem punctum notabo verbi gratia in pariete dealbato: id facile erit ſigno aliquo cócepto, aut facta collimatione ſecũdum ſigna duo in loco menſoris concepta: manum admove & facile videbis.



Sic in ſequenti ſchemate ex eadem diſtantia cape m s latitudinis dextræ angulum m a e verbi gratia 20. Sitque eadem diſtantia quæ ante fuit nempe m a pedum $70\frac{2}{3}$. & ex canone datur tangens g 20.

 nempe

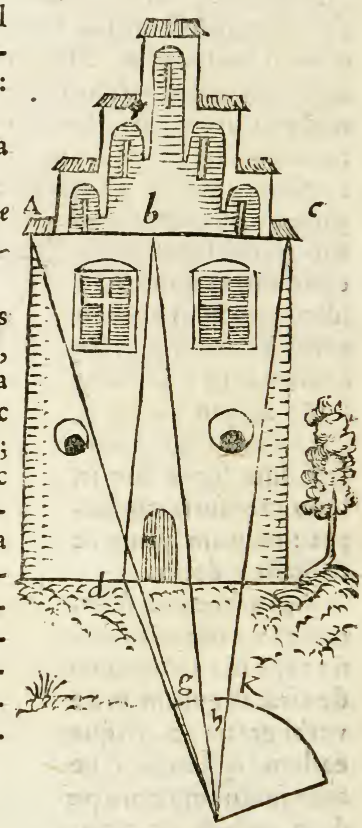
nempe anguli dextri m a e 3,640.qualium radius vel ſegmentū indicis eſt 10,000.Ergo ut radius ad 3,640 ſic $70\frac{2}{3}$ ad 26 ferè.

1,000. 364. $70\frac{2}{3}$ 26 ferè.

Rurſus ſit angulus m a s ſiniſter gr.10.hac ratiocinatione ex diſtātia m a invenietur s m ſiniſtra latitudo pedū 12 & paulo ultro.

10,000. 1,763. $70\frac{2}{3}$ 12.

Ergo additione particularium altitudinum, s s erit pedum 38. Sic latitudines domorum capi poſſunt: ut in hoc ſchemate patet.



Atque ſic demenſio & geodæſia unius diſtantiæ eſto.

12. *Geodæſia duplicis diſtantiæ eſt ubi menſor locum collimationis mutat.*

Hæc geodæſia ſæpe neceſsitatis eſt:ubi propter montiū, murorum, domorum, aut alia impedimenta unica ſtatio nec longitudinem nec altitudinem ſcrutatur, nec deniq; latitudinem menſurat. Quare hic quaſi duplex eſt ſimplicis illius diſtantiæ calculus.& recidunt huc ea quæ de angulis magnitudinis metiendæ eorumque tangentibus ſupra dicta ſunt. Et notabis hic differentiam tranſverſariorum aut ſegmentorum indicis eſſe differentiam tangentium angulorum magnitudinis ad metiendam rectæ.

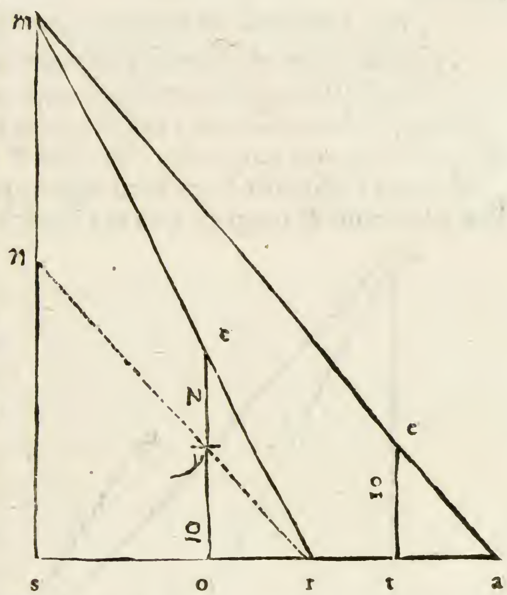
Hic

Hic

13. Si visus sit in metam altitudinis: erit ut differentia transversariorum ejusdem indicis segmenti ad minus, sic differentia stationum ad longitudinem.

Hic rursus operam dabis, quod etiam deinceps facies, ut transversaria habes paucarũ partium. Esto ergo metienda longitudo s r.

Collimatio itaque prima sit remotior ex a. secunda ex r in eandem altitudinis licet ignotæ metam m. Sitq; distantia stationum r a pedũ 20. erit m s tangens anguli a. si a s sit radius: tangens vero anguli r si r s sit radius. Erit proinde e t, hoc est o y tangens anguli a. & e o tangens anguli r.



Dico jam esse ut e y ad y o, hoc est differentiá tangẽtium ad tangẽtem minorem sic a r distãtiam ad a s longitudinem quæsitam. Nam ducta parallela n r ad m a. ducta inquam per y. ut y o æquetur rectæ e t.

ut m n ad e y sic n r ad y t per 14. e. 5. R.

ut n r ad y r sic n s ad y o

Ergo per æquationem rationũ & inversis alternisq; terminis

ut e y ad y o sic m n ad n s.

At ut m n ad n s sic a r ad r s per 13. e. 5. R.

Quod si jam y o fuisset 10, & e o 12. erit longitudo pedum 100. Nam ut 2 ad 10, sic 20 ad 100.

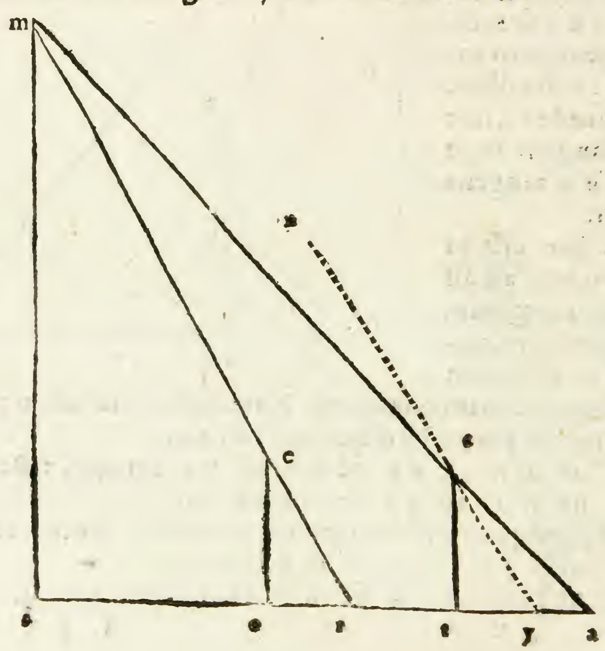
Eadem res erit si tangentes singulorum angulorum altitudinis oblatæ in consilium voces.

Quare hæc est geodæsia duplicis distantiæ in capienda longitudine: ubi ob interjectum impedimentum meta longitudinis videri non posit: nec altitudo in cujus terminum visus exit, nota sit. Et

14. *Si visus sit in metam altitudinis per idem transversarium erit ut differentia segmentorum indicis ad differentiam stationum, sic transversarium ad altitudinem.*

Sit itaq; collimatio prima ex a in m. erit angulus e a s altitudinis quæsitæ ejusq; tangens m s si radius sit a s.

Secunda collimatio sit ex r. erit ergo angulus e r o etiam quęsitæ altitudinis & tangens ejus m s. si radius sit r s.



Duca-

Ducatur perpendicularis rectæ m r. ipsa n y quæ secet t y æqualem ipsi o r ut y a sit differentia segmentorum indicis: hoc est differentia tangentium complementorum quæsitæ altitudinis angulorum.

jam reliqua sic erunt.

ut a t ad a s sic t e hoc est o e ad m s.

ut o e ad m s sic o r hoc est t y ad r s. Quare ut totus a t ad a s sic ablatus t y ad r s ablatum: erit ergo ut totus ad totum sic reliquus y a ad reliquum a r, hoc est ut o e ad m s.

Quod si transuersarium sit 9, & distantia stationum pedum 30. indicis segmentum minus 9. majus 12. erit altitudo quæsita pedum 90. Nam

3. 9. 30. 90.

in quadrante res eadem est: est enim ut differentia tangentium complementorum angulorum altitudinis ad minorem sic distantia ad altitudinem.

Esto enim angulus primæ collimationis e a t gr. 36, 52'. erit angulus t e a gr. 53, 8'. Ejusq; tangens o r vel t y 1,333 resectis scilicet dextris notis quatuor. Et angulus secundæ collimationis e r o sit g. 45 erit & angulus lōgitudinis totidē ejusq; tangens 1,000. Quare t y de t a relinquet y a nempe 1, 000 de 1,333 relinquit 333 jam distancia stationum a r sit pedum 30. altitudo erit 90.

333 ad 1,000. sic 30 ad 90.

Ramus ex superioribus multa hinc deducit postulata ad 12.e. 9. primo de differentia duarum altitudinum: ex dimensarum nempe subductione.

Secundo: ex inæqualium altitudinum exempli gratia turrium altera licet alterius altitudinem metiri. quod etiam hinc expeditum est: primo enim si ex minori velis metiri majorem cape per 8 e longitudinem inter minorem & majorem. tum enim 9 e dabit altitudinem supra minorem quæ cum nota minore addita restituit majorem.

Secundo si è majore velis metiri minorem: rursus è nota majori altitudine capies longitudinem inter altitudines: tum 9 e

dabit

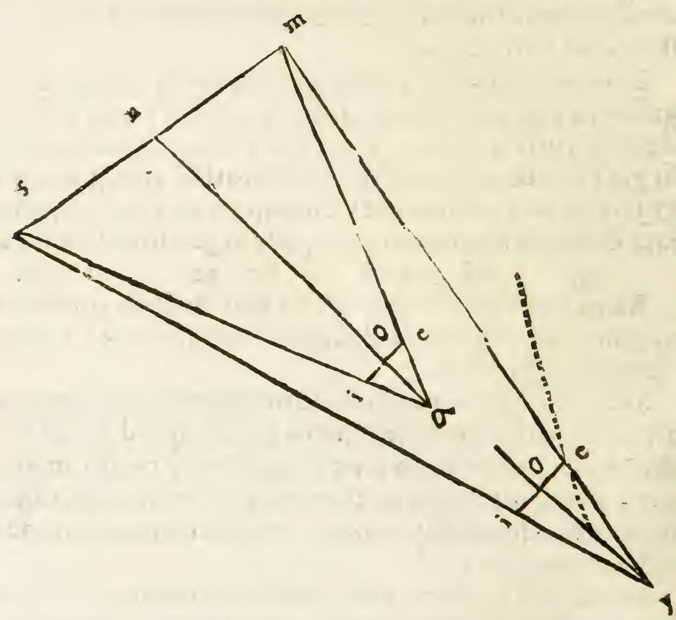
dabit altitudinem qua major ſuperat minorem inde reliqua pars de majore eſt altitudo minor. Schema illic Rami additum huc referri poterit.

Tertium ipſi poſtulatum eſt de inventione diſtantiæ turrium è vertice alterius: quod in præmiſsis innotuit poſtulatis per 8.e. Figura Rami huc etiam accommodari debet.

Et

15. Si viſus ſit in terminos latitudinis per idem tranſverſarium: erit ut indice differentia ad differentiam ſtationum, ſic tranſverſarium ad latitudinem.

Hic rurſus, ut in ſuperiori latitudinis dimenſione, index perpendicularis latitudini, eſt ad terminũ ejus perpendicularis aut ad aliud punctum: & tum ſumma particularium latitudinum to-



tam

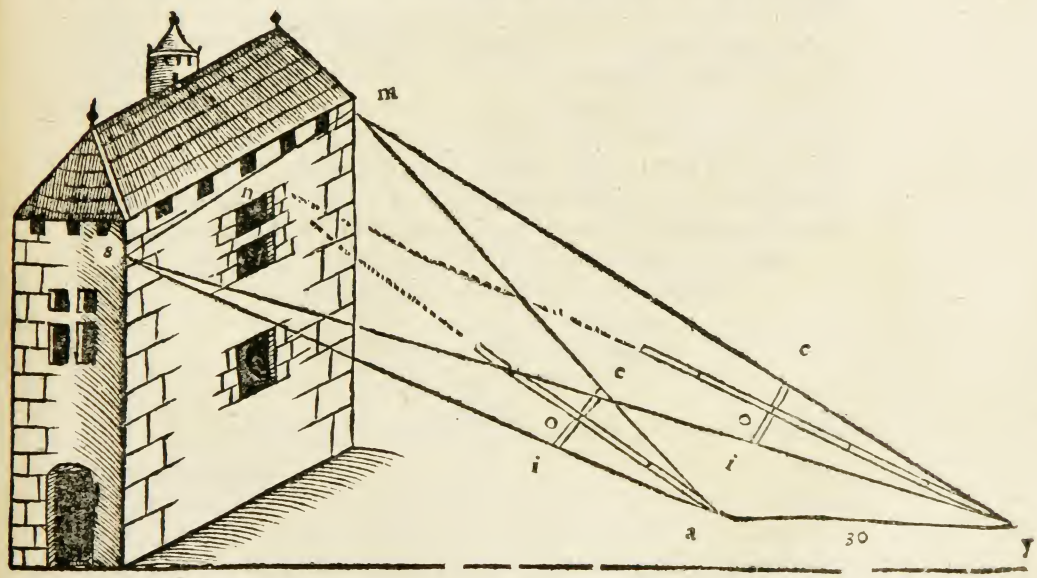
tam colligit. Sed tamen expeditior ratio eſt, ſi tranſverſarium imponatur ita ut in metiendæ latitudinis terminos immoto radio viſus deferatur. ut ſit metienda latitudo s m.

jam ſit prima collimatio ex a per e in m indice perpendiculari ad punctum m: & ex a manente indice per i viſus diducatur in s. Quod ſi non ſit in promptu unum tranſverſarium tale: imponantur ſane duo, ſed ita ut idem in eodem ſtationis loco ſecent indicis ſegmentum.

Secunda collimatio ſit ex y manente tranſverſario e i ſimplici aut duplicato: ſed longius tamen ab oculo remoto: & ſit viſus per e & i in m & s.

Dico eſſe ut indicum differentiam nempe differentiam a o & y o ad y a ſic tranſverſarium e i ad s m latitudinem.

Demonſtratio cum priori eadem eſt. Nam ducta parallela ad a m quæ ſecet ex o y ſegmentum æquale ipſi o a: erit ut differen-



tia segmentorum indicis ad y a sic e o ad m n.& i o ad n s:ut ante patuit:Ergo per terminorum proportionalium additionem,erit ut differentia segmentorum indicis ad y a sic i e ad s m.

Sic si sit metienda latitudo domus s m,& sit a o 12.y o vero 20. transversarium i e 6.ita ut ab utraq; indicis parte 3 sint.

Sitque distantia collimationis a y pedum 30. Quæritur s m. Differentia segmentorum indicis est partium 8. Ergo s m erit pedum $22\frac{1}{2}$. Nam

8. 6. 30. $22\frac{1}{2}$.

Quod in particularium quoq; latitudinum dimensione sic patet:ut se habet 8 ad 3,sic 30 ad $11\frac{1}{4}$ pro n m itemq; pro s n. Nam transversarii segmenta utrinq; æquantur.

Et hinc intelligitur quomodo quadrante capienda sit. Nam ex antecedenti datur primo n m, deinde n s tanquam altitudines:hinc summa m s. Et idem fuerit si visus sit in superiori loco. Situs enim geodæsiam non mutabit.

Quare geodæsia nobis sic exposita sit. ubi, si quis requirat,radius quidem accommodatissimũ est instrumentum : quadrans tamen sæpè magis tractabilis. Quod si radio uti vis: vide ut quemadmodum monui distribuas. in geodæsia duplicis distantiæ facilior per radium contra aberrationes visus munitum est geodæsia.in geodæsia unius distantiæ nisi radius dicto modo & correctus & distributus sit quadrans præstabit. Hinc itaq; pictor Architectus cæteriq; artifices locorum omnium dimensionem instituent:fenestras,statuas pyramides metientur: & quicquid omnino in loco insigni notabile est scrutabuntur.

TH. FIN.

TH. FINKII GEOMETRIAE ROTVNDI,

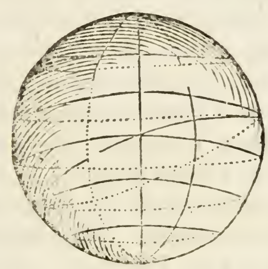
LIBER DVODECIMVS.

De ſphæra eiusq́ꝫ circulis.

ROtundi nobis Geometria eſt propoſita: ejus duo fecimus genera circulum & ſphæram. Circularem itaque Geometriam ſphærica ſequetur quam breviſsime delineata.

1. *Sphæra eſt rotundum ſolidum.*

Poſſet definiri corpus comprehenſum à ſphærico ſeu ſuperficie ſphærica:verum rotundum dici rotundius eſt. Vulgo corpus ſphęricum,Latine globus dicitur: ſed tamen ſphæræ vocabulum græcum retentum in uſu eſt.



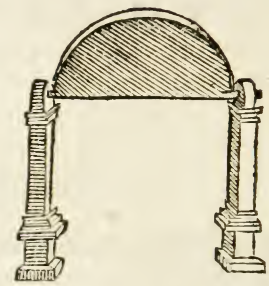
Proclus in hypotypoſi aſtronomicarum hypotheſeon duplicem ſphæram facit. Alia enim ipſi στερεά eſt alia κρικωτής,hoc eſt armillaris, diſtinctis circulis conſpicua: cujuſmodi eſt ea quam Elementarii Aſtrologi in Sphæricis ſuis libellis explicant. Sed hæc diviſio ſeu diſtinctio non uſq; adeo eſt neceſſaria. Nam armillaris eſt tantum effigies certarum ſphæræ partium.

Itaq;

2. *Sphæra fit converſione ſemicirculi, manente diametro.* 14.d.11.

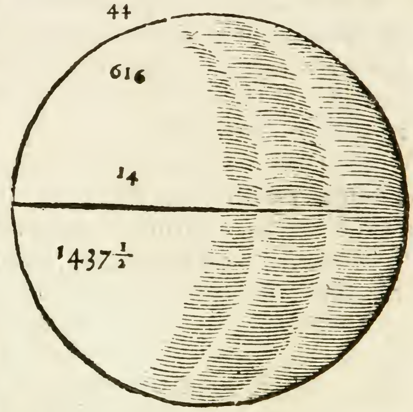
Sic peripheria generatur motu puncti lineæ unius reliquo quieſcente: circulus motu lineæ quieſcente ejus termino uno. Sic conus Euclidi 18.d.11.fit converſione trianguli rectanguli manente altero crure. Sic 21.d.11.è rectangulo cylinder fit,& ſic de-

inceps: & hoc modo ſphæra fit converſione ſemicirculi manẽte ejus baſi nempe diametro. nimirum ſi concipiamus ſpacium illud quod in converſione ſemicirculus attingit ſolidari & quaſi corporari. id quod in officinis tornariorum videre licet. ibi enim corpus circa polos & axem motum, ſpacium rotundat.



3. Planus è diametro & ſextante ſphærici eſt ſphæra.

Sphæricum metieris per 5.e.20.R. Hic ut circulo & ſphærico, ſic culo & ſphæræ ſua eſt analogia. Cubicum ſeu cubica ſuperficies comprehenditur ſex baſibus quadratis & proinde æqualibus. Sphæra item continetur ſex baſibus ſphæricis æqualibus cubicas baſes ambientibus. jam cubus eſt planus è ſexta cubici parte per latus. Sic etiam ſphęra eſt planus è ſexta ſphęrici parte per latus nempe diametrum.



Exemplum. Eſto diameter ſeu axis ſphæræ 14. hoc eſt diameter maximi circuli. jam per 1 e 8. peripheria maximi hujus circuli eſt 44. Ergo planus è 44 per 14 nempe 616 eſt ſphæricum per 5.e.20.R. Nunc planus è $\frac{616}{6}$ per 14, hoc eſt $1,437\frac{1}{3}$, eſt ipſa ſphæræ ſoliditas.

Itaq;

4. ut 21 ad 11, ſic cubus diametri ad ſphæram.

ut in præſenti exemplo cubus axis eſt 2,744. Nam 14 per 14 facit 196 & hæc per 14 faciunt 2,744 cubum ſcilicet diametri.

jam

jam ratio 2,744 ad sphæram 1,437$\frac{1}{3}$ est in minimis terminis ut 21 ad 11. Maxima enim communis mensura 8,232 & 4,312 integrorum datis proportionalium est 392. per quam divisi redeunt ad 21 & 11. Et

5. *Planus è sextante sphærici & radio est hemisphærium.*

ut planus è 61$\frac{6}{6}$ & 7 est 718$\frac{2}{3}$. Sed accuratius est sumere dimidiam sphæram.

6. *Sphæræ sunt ut à diametris cubi.* 18.p.12.

Solida similia habent triplicatam rationem homologorum laterum 5.e.20. R. & axes sphærarum sunt earum latera homologa: rursus cubare (ut ita diaam) diametros est triplicare rationem homologorum laterum. Ergo ut cubi diametrorum sic sphæræ. Euclidea demonstratio paulo est laboriosior. Exempla fructus hujus propositionis apud Astrologos multa reperiuntur.

Stellas primi luminis atq; honoris ita ad terræ globum se habere ajunt: ut sit ratio diametri earum ad diametrum sphæræ terræ quadrupla major tribus quartis qualis est 19 ad 4. Hinc inquiritur differentia magnitudinis stellarum earum & terræ: & inveniuntur eæ ad terram ut 6,859 ad 64. nempe si cubicè termini multiplicentur.

$$\begin{array}{rr} 19 & 4 \\ 19 & 4 \\ \hline 361 & 16 \\ 19 & 4 \\ \hline 6{,}859 & 64 \end{array}$$

Hoc est terram superari à stellis illis magnitudine & crassitie centies septies & $\frac{11}{64}$.

Sic Ptolemæus deprehendit Solis diametrum ad globi terreni axim esse ut 11 ad 2. Ergo ex cubica multiplicatione Sol est ad terram ut 1,331 ad 8.

$$\begin{array}{rr} 11 & 2 \\ 11 & 2 \\ \hline 121 & 4 \\ 11 & 2 \\ 1{,}331 & 8 \end{array}$$

S 3 Ergo

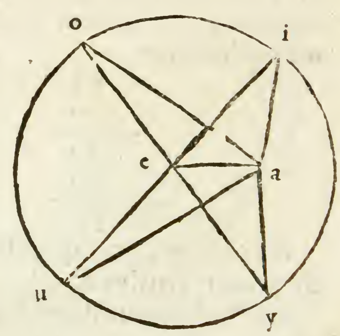
Ergo Sol terram ſuperat centies ſexagies ſexies & $\frac{3}{8}$, Copernicus noſter paulo majorem invenit diametrum. Deprehendit enim ἐκκεντρότητα ſolarem à tempore Ptolemæi decreviſſe. itaque ex opticis diametrum auctam eſſe. ut hæc in Aſtronomicis docentur.

Atq; hæc ferè ſunt quę apud Euclidem atq; Ramum habemus de ſphæra ſigillatim. Nos ex Regiomontano quędam addemus: poſt calculum novum exponemus. Hoc tamen admonitum Lectorem Philomathen volo: ut ſibi ſphæram comparet diſtinctam circulis: qualis eſſe poteſt quo vulgo ab Aſtronomis decem circulis componitur. in ea enim ſequentia melius videri poſſunt quam in charta ac ſuperficie depingi.

7. *Si axis ſphæræ eſt per centrum circuli: eſt perpendicularis dicto circulo: & contra.* 2.1.p.3. *Regiom.*

Regiomontanus ſegmentum tantum axis adhibet: quod eodem redit. Nam ſegmentum axis in axem continuari poteſt.

Eſto itaq; centrum ſphæræ a. circulus ſphæræ i o u y. eiusque centrum e. jam recta ſit ab a in e, hoc eſt axis aut ſegmentum ejus.



Dico a e eſſe perpendicularem circulo i o u y. Ducantur enim circuli diametri i u & o y earumq; termini cõnectantur cum centro ſphæræ rectis a o, a u, a y, a i, quæ ſunt ſphæræ radii & proinde per 2. e. 1. æquales. Sed & radii e u, e o, e i, e y ſunt æquales: & a e latus commune eſt. Cum itaq; triangula hæc ſint æquilatera, erunt æquiangula per 1.p.7.R. & anguli ad e æquales.

Quare cum a e diametris circuli rectè interjaceat per 10.e.2.R. erit iis perpendicularis in communi diametrorum ſectione nempe ad centrum.

At ſi recta eſt rectis in ſubjecto plano interſectis perpendicularis

laris in communi ſectione: eſt perpendicularis ſubjecto plano per 3.e.21.R.

Quare a e axis eſt perpendicularis circulo i o u y.

Converſa patet: Nam ſi axis perpendicularis circulo non eſt per centrum: erit extra.

At ſi extra centrum ſit: jam recta duplex ex eodem termino nempe centro ſphæræ erit perpendicularis eidem plano.

At perpendicularis ab eodem termino & eadem parte non eſt duplex per c.10.e.2.R.

Ergo perpendicularis axis in circulũ nõ eſt extra ejus centrũ

Itaq;

8. *Perpendicularis è centro circuli utrinq; in ſphæricum continuata eſt axis ſphæræ.* *3.p.3.Regiom.*

Demonſtratio eſt Regiomontano per abſurdum. ut è centro circuli e egrediatur perpendicularis e a: in ſphæræ centrum cadet: hoc eſt continuata erit axis ſphæræ. Si enim non eſt axis. jam è centro egreditur duplex perpendicularis: hæc recta continuata ex theſi: deinde axis alius ex theſi adverſarii. At id fieri nequit: ut jam antè patuit.

9. *Axis ſphæræ eſt etiam axis circuli, cui ille eſt perpendicularis.*

Id vulgo ſic ſumitur: unde & poli ſphæræ ſunt poli circuli eodem modo. Hinc ex præmiſsis elementis ſunt 4,5,6,7,8,9.p.3.Regiomontani. quas huc referes.

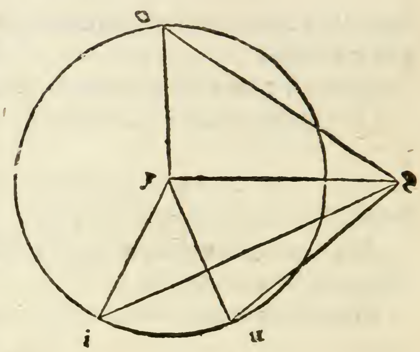
Itaq;

10. *Si ſphærici punctum eſt terminus rectarum trium in peripheriam circuli æqualium: eſt circuli dicti polus.* *13.p.3. Regiom.*

Hoc reſpondet circulo ad 27.e.1.

A puncto enim ſphærici a. in peripheriam o u i ſint æquales rectæ tres. ajo circuli dicti polum eſſe in a. Etenim ab a in circulum dimittatur perpendicularis a y. Erunt ergo anguli ad y recti,

recti. Et cum a y sit commune latus & a o, a i, a u ex thesi æquentur: per 5. e. 12. R. o y, y i, y u æquantur: & per 27 e i. y est centrũ circuli. Quare ex superioribus y a axis sphæræ est axis circuli, cui est perpendicularis ex thesi. ideo ejus terminus a est polus ut sphæræ sic circuli o u i.

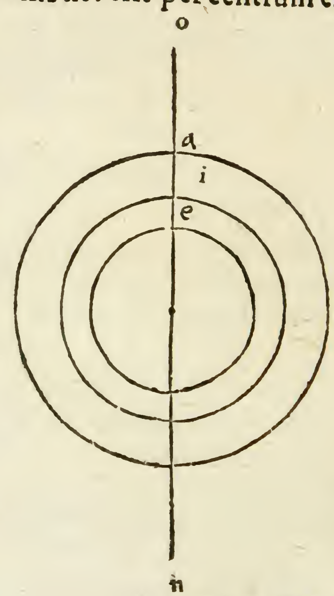


Et

11. Circuli quorum axis communis est sunt paralleli. *22.p.3.Regiom.*

Nam cum ex thesi axis communis sit: erit per centrum circulorũ ipsis perpendicularis per 9 & 7 e.

jam vero si plana communi perpendiculo dividuntur per 12.e.21.R. sunt parallela. ut hic circuli a, i, e. habent axem communẽ o u. Ergo sunt paralleli.



Exemplum sumere potes in sphæra astrologica ex Æquatore & tropicis: his enim communis axis est. unde & paralleli sunt.

jam contra si sint paralleli per 12. e. 21. R. communi perpendiculo dividuntur. Id continuatum utrinq; incidit in polos circulorum communes.

12. *Maximus*

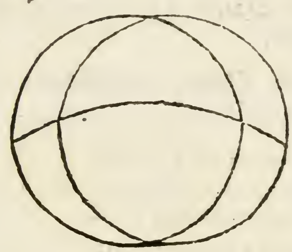
12. *Maximus sphæræ circulus est qui sphæram bisecat.*

Id Euclides seu Euclidis demonstrator ad demonstrationem 17.p.12.sic postulat: nempe circulum maximum esse eum qui per sphæræ centrum sphæram secet: hoc est qui sphæram bisecet.

Itaq;

13. *Circuli maximi se bisecant.* *19.p.3.Regiom.*

Nam axis sphæræ est ipsorum communis sectio & utriusque communis diameter. Secant enim sphæram per centrum. jam diameter eadem secat ex æqualibus ęqualia segmenta, nẽpe semicirculos per 2.e.4.



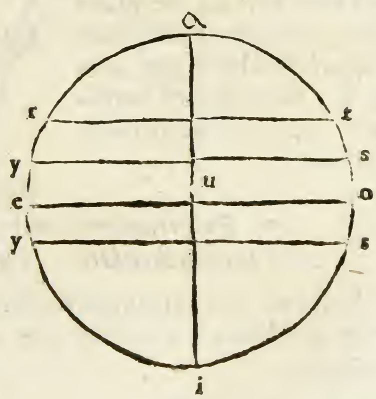
Et

14. *Circuli æquidistantes à maximo, sunt æquales.* *28.p.3.Regiom.*

Et

15. *Propior maximo, est major remotiore.*

Causa eadem est ex 7 & 25.e.1. Nam circuli sunt ut à diametris quadrata: Et diametrorum æqualium, inæqualium quadrata sunt æqualia, inæqualia per 1.c.2.e.12.R. Et rursus circulorum ęquidistantium à maximo diametri sunt ęquales: propioris diameter major est quã remotioris. Quod hinc patet.



Ducatur circulus magnus secans æquidistantes à maximo item remotiores & propiores per polos.

Sitq; communis sectio maxi

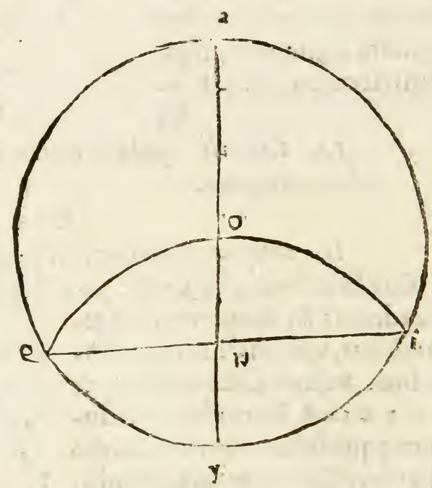
mi cum descripto maximo recta e o. diameter nempe utriusque sectio vero communis descripti cum æquidistantibus à maximo duæ rectæ y s & y s. cum propiore itidem y s cum remotiore r t. Dico y s, y s æquari. Nam per thesin æquidistant à centro. Ergo per 24 e i. sunt æquales.

Rursus y s major est quam r t per 25 e i. itaque circuli quorum diametri sunt y s nempe æquidistantes à maximo sunt æquales: & circulus diametri y s propioris est major circulo diametri r t remotioris.

16. Si maximus sphæræ circulum maximum per polos secet: rectè secat: & contra. 20. p. 3. Regiom.

Maximus enim e o i secet maximum a e i per polos. dico secare rectè.

Nam si recta in altero intersectorum planorū perpendicularis reliquo, ut hic à polo in centrum recta o u perpendicularis circulo a e i, est perpendicularis communi sectioni e i per eandem causam: plana ipsa, nempe circuli, sunt perpendicularia per 9. e. 21. R. Conversam similiter veram ex superioribus probabis.



Itaq;

17. Quadrans maximi circuli à polo maximi in ipsum, ei est perpendicularis. 1. p. 4. Regiom.

Secat enim maximum circulum per polos. ut in præmissa figura quadrans o e à polo o in circulum a e y ei est perpendicularis.

Et

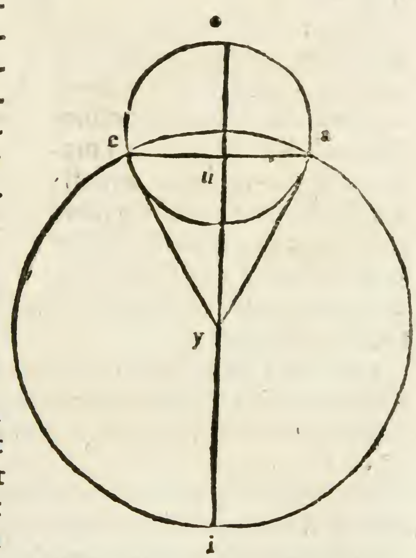
18. Si quadrantes maximorum circulorum perpendicu lares maximo circulo concurrant: punctum concurſus in ſphærico eſt polus dicti circuli. 2.p.4.Reg.

Eſto in præmiſſa figura quadrantes e o, i o perpendiculares ad circulum a e i concurrant ad punctum ſphærici o. dico ibi eſſe polum circuli a e i.

Nam maximus circulus maximum rectè ſecans, ſecat per polos. 18. e.

At ex theſi ſigillatim quadrans maximi circuli ſecat rectè maximum. Ergo ſigillatim ſecat per polum. hoc eſt concurrunt in polo maximi circuli.

19. Si maximus ſphæræ circulus biſecet minorem circulum: rectè ſecat: & contra. 21.p.3. Regiom.



ut hic ſi maximus circulus a e i biſecet circulum a o e: recte eum ſecat: Nam cum biſecet per centrum ſecat. copuletur itaq; centrum u cum centro ſphæræ y. erit axis y u perpendicularis circulo a o e per 7 e. Ergo per 10.e.21.R. circulus per axem ſecans ſecto circulo eſt perpendicularis.

Converſa, Si rectè ſecat biſecat, patet. Secat enim per centrum. Eſto enim communis ſectio e a, jam ſit perpendicularis y u à centro ſphæræ in cõmunem ſectionem: hoc eſt recta à centro ſphæræ biſecet communem ſectionem. erit ergo u ipſum circuli centrum & e a ejus diameter, quæ cum

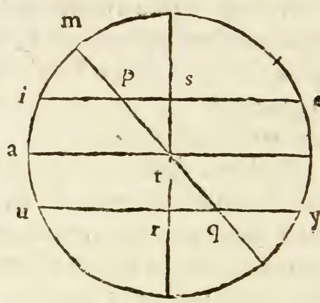
meter maximi circuli per 8 e. quæ cõtinet polos ſectorum. Quare tertius ſecans tranſieris per ſuæ diametri terminos tranſit etiam per polos interſectorum. Sed hæc, ut dixi, in ſolida figura & globo ipſo plenius videntur & cognoſcuntur, quam in charta depinguntur.

22. *Si maximus ſphæræ circulus parallelos & æquales extra polos ſecet: alterna peripheriarum ſegmenta æquantur. 19. p. 3. Reg.*

Hoc elemento Aſtrologi docent diem æſtivum longiſsimum & noctem hyemalem longiſsimam æquari. Et cæteras ſtellarum obvolutiones in parallelis & æqualibus cœli partibus ſimiliter perfici.

Demonſtratio ex inſpectione globi non eſt obſcura.

Sint enim diametri parallelorum o i, y u: erunt æquales, circulorum ſcilicet æqualium: & ſit diameter ſecantis extra polos m n. dico o p & u q æquari. Nam æqualium ſemiſſes æquantur o s, u r. His addantur æquales s p. r q. Nam per 14. e. 5. R. ut s t æquatur ipſi t r ex theſi ſic s p, r q. ſibi æquabuntur. Summæ ergo erunt æquales.



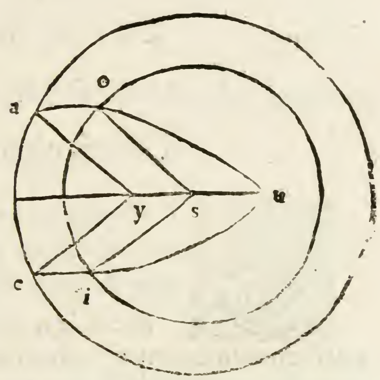
Hinc ſecundo ſequitur quod erat propoſitum. Sinuum ſecundorum æqualium æquales ſinus primi ſunt. Ergo o p, u q ſinus ſecundi ſemiſsium ſegmentorum habebunt æquales primos ductos à p & q. in commune punctum ſectionis ſecantis & parallelorum. & proinde per 15 e s. ſemiſſis peripheriarum alternarum ſemiperipheria aut majores aut minores æquabuntur. Semiſsium autem æqualium aſſes exiſtunt æquales. Itaq;

23. *Peripheria circuli maximi ſecantis inter parallelos biſecatur parallelorum maximo.*

Nam

Nam ſi ducatur circulus magnus ſecans parallelos & maximum ſecantem parallelos, ſecans (inquam) per polos: ex 12 e conſtabit ſegmenta & parallelorum & ſecantis maximi à ſecante in reliquum aut parallelum aut datum ſecantem æquari. Hæc ergo ſegmenta æqualia de quadrantibus æqualibus, qui ſecantur per parallelorum maximum, relinquunt ſegmenta peripheriæ æqualia inter maximum parallelorum & utrinque parallelos æquales. Quare tota peripheria inter parallelos eſt biſecta.

24. Si duo maximi ſphæræ circuli in parallelorum polis concurrant: ſecant parallelorum peripherias ſimiles. 23, p. 3. Regiom.



Sint circuli paralleli a e, o i. eos duo maximi circuli ſecent p polos a u, e u. Dico peripherias a e, o i eſſe ſimiles. Ducatur em̃ cõmunis ſectio ſecantium à polo u. Ea continebit parallelorũ centra per 9 & 7 e. & ſint ea y & s. Ab his centris ſint ſui radii y a, y e & ſo, ſi. jam quia ex eadem ſectione egrediuntur in planis parallelis binæ conterminæ o s s i. & a y, y e: erunt o s & a y itẽ s i, & y e parallelæ per 13. e. 21. R. Et proinde per 4. c. 12. e. 5. R. anguli a y e, o s i æquantur: & ideo ſunt proportionales ad quatuor rectos: ſed ut y vel s ad 4 rectos, ſic a e vel o i ad ſuam peripheriam. Quare a e ad o i ſunt ſimiles.

Hinc vulgò Geographi magna ſumunt ſubſidia in quærendis locorum diſtantiis adhibito 8. e 1.

Sic breviter excerpſimus ea & collegimus: quæ uſui fore videbantur. pleraq; enim alia ex his conſequuntur cognita doctri-

na

na ſinuum: Et ſi quæ præterea ab Aſtronomis à Geographis & omnino à Coſmographis deſiderari videbuntur: ea explebit liber ſequens: qui triangulorum ſphæricorum continebit doctrinam. unde ea breviſsimè Aſtronomis probari poterunt: quæ per longas & propoſitiones & propoſitionum demonſtrationes libri 3 Theodoſii Vogelinus in Aſtronomia demonſtrari putat. Sed Vogelini ſtudium laude eſt digniſsimum: qui de uſu propoſitionum quas doceret cogitavit: utinam multos haberet ſucceſſores qui non modo propoſitionum verba, ſed & uſum aliquem ſubindicarent: nec tam veterum propoſitiones vellent retinere: quam in pulvere uſui accommodatas exercere.

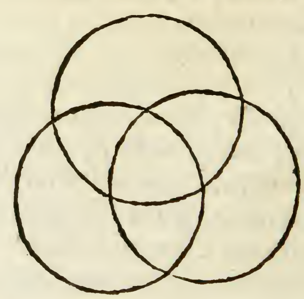
TH. FINKII GEOMETRIAE ROTVNDI,

LIBER DECIMVSTERTIVS.

De triangulis ſphæricis.

SI trium maximorum ſphæræ circulorum peripheriæ ſigillatim ſemiperipheriâ minores concurrant: triangulum conſtituunt ſphæricum.

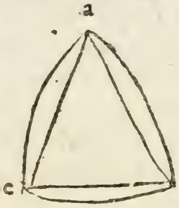
Sic vulgo definitur triangulum ſphæricũ concurſu maximorum circulorũ pyramidẽ triangularis baſis, quam faciunt inſcriptæ arcuum, facientium, ut deinceps magis apparebit. poſſet etiam definiri concurſu æqualium circulorum: at tum ſi minores ſint rectangulum triangulum non facient: ſin majores & minores: jam calculum nõ admittunt. Arcus autem ſectos per



pyramidis dictæ basis latera, latera triangulorum deinceps, jam usitato modo, dicemus.

2. Trianguli duo qualibet latera sunt majora reliquo. 37.p.3.Regiom.

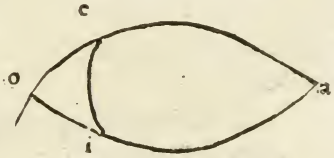
Hęc affectio cómunis est & triangulo plano, ut & insequentes nonnullæ: catholica ergo doctrina requireretur. Causa autem est ex triangulo plano: nempe ex basi pyramidis ad centrum sphæræ factæ. ut esto triangulum a e i sphæricum. sint etiam singulorum arcuum inscriptæ nempe latera basis pyramidis: Hic a i, & i e inscriptæ sunt majores quam a e inscripta per 7.e.6.R. Ergo secant a i & i e majores peripherias quam a e per 14. e. 2. Eodem modo a e & e i sunt majores quam a i &c.



Itaq;

3. Trianguli tria latera sunt minora duabus semiperipheriis. 39.p.3.Regiom.

Esto enim triangulum a e i. dico tria ejus latera esse minora duabus semiperipheriis. concurrant enim latera a e, a i in o. Erunt a e o, a i o semiperipheriæ duæ. per 1.e & 13 e 12. jam per præmissum o e & o i sunt majora latera quam e i. Quare adjectis communibus a e, a i: erunt a e, e i, i a minora quam a e o, & a i o.



4. Anguli trianguli sphærici amplitudo est penes arcum maximi circuli qui ex angulo tanquam polo descriptus angulum subtendit,

Hoc sic sumimus. Et tamen angulus est ad centrum sphæræ si ex quadrantibus admittantur radii ex terminis arcus subtendentis angulum. Vel si ex cruribus anguli æqualibus in communem sectionem sinus cadant. prout hæc in ipso globo apparent. Et

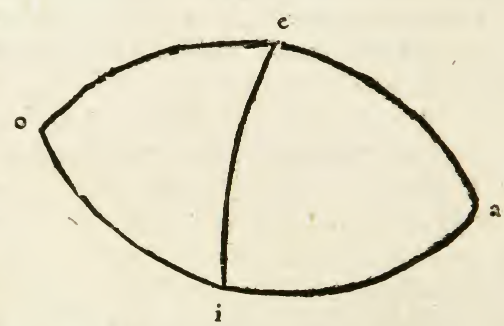
V sic

ſic 1 & 13 propoſitione Copernicus angulum trianguli metitur item Regiomontanus 34.p.4.de triangulis. Et hinc etiam Regiomontanus 4.p.5.angulum per inſcriptas metitur, ut illic videre licet.

Hinc patet & illud quod Regiom.30.p.3. habet: anguli ad 4. rectos rationem eam eſſe quę ſit peripheriæ metientis angulum ad totum. Itaq;

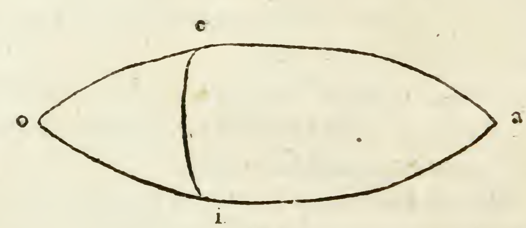
5. Si anguli crura continuata concurrant: angulum comprehendent prædicto æqualem.

In ſphæra hoc ſtatim apparet. Nam rectitudo, inclinatio, declinatio circulorum majorum manet ęqualis ab uno termino communis ſectionis in alterum.



Et etiam hinc patet: quia idē arcus magni circuli eos ſubtendit: ut hic e i metitur & angulum o & angulum a.

6. Trianguli tres anguli duobus rectis ſunt majores. 49.p.3.Regiom.



Eſto triangulum a e i. dico ejus angulos eſſe majores duobus rectis. Continuentur enim crura a e, a i in o. Erit angulus o i e angulo i e a, aut æqualis aut inæqualis. Tertium enim dari nequit. Si æqualis addatur ipſis communis

nis e i a fient æquales. At o i e, e i a sunt duobus rectis æquales. Nam recta in communem sectionẽ incidit ergo per 1. c. 8. e. 5. R. facit angulos duobus rectis æquales. itaque & anguli a e i. e i a. sunt æquales duobus rectis : addito ergo angulo e a i sunt majores.

Si jam angulus a i e sit inæqualis esto exempli gratia minor. Ergo cum inæqualibus o i e, i e a additur idem e i a. & anguli ad i æquentur duobus rectis: duo a e i, e i a erunt majores duobus rectis: & proinde tres multo magis majores. Eodem res redibit si a i e sit major: tum enim a e i erit minor: & angulus o æquatur angulo a per s e. unde concludes propositum.

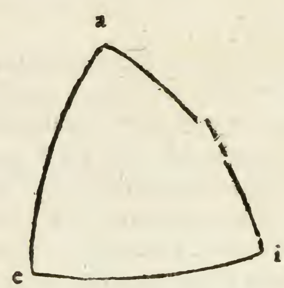
Et sic diversum aliquid hic habemus à triangulis planis. Illic enim tres æquantur duobus rectis 9. e. 6. R. hic majores sunt. Ergo non omne triangulum æquat suos angulos duobus rectis.

7. *Si triangulum est æquicrurum est in basi æquiangulum: & contra.* 41. 40. p. 3. Regiom.

His diu immorari non est necesse: Hæc ut communia sunt um planis: sic communem ha cent demonstrationem.

Vt hic duo anguli e & i sunt æquicruri & æquibases. Nam illic a e, e i hic a i i e crura æqualia, basis illic a i, hic a e ex esi æquales. Ergo per 1. c. 6. e. 3. R. anguli e & i sunt æquales.

Conversa similiter per absurdum patet ut 10. e. 6. R.

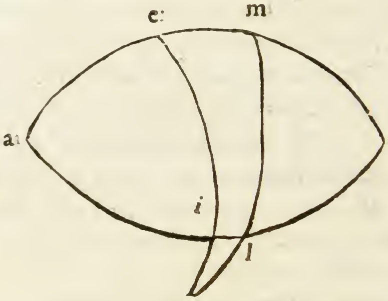


8. *Trianguli majus latus subtendit majorem angulum: & major angulus à majori latere subtenditur.* 43. 42. p. 3. Regiom.

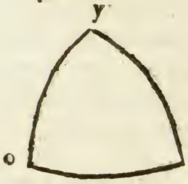
His diu immorari non est consultum. debebant alibi & generalius doceri. Demonstratio ex doctrina angulorum contexitur ut 11. e. 6. R.

9. Si duo triangula æquentur angulis aut duobus æquicruris, aut binis duorum æqualis cruris aut baſis, ita ut reliquorum duorū æqualium baſes ſemiperipheriæ inæquales ſint: ſunt æquilatera. 36.52.53.p.3.Regiom.

Et hoc maxima ex parte commune eſt planis. primæ partis & ſecundæ demonſtratio eadem eſt cum planis: nempe triangula æquicruro angulo æqua æquari baſi & triangula æqua binis angulus æqualis cruris adjacentis reliquis lateribus æquari. Quare inde huc transferantur.



Tertia pars ab illis aliquid diverſi habet. Sint ẽm duo triangula a e i, o y u. Sintq; anguli a & i æquales angulis o & u: æqualis baſis duorum nempe e i & y u. reliqua quoque latera a e, y o non ſint æqualia ſemiperipheriæ. Dico a e, o y & a i, o u æquari.



Sequitur ex ſecunda parte: hoc demonſtrato quod angulus a e i ſit æqualis angulo o y u. id ẽm ſi demonſtretur jam angulorum e & y, i & u æqualium æqualis cruris e i & y u reliqua latera a e, o y & a i, o u erũt æqualia. Demonſtretur itaque. Continuetur ergo a e & a i in s. fient ſemiperipheriæ. jam ex a s ſcindatur s l æqualis ipſi o u: & ex a e s auferatur s m æqualis y o. Diſtant igitur a & m. Nam ex theſi o y hoc eſt s m & æ non æquantur ſemiperipheriæ. Et y u & m l æquantur ex pri-

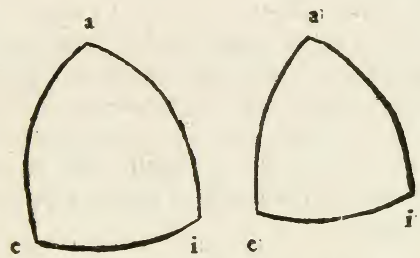
ma

ma parte. Deinceps e i & m l concurrant in n. Cum itaq; anguli i e & s l m æquentur. Nam eidem ad u æquantur illic ex thesi. hic vero quemadmodum mox patebit: erunt anguli trianguli i n l ad i, l æquales, & n i, n l æqualia latera per 7 e. Et cum e i & m l æquentur. nam æquantur eidem nempe y u illic ex thesi: hic ex prima parte. Ergo additis æqualibus n e, n l æquabuntur & anguli n e m, n m e per 7 e. proindeq; reliqui de æqualibus nempe duobus rectis a e i, l m s, hoc est ut patebit jam o y u. His jam æqualibus per secundam partem a e, o y & a i, o u æquabuntur.

10. *Triangula æquilatera sunt æquiangula: & contra.*

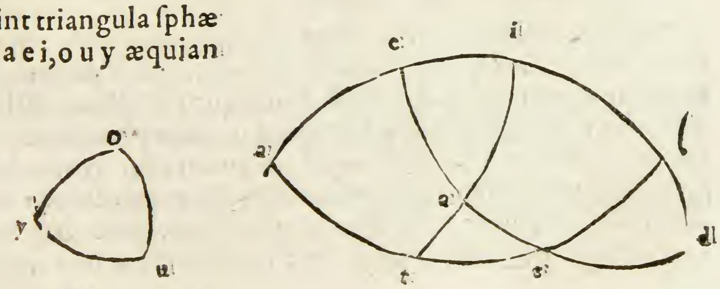
36.54.p.3. Regiom.

Propositio cum planis ut communis est sic communem cum iis demonstrationem habet ex 1.c.6.e.3.R. ut hic si a e & a e item a i & a i æquentur & sit basis e i basi e i æqualis: anguli ad a æquabuntur, sic anguli ad e & i. eadem ratiocinatione.



Conversa sphæricis peculiaris est. Nec enim plana triangula æquiangula semper sunt æquilatera. Ejus ergo demonstratio apud Regiomontanum hæc est.

Sint triangula sphærica a e i, o u y æquian



gula: nempe o & a, i & u, e & y æquales. Continuentur ergo i a & e a in t & s, ut a t & o u, a s & o y æquentur per 15 e 2. Erunt proinde t s & y u æquales. Nam angulus e a i hoc est y o u æquatur angulo verticali t a s. Et crura a t, o u & a s, o y æquantur. Ergo per 9 e bases æquantur t s & y u. Et proinde per primam hujus partem triangula sunt æquiangula t a s & y o u. Nempe anguli s & y, t & u. Sed angulo u æquatur angulus a i e, & angulo y æquatur angulus i e a. Ergo angulus a s t angulo i e a. & angulus a t s angulo e i a est æqualis.

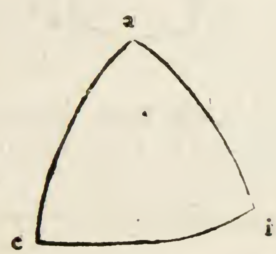
His jam datis continuentur e i, t s ut concurrant in m & l. dico e l & l s æquari semiperipheriæ itemq; i l & l t. quod facilè patet. Nam si concipias angulum l e s æqualem a s t. & continues e s, e l donec concurrant in d. fient anguli l d s & l s d æquales. cum æquentur æqualibus d quidem angulo ad e per s e. s vero suo verticali: erit & latus l s lateri l d æquale per 7 e. itaq; cum e l, l d sit semiperipheria. erit & e l, l s similiter semiperipheria. Et id est quod Regiom. 48. p. 3. demonstrat. Eadem ratione i l, l t semiperipheriæ æquabuntur. Quare ab æqualibus semiperipheriis ablata communia i l, l s, relinquuntur æqualia t s, hoc est y u & e i. Et proinde per 9 e. triangula hæc æquiangula cum æquentur crure duorum æqualium y & e, i & u æqualibus e i, y u erunt æquilatera.

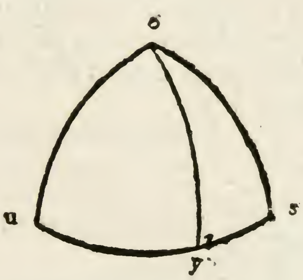
11. Si duo triangula æquantur angulorum æqualium duorum, duorum verò similiter obliquorum basibus: sunt æquiangula. 55. p. 3. Regiom.

Similiter obliquorum. intellige ut uterq; aut acutus aut obtusus sit. Nescio sanè quid p. Nonius hic in Menelao desideret. ait Menelaum aberrare quod dicat duos, quos similiter obliquos dedimus, non rectos. posse enim fieri ut alter obtusus reliquus acutus sit. & tum triangula æquilatera non esse. Non arbitror ipsum Menelao aliquid affingere: sed credo exemplorum discrepantiam hæc parere. ipse citat 13. p. 1. Maurolycus inde habet 22. p. 1. Regiomontanus 55. p. 3. Hic conditionem non omittit.

Nec

Nec Maurolycum, si rectè memini omisisse puto. Nescio quale ipse exemplar habuerit. Quod si litera ut refert habuit. Mathematicè certè fecit acrius propositiones examinans: & benigne fecit, benigne authorem interpretans.



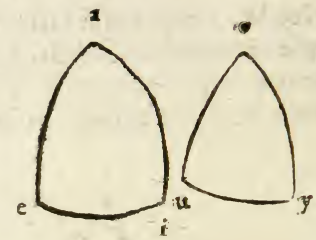


Sint itaque duo triangula a e i, o u y: & æquentur anguli duo e & u æqualium basium a i, o y. rursus anguli i & y sint similiter obliqui nempe acuti aut obtusi & subtendantur ab æqualibus basibus a e, o u. Dico triangulorum reliqua latera e i & u y æquari. Secus si non æquentur sit e i majus. & protrahatur u y in s ut u s æquetur e i. Ergo cum anguli e & u æquentur & e a, u o, itemque e i & u s, ex adversarii thesi, per 9 e a i & o s æquabuntur, & anguli i & s. Sed arcui a i æquatur ex nostra thesi o y. quare o y & o s æquabuntur & per 7 e anguli ad s & y æquales erunt: & quidem cum o s y sit acutus: est enim acuto i æqualis: etiam o y s erit acutus: & proinde reliquus de duobus rectis o y u erit obtusus. At hoc contra thesin elementi est: qua acutus dabatur: Ergo thesis adversarii falsa fuit.

Eodem modo si i & y dentur obtusi propositio sibi constabit. Sunt itaq; triangula æquilatera: Ergo & æquiangula.

12. Si triangulum triangulo æquicrurum est majus basi: est majus angulo: & contra. 51.50.p.3.Regiom.

Sint



Sint enim triangula duo sphę-rica a e i, o u y æqualium crurum a e, o u & a i, o y. basis autem e i majoris basi u y. Erit angulus a major angulo o. Et si hoc: nempe si angulus a æquetur angulo o. quod ad crura & tamē a major jor sit quam o. erit e i basis major quam u y.

Est & hoc commune cum planis 4.e.7.R. Et quemadmodum veritas in planis è 3.c.6.e.3.R. deducitur: sic eodem argumento hæc angulorum ac basium inæqualitas patet. Et sic Regiomontanus in demonstratione hujus elementi nos ad 25 & 24.p.1. Euclidis remittit.

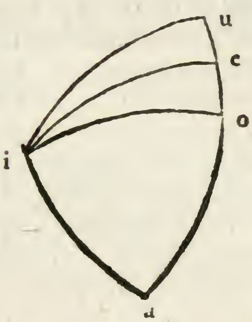
Atq; sic habuimus quæ generalia erant, & triangulo sphærico simpliciter & triangulis comparatè: deinceps de triangulorum speciebus agendum erit: Et primo quidē de rectangulo. Rectangulum autem hic ut & obtusangulum non astringitur ad rectum unum, unumq; obtusum. fieri enim potest ut in eodem triangulo plures sint. Et quidem tres recti rectanguli esse possunt: geodæsia tamen & calculus unum rectum plerunq; postulat.

13. Si crus recti quadrans sit, angulum subtendit rectum: quadranti inæquale similiter recto inæqualem angulum subtendit: & contra. 3.p.4. Regiom.

Similiter inæqualis angulus dicitur ut recto major aut minor prout crus recti majus aut minus quadrante est. Quadrantem autem voco pro peripheria quadrantis.

Esto ergo triangulum rectangulum recti anguli ad a. sitq; crus ejus a e quadrans. dico angulum ad i esse rectum. Est enim per 17 e 12. e polus circuli i a. jam per polum e secat circulum a i. Ergo per 16 e 12 rectè secat. & proinde angulus i rectus est. Quod si a e majus crus sit quam quadrans erit angulus i obtusus. Nam fiat polus in o. circuli i a & per polum secet circulus o i. fiet angulus

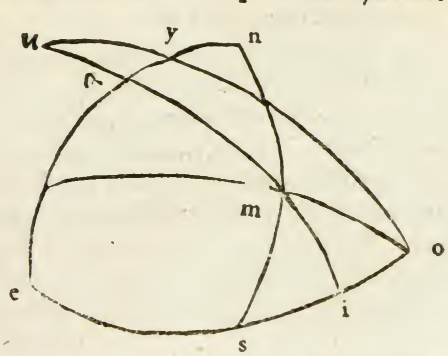
lus o i a rectus.qui minor eſt angulo e i a dato. Eodem modo ſi e a ſit minus crus quadrante:fiet u i a rectus: & proinde major quam e i a.



Converſa ſimiliter patet. Si enim angulus e i a rectus eſt: e a erit quadrans. Quia enim i e rectè ſecat circulum i a: per 16 e 12. per polum ſecabit. Eodemque modo cum ex theſi e a rectè ſecet circulum etiam per polum ſecabit. itaq; concurrent in polo per 18 e 12. Quare erunt quadrantes maximorum circulorum. Eodem planè modo probabis e a ſuperare quadrantem ,ſi angulus i obtuſus ſit: ſuperari ab eo,ſi acutus. Nam ut illic o a erit quadrans & tum minor quam a e:ſic hic a u erit quadrans major quam a c.

14. Si trianguli rectanguli crus unum quadrans fuerit, & baſis quadrans erit:ſin crura quadranti ſimiliter inæqualia fuerint:baſis quadrante erit minor,ſi diſſimiliter:baſis erit major.& contra. 4.5.p.4. *Regiom.*

Similiter inæqualia intellige ut crus utrumque aut majus aut minus quadrante ſit.diſſimiliter ſi crus unũ majus reliquũ minus quadrante extiterit.



Eſto itaq; triangulum rectangulum a e i. recti i cujus crus a i ſit quadrans. dico a e eſſe quadrantem. Nam per 13 e. angulus e eſt rectus. Quare perpendiculares

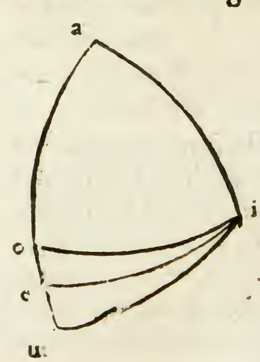
i a & e a circulo e i in ejus polo concurrent per 18 e 12. Ergo a e & a i sunt quadrantes per 17 e 12.

Rursus crus utrumq; a i, e i inæquale sit quadranti. erit e a basis minor quadrante. Sint autem crura primò minora. Continuetur ergo e i in o. ut e o sit quadrans. & i a continuetur in u. ut i u fiat quadrans. tum per u & o sit peripheria maximi sphæræ circuli. in eam continuetur e a in puncto y. Quia itaque u i quadrans perpendicularis est circulo e i. erit polus e i circuli in u per 17 e 12. & per idem angulus y o i rectus. jam cum ex fabrica e o sit quadrans, erit per primam hujus partem e y quadrans. at e y est basis major quam a e basis data.

Deinceps crura sint majora quadrante. Tum secentur quadrantes e s, i m: tum per s & m magnus circulus secet circulum e a in n. jam quia angulus i rectus est ex thesi & i m quadrans. erit per 17 e 12 m polus circuli e s i & angulus ad s rectus per 16 e 12. & cum e s sit quadrans erit per primam partem hujus e n peripheria quadrantis major basi dati trianguli.

Sint deinceps crura inæqualia quadranti dissimiliter & i a majus, i e vero minus. Erit a e basis major quadrante. Continuando secandoq; crura itaq; fiant quadrantes e o, i m: & per o ac m sit maximi circuli peripheria o t in basin. Hic ut ante m erit polus circuli e o. & quia per eum t o secat circulum e o per 16 e 12. angulus t o e erit rectus. & quia e o quadrans: trianguli o e t basis e t quoq; quadrans erit, minor quam a e basis dati trianguli. Conversa quoq; facilis est.

Sit enim trianguli a e i rectanguli ad e basis i a quadrans, dico alterum crus esse quadrantē. polo enim a in i describatur circulus qui erit maximus: cum a i ex thesi sit quadrans. jam si transeat p r e constat a e esse quadrantem per 18 e 12. Quod si non secet circulum a u per e secet sane alibi aut supra e in o aut infra in u. Si in o. itaque a o, o i rectè secantur



cantur per 16 e 12. Et a e perpendicularis est ad e i ex thesi. Quare i est polus circuli a e per 17 e 12. Quare per idem e i crus alterum nempe trianguli erit quadrans.

Eodem modo colliges e i esse quadrantem si polo a in i. descriptus circulus secet a e in u.

Rursus si sit a i minor quadrante crura erunt aut majora eo aut minora. Si sit major quadrante erit crurum alterum majus alterum minus ut simili ratiocinatione facilè colliges, sed impossibile Regiomontanus adhibet, ut & Euclides plerasq; conversas demonstrat. Nam si a i est quadrans erit crurum e i, e a alterutrum quadrans. secus si non sit: erit utrumque aut majus aut minus: aut alterum majus alterum minus quadrante: At si majus, si minus utrumque, basis a i minor quadrante: si unum majus reliquum minus a i erit major quadrante. At hoc est contra thesin.

Rursus si a i basis sit minor quadrante: erit crus utrumque aut majus aut minus. Secus si non sit erit unum quadrans aut unum majus reliquum minus quadrante. Quo posito a i illic erit quadrans: hic major quadrante per ante demonstrata. At a i nec quadrans nec major quadrante ex thesi: Ergo &c.

Eodem modo si a i basis sit major quadrante, alterum crus quadrante majus alterum eo minus erit. Nam si non sit: basis a i aut erit quadrás aut eo minor, ut jam patuit. At nec quadrás nec eo minor. Ergo crus alterũ quadrante majus, reliquum minus.

Itaq;

15. Si trianguli rectanguli angulorum in basi alter rectus sit, basis quadrans erit: sin uterq; æqualiter obliquus, basis quadrante minor est: sin inæqualiter obliquus, major est: & contra. 6.7.p.4.Regiom.

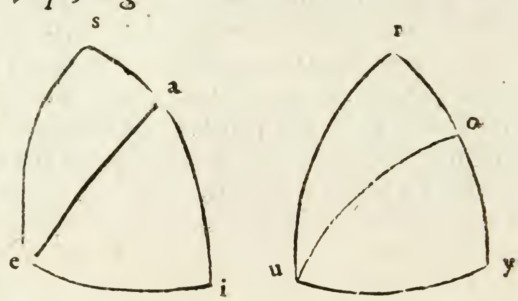
Hoc elementum ex præmissis duobus sequitur. Nam si trianguli rectanguli crus unum quadranti æquatur, basis quadrans est per 14 e.

At si trianguli rectanguli angulus in basi rectus est crus recti, recto in basi oppositum est quadrans. 13. e.

Ergo si trianguli rectanguli angulus in basi rectus est, basis est quadrans.

Sic reliquæ ejus partes ex præmissis duobus elementis syllogismi complexiones sunt. Et novis hic elementis nō esset opus: dato enim antecedente syllogismi: vel non enunciata complexio vera est & intelligitur.

16. Si duo triangula rectangula crurum recti sigillatim quadrante minorum æquentur binis lateribus: sunt æquilatera. *56.p.3.Reg.*



Sint enim rectangula a e i, o u y. rectos habentia ad i & y & e i, i a rursus u y, y o. sigillatim quadrante sint minora. jam bina latera æquētur. Erūt ea aut crura: aut crus recti & basis n utroq; triangulo. Æquentur ergo primo e i, u y & i a, y o. per 9 e æquabitur e a & u o. Quare triangula erunt æquilatera.

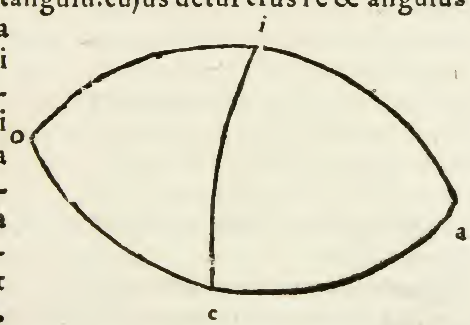
Secundo e a, u o, & a i, o y æquentur. dico e i & u y æquari. Continuentur enim i a, y o in s & r ut fiant quadrantes s i, r y. erunt in s & r circulorum subjectorum e i, u y poli per 16 e 12. Rursus ab s & r descendant perpendiculares quadrantes s e, r u æquales: sunt enim quadrantes æqualium scilicet maximorum circulorum. Sed & a s æquatur arcui r o. sunt enim complementa æqualium i a, y o. & e a, u o ex thesi æquantur. Quare triangula æquilatera cum sint erunt æquiangula & s a e, r o u æquales anguli: & reliqui e a i, u o y de æqualibus nempe duobus rectis. jam in triangulis e a i. u o y anguli æquicruri æquales sunt. Ergo per 9 e. bases e i, u y æquabuntur.

Sic de triangulo rectangulo hactenus diximus: quod ut nec ad quadrantem unum sic nec rectum angulum unum est redactum

ctum. pro lateribus tres habere quadrantes poteſt & tum rectos tres habebit. aut duos habet quadrantes & tum totidem rectos: aut nullum habet quadrantem & tum tantum unũ rectum obtinet: & latera omnia quadrante aut minora ſunt, aut duo majora tertium minus: Et illic, reliqui duo anguli erunt acuti: hic vero alter acutus alter obtuſus: ſi modo rectus uni inæqualium laterum opponitur. Si enim rectus ſubtendatur à baſi minore quam ſit quadrans, anguli reliqui ſunt obtuſi. ut jam ante per aliquot elementa patuit.

Et hinc jam patebit quot in triangulo rectangulo data eſſe oporteat ut tertius aliquis & optatus terminus inveniatur. Certè conſtat datis præter rectum angulum quibuslibet duobus terminis tertium inveniri poſſe: abſque alia aliqua laterum aut angulorum determinatione. Si tamen ex dato crure recto & angulo oppoſito baſin inveniendam poſuerimus: ſit'ne ea futura major an minor quadrante præſciamus oportet. id itaq; ex theſi dabitur: Calculus enim hoc non judicabit, poteſt enim fieri ut crure minore aut majore baſis tamen major aut minor ſit. Ita ſi dato crure cum angulo oppoſito crus reliquum inveniendum ſit. aut angulus reliquus, ut in ſequenti figura vides.

Eſto triangulũ i e a rectangulũ. cujus detur crus i e & angulus a. & ſit baſis i a minor quadrante niſi eñ hoc conſtiterit dubiũ eſt: ſit ne aſſumẽda o i an i a. Quia in triangulo i o e eadẽ data manent. crus i e cõmune. angulus o ęqualis angulo a p 5 e 13. & ſinus i a itẽ tangẽs ac ſecãs conveniunt cũ dictis baſis o i p 8 e 5.



Eodem modo ſi his datis velis inquirere crus alterum e a. niſi definias ſit'ne majus aut minus quadrante: calculus id non inveniet. Nam trianguli i e a & i e o communia data ſunt. ſed unius

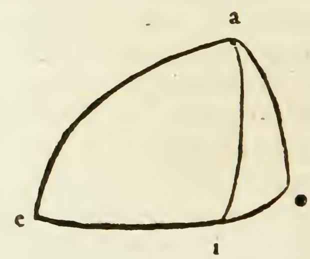
crus reliquum majus alterius minus quadrante existit:communi etiam sinu,tangente,secante.ita si iisdem datis angulum i investigare velis nisi de ejus specie constet calculus eum non definiet. Nam duorum triangulorum data sunt communia & tamen angulus i unius est obtusus,alterius acutus & utriq; interim respondet idem sinus,secans eadem atq; etiam tangens.

Sequitur de obliquangulo triangulo: quod ut in planis est obtusangulum aut acutangulum.jam fieri potest ut obtusangulum tres obtusos habeat,duos,etiam unum tantum. acutangulum itaq; id tantum erit quod tribus est acutis donatum. De his quædam sunt apud Regiom.9.10.11.12.13.14.p.4 .Quæ tamen ex rectangulis cognosci possunt.

Ne tamen quis hic quicquam desideret:breviter ea subiiciam: demonstranda tamen cuiq; ex superioribus modo jam prædicto relinquam:præmisso hoc elemento.

17. *Si triangulum similiter in basi est obliquangulum: perpendicularis arcus à vertice cadit intra triangulum: extra si dissimiliter 8.p.4.Reg.*

Esto enim triangulum a e i in basi similiter obliquangulum nempe ut e & i aut acuti sint aut obtusi.Dico perpendicularem arcum ab a cadere in basin e o intra triangulum. Secus: si non cadit intra,coincidit lateri aut extra cadit. At neutrum. Nam si coincideret lateri. fieret angulus ad o exempli gratia rectus: quod est contra thesin.



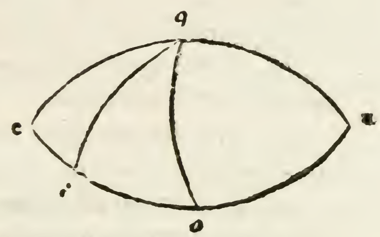
Quod si extra cadat:verbi gratia ad o basis continuatæ,erit a o latus commune duobus triangulis rectangulis.jam in triangulo rectangulo a e o. quia angulus e.exempli gratia est acutus per 13 e.a o. arcus perpendicularis erit minor quadrante.Rursus quia a i e est acutus ex thesi.reliquus

liquus ad duos rectos a i o erit obtusus.itaq; a o per 13 e erit major quadrante. & proinde major & minor eodem: quod est absurdum.

Ergo perpendicularem dicto modo angulis in basi affectis extra cadere est imposibile.

Regiomontanus ut primam partem induxit: sic secundã collegit:nempe perpendicularem non lateri concidere nec cadere intra:si scilicet angulorum unus in basi acutus reliquus obtusus sit:proindeq; cadere extra. Potest tamen & hoc modo ex prima parte colligi.

Sit enim rursus triangulum a e i,& angulorum in basi e sit acutus,reliquus vero i obtusus.Dico perpendicularem à vertice a. cadere extra datum triangulum.



Concurrant enim e a,e i in u. Erit angulus u trianguli a i u æqualis angulo e dato per 5 e.& proinde acutus.

Deinde quia angulus e i a est obtusus reliquus a i u erit acutus. Ergo cum anguli i & u in triangulo a i u sint ejusdẽ speciei nempe acuti: perpendicularis cadet intra triangulum factum.& proinde extra datum triangulum.

Sed hoc est syllogismi demonstrare complexionem: quæ tamen præmissis datis necessario concluditur.Quare syllogistico judicio hæc pars contenta erit isto.

Si perpendicularis à vertice trianguli cadit intra: anguli in basi specie iidem sunt.ex prima parte.

Adde assumptionem quam thesis adfert: At si angulorum alter acutus alter obtusus sit:anguli specie non sunt iidem.

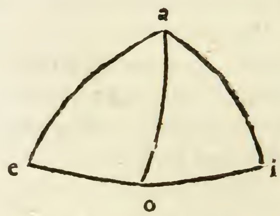
Ergo si anguli in basi disimiliter sint obliqui: perpendicularis non cadit intra. Ergo extra. Nam lateri coincidere nequit,ut in prima parte est demonstratum.

Hoc

Hoc itaque elemento expoſito: ſic de obliquangulis Regiomontani ſunt theoremata.

18. Trianguli acutanguli latera ſigillatim ſunt minora quadrante. 9.p.4.R.

Eſto triangulum acutangulū, hoc eſt trium acutorum angulorum a e i. deſcendat arcus perpendicularis a o. fient triangula a o i, a o e rectangula. jam utrinq; cum ex theſi o a i, & angulus i. item o a e & ang. e. ſint acuti: per 13 e. o a & o i item o a & o e ſigillatim ſunt minora quadrante. Quare per 14 e. & a i & a e ſigillatim ſunt minora quadrante. Eodem modo de e i conſtabit ſi ex alio angulo cadat perpendiculares. Et ſic ſequentes propoſitiones induci poterunt, quod monuiſſe ſatis ſit.



19. Si trianguli duo acuti æquales ſint: æqualium baſes quadrante ſunt minores. 10.p.4.

20. Si trianguli duo acuti inæquales ſint: minores baſis quadrante minor erit. 12.p.4.

21. Si duorum acutorum alterius baſis non ſit minor quadrante: tertius angulus obtuſus erit ejusq; baſis major quadrante. 14.p.4.

Baſis poſtulatur non minor quadrante: ſed aut ei æqualis aut major. Et demonſtratio facilis eſt ex ſuperioribus. Si enim in præmiſſo ſchemate a i ſit quadrans jam aut a o aut o i erit quadrans per 14 e. ſed a o non eſt per 13 e. Ergo o i & proinde e i major & angulus e a i obtuſus. Quod ſi a i ſit major quadrante: erit, quia a o minor, per 14 e o i major &c.

22. Si trianguli duo obtuſi æquales ſunt: æqualium baſes quadrante ſunt majores. 11.p.4.

23. Si trianguli duo obtuſi inæquales ſint: baſis majoris erit major quadrante. 13.p.4.

Hæc ſunt quæ Regiom. habet de obliquis angulis. quæ ex rectangulis huc tamen redundant. TH.

TH. FINKII GEOMETRIAE ROTVNDI,

LIBER DECIMVSQVARTVS.

De calculo triangulorum sphæricorum.

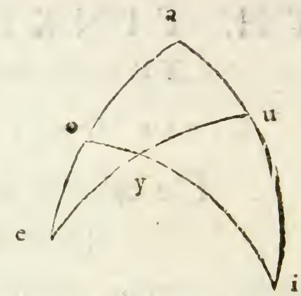
PErvenimus tandem Dei gratia ad id quod inter cætera præcipuè nobis propositum fuit: pervenimus, inquam, ad doctrinam calculi triangulorum: quæ tam est latè dissemita ac diffusa: quam latè cœlum & terra patent. Majori itaq; animi alacritate, studio & diligentiâ summâ ea erit perdiscenda: si modo ad cœlum evolare, arcus globi terreni metiri bene voluerimus: si deniq; hinc in omnia vitæ genera redundantem usum atq; fructum cognoscere cupiverimus.

Calculus erit facilis & expeditus: deductus & ex lemmatis Ptolemęi cyclicis auxilio tabularum Directionū Erasmi Rheinholdi, & ex Regiomontano nostro. Quare primo Ptolemæum sua docentem è Menelao paulò plenius audiamus.

1. *Si quatuor peripheriarum maximorum sphæræ circulorum sigillatim semiperipheriâ minorum, duæ faciant angulum reliquæ ab harum terminis reflexæ se & priores secent: ratio sinuum unius ad sinum sui segmenti vel segmentorum inter se fit è ratione sinuum ita conterminarum peripheriarum, ut prima facientium conterminetur principio antecedentis factæ, secunda hujus consequentis fini conterminata terminetur in finem consequentis factæ.*

Sint quatuor peripheriæ maximorum sphæræ circulorum sigillatim semiperipheria minores, quarū duæ e a, i a faciant angulum, reliquę sint in se reflexę ab i & e secantes priores in o & u

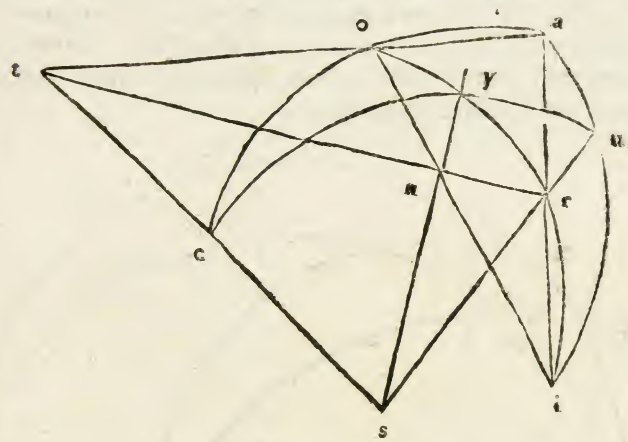
ſe ipſas vero in y. Hinc jam variè ratio ſinuum arcuum particularium componitur: prout in elemento eſt indicatum: Et ſequens quorundam caſuū deſcriptio oſtendit: ubi tamen pro ipſis arcubus, arcuum ſinus intelligantur: Et antecedentibus rationum terminis conſequentes ſubjecti intelligantur. Et ſunt prima duo exempla Ptolemæi.



	Facientes.		Facta.	
A.	i y	o e	i u	u a
C.	y o	e a		
		I I.		
	i o	y e	i a	a u
	o y	e u		
		I I I.		
	a e	o y	a u	u i
	e o	y i		
		I I I I.		
	a o	e y	a i	i u
	o e	y u		
		V.		
	i u	a e	i y	y o
	u a	e o		
		V I.		
	o a	e u	o i	i y
	a e	u y		
		V I I.		
	i a	u e	i o	o y
	a u	e y		

Et

Et ſimiles caſus in reliquis duab. peripheriis colligere potes.
Ptolemæus primum ſuum exemplum ſic demonſtrat.
Sint inſcriptæ arcuum o a, a i, i o, Et ſit centrum ſphæræ s ducanturque radii, s u quidem ſecans i a in r: & s y ſecans i o in

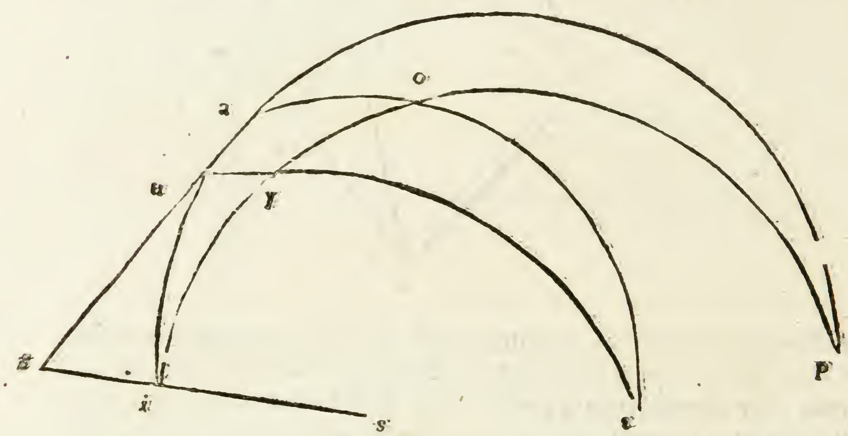


n: & s e. Continuatus concurrat cum a o inſcripta continuata in t. Erit tum s r u pars communis ſectionis a u i & u y e. Recta autem s n y circulorum u y e & i y o. Recta vero s e t circulorum u y e & e o a. Porro inſcriptæ o a, a i, a o t ſunt ſimul in triangulo a o i. Suntq; communes ſectiones trianguli dicti cum dictis circulis. itaque cum r, n, t, ſint in dicto triangulo & circulo u y e. erunt in communi ſectione dictorum planorum trianguli nempe atque circuli hoc eſt in una recta. Nam plana terminantur linea: ergo linea ſecantur per c. 5. e. 1. R.

jam cum quatuor rectæ ſint duæ facientes angulum a t, a i. reliquæ ab harum terminis i & t reflexæ nempe i o, t r ſecantes ſe & priores: per 15. e. 5. R. ratio i r ad r a, hoc eſt per 13 e 5. ſinus arcus i u ad ſinum arcus u a, ſit è ratione i n ad n o, hoc eſt per dictum elementum è ratione ſinus arcus i y ad ſinum arcus y o, & ratione o t ad t a, hoc eſt per 14 e 5 è ratione ſinus arcus o e ad ſinum arcus e a.

Ptolemaicam demonſtrationē omnibus inductionibus non ſufficere arbitratus eſt Thebitius Arabs Corę filius: ſic enim Ben corat vocant: itaq; eam auxit. ut refert Maurolycus ad 1.p.3. Menelai: qui etiam adfert Thebitii lemmata & demonſtrationes.

Rectas itaq; a o, s e non ſemper concurrere in partem averſam peripheriis quarum rationes fiunt, ſed aliquando in eam ipſam, imo aliquando æquidiſtare exiſtimat. Et tum Ptolemęi demonſtrationem non ſufficere arbitratur.

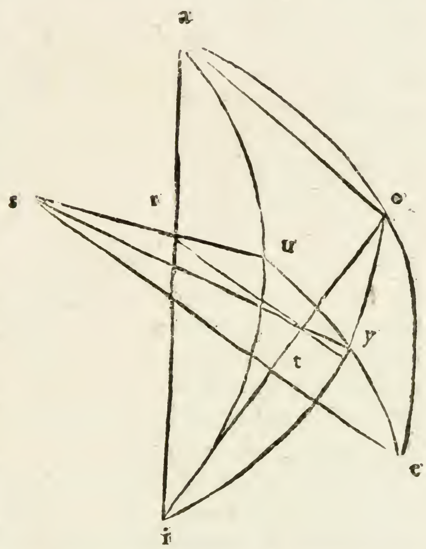


ut in hac figura ſit centrum s. Hic ſi a o continuare velis in dextram partem nunquam concurret cum radio: at verſus ſiniſtram a u coincidit cum eo ad punctum t. itaq; per Ptolemæi demonſtrationem ratio ſinus e o ad ſinum o a componitur è ratione e y ad y u, & u i ad i a.

Reliqua pertexit ex lemmatis ſuis & modis ſex quantitatum: hoc eſt per multiplicationem proportionum: Nec quicquam modi ſunt aliud quam exempla iſtius multiplicationis. Sed reliqua ſic pertexentur: inverſis prioris proportionis terminis: & completis ſemicirculis i a p, i o p. Ratio ſinus arcus i a, hoc eſt ſinus p a per 8 e 5 ad ſinum a u, fit è ratione ſinus arcus i o, hoc eſt

p o ad

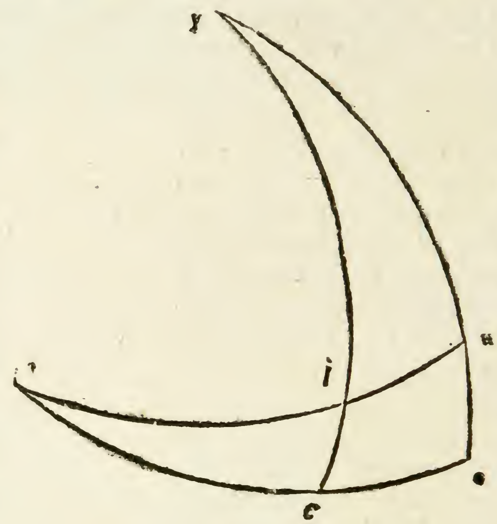
p o ad ſinũ arcus o y, & ratione ſinus arcus y e ad ſinũ e u. Quod Ptolemæi primum eſt exemplum: at dices quis dedit i a ad a u ſic fieri? Secundum Ptolemæi exemplum id habet. itaq; ſpecialibus ejuſdem generis omnibus ſatisfeciſſe Ptolemæum credo. Nam ſi radius & inſcripta dicta parallelæ ſint demonſtratio cuivis obvia eſt: ut hic ſi a o & s e ſint parallelæ. Quia itaq; ſunt in eodem plano nempe circulo a o e: & rurſus a o atque r t ſunt in eodem triangulo a i o. itemq; r t & s e in eodem circulo u y e: & harum duæ a o, s e ſunt parallelæ ex theſi. Ergo utraq; parallela eſt ad tertiam r t docente id 6.e.21. R. itaq; per 13. e. 5. R. ut i r ad r a, hoc eſt ſinus arcus i u ad ſinum arcus u a, eſt ut i t ad t o, hoc eſt ut ſinus arcus i y ad ſinum y o. jam ſinus o e & a e æquantur per 8 e 5. Ergo ratio ſinus i u ad u a fit è ratione ſinus arcus i y ad ſinum y o & ratione ſinus arcus o e ad ſinum arcus a e. Nam ratio hæc ultima ſinuum æqualitatis eſt, ideoque rationem non mutat.



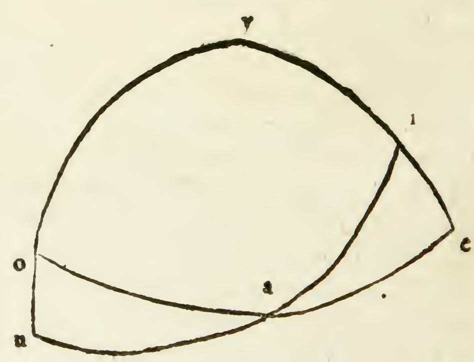
Secundum ſuum exemplum Ptolemæus demonſtrat ad primi modum. in quo eadem Thebitius deſiderat. At nos Theonis demonſtratione contenti ſumus quam paulo ante attulimus. ut in ſecundo ſchemate ratio p a ad a u hoc eſt i a ad a u fit è ratione p o, hoc eſt i o ad o y, & ratione y e ad e u: Et hæc demonſtratio omnibus caſibus hujus generis exemplorum convenit. Reliquos caſus ex hactenus dictis ſimiliter expedies.

2. Hinc quatuor conſequuntur. primum: Si maximus ſphæræ circulus maximum ſecet, & à ſecante duo arcus perpendiculares reliquo ducantur: ſinus ſegmentorum ſecantis à ſectionis puncto directè ſunt proportionales ſinibus arcuum perpendicularium. 15.p.4.Regiom.

Eſto ſecet maximus ſphæræ circulus u a, maximũ o a. ſitq; terminus interſectionis peripheriarum ad a. Et à ſecante ſint duo arcus i e, u o perpendiculares ſecto. Dico ſinum u a ad ſinum arcus a i eſſe ut eſt ſinus arcus u o ad ſinum arcus i e. Continuentur enim & concurrãt perpendiculares ad y. erit illic per 18 e 12 polus circuli ſecti o a. Quare per 17 e 12 y o, y e ſunt quadrantes & proinde æquales, & quia æquales, æquales ſinus habent, nempe radium ut ante patuit.



Hinc conclude.



Ratio ſinus u a ad ſinũ arcus a i fit è ratione ſinus arcus. u o ad ſinũ peripheriæ o y. & ratione ſi-

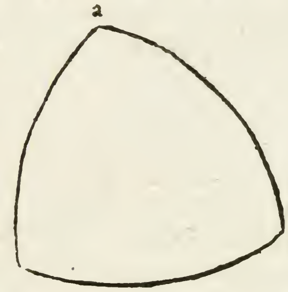
nus arcus y e ad ſinum e i. At hæ inter ſe multiplicatæ ſunt ratio ſinus arcus u o ad ſinum arcus i e. per multiplicationem rationum. heterologi enim æquales ſunt. nempe ſunt radii.

Sin. u o. Radius
Radius ſin. e i. u a. a i.

Ergo hinc jam ſequitur elementum ipſum. cujus vires in calculo triangulorũ multæ ſunt: maximæ in calculo rectangulorum: quorũ ea Geodæſia hæc recipit quæ unum habent rectum.

In ſequentibus autem demonſtrationibus nos ejuſmodi adhibebimus triangulum cujus crura recti ſint minora quadrante, & proinde anguli in baſi acuti. ut triangulũ a e i. cujus angulus e rectus ſit.

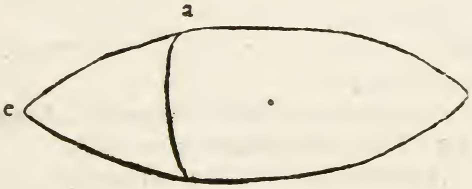
a



e i

Et quicquid de ejuſmodi triangulo demonſtratum fuerit: uerum erit etiã ſi crura ſint quadrante majora, proindeq; anguli in baſi obtuſi. ut hic quod de ſiniſtro triangulo fuerit monſtratũ: id verum erit de dextro. Nam cruris e a illic & hic, ſinus idẽ eſt: item e i illic & hic. Sic angulorum ad a & i acutorum & obtuſorum ſinus iidem ſunt per 8 e 5.

a



e c
i

Eodemq; modo ſi crurũ illud quadrantem ſuperet hoc vero à quadrante excedatur. Quia ſinus, ut jam dictum eſt, & proinde etiam tangentes ac ſecantes non mutantur. Quare jam matris fœcundæ filias amabiles videamus.

Itaq; in triangulo rectangulo.

3. Radius eſt ad ſinum baſis, ut ſinus anguli ad ſinum cruris dicto angulo ſubtenſi. 16.p.4.Regiom.

In

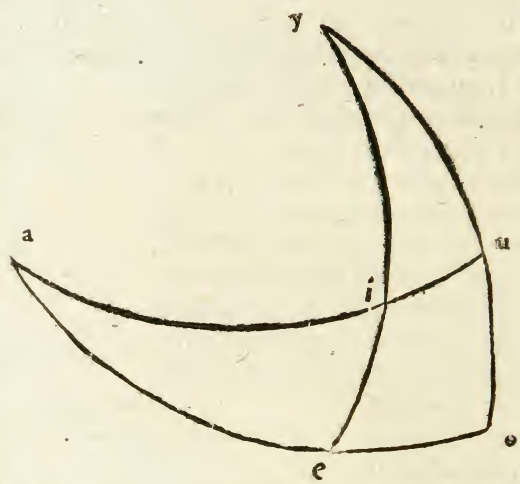
In rectangulo crus recti ut & baſis recti intelligitur.

Eſto triangulum rectangulum i e a.

Dico eſſe proportionales

Rad. ſin. a i. ſin. a. ſin. i e.
Rad. ſin. a i. ſin. i. ſin. e a.

Facto enim polo in a, deſcribatur circulus maximus a u y. cũ quo cócurrat e i in y. Erit per 18 e 12 ad y polus circuli o e a. Sed & a o ſecet circulum y o ad o. & a u in u. Erit ergo primo u o. amplitudo anguli a per 4 e 13. & ſecundo u o perpendicularis



ad o a per 16 e 12. cui etiam perpendicularis eſt ex theſi i e. Tertio a u, a o ſunt quadrantes per 17 e 13.

Jam per prædictum elementum

ut ſinus u a hoc eſt ex fabrica radius (Eſt enim quadrans) ad ſinum arcus a i, hoc eſt baſis dati trianguli, ſic ſinus u o, hoc eſt ſinus anguli a ex fabrica ad ſinum i e cruris nempe dicto angulo ſubtenſi. Eodem modo res patebit de angulo e i a. & ei oppoſito crure e a.

Hinc jam patet inventio horum trium terminorum baſis, anguli cum crure oppoſito, datis duobus: idque multiplicationis aut diviſionis via. Verum ut multiplicationis ratio tanquam facilior retineatur: adhiberi tantum elementum in expreſſo caſu ſatis eſt: reliquis enim ſua etiam erunt elementa: ut radius primus aureæ regulæ terminus exiſtat. Sunt

Sunt ergo hujus elementi data præter angulum rectum basis recti & angulus. inventũ erit crus recti angulo dicto oppositum. Et ne res exemplo careat:

Detur basis i a gr. 30.

Angulus a. gr. 23. 28′. Quantus est minimæ inclinationis Zodiaci ad Æquatorem Copernico.

Quæritur i e. arcus nempe quem Astrologi vulgo declinationem vocant puncti Zodiaci i ab Æquatore.

Ex canone i a sinus datur 5,000,000. sinus vero anguli a. 3,982,155. Ergo sinus i e invenietur 1,991,077½.

Rad.	i a	a.	i e
2.	1.	3,982,155	1,991,077½.

Sinus inventus in canone sinuum monstrat arcum gr. 11. 29′ 5″. ut etiam in tabula sua Rheinholdus annotavit. Eodem modo si angulus a esset obtusus gr. 156. 32′. sinus quidem maneret & inveniretur idem. verũ e i non amplius esset gr. 11. 29. 5. Sed 168. 30′, 55″, major scilicet quadrante per 13 e 13. & per 8 e 5. reliquus ad semiperipheriam arcus gr. 11. 29. 5.

4. Radius est ad sinum complementi cruris, ut sinus complementi reliqui cruris ad sinum complementi basis.

19. p. 4. Regiom. 3. theor. Gebri.

ut retento triangulo nostro sunt proportionales

Radius. sin. compl. i e. sin. comp. e a. sin. comp. i a.

Manente enim fabrica prioris figuræ erit u i complementum basis. i y complementum cruris i e, e o complementum cruris a e. Sunt enim ex præmissa fabrica y e, a u, a o quadrantes. At vero per 2 e. ut sinus e y radius ad sinũ y i, sic sinus e o ad sinum i u.

Quare data hujus elementi præter primum aureæ regulę terminum sunt crura: quæritur basis.

Detur exempli gratia e a. grad. 27. 54′. 20″ at i e grad. 11. 29′. 5″. Quæritur basis.

Ex canone igitur sinuũ datur sinus cõplementi e a. 8,837,203. complementi i e 9,799,778. Hinc elementum dictum inveniet sinum complementi i a. 8,660,263.

Z Comple-

	Complementi		
	i e	e a	i a
10,000,000.	9,799,778.	8,837,203.	8,660,263.

Sinui invento respondet arcus grad.60. ejus complementum est basis grad.30.

Sed cur basis non est gr.150? Quia crura sunt minora sigillatim quadrante ex thesi. Ergo per 14 e 13. basis est minor quadrãte.

Ac si crus i e fuisset 168.30'.55''. & crus e a. gr. 152.5'.40''. Et tum basis etiam esset gr.30.

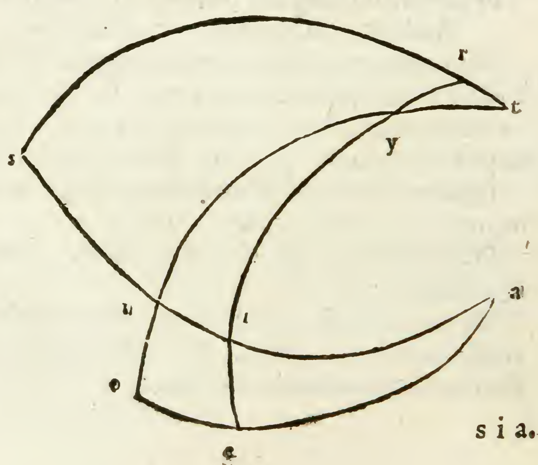
Verum si crus i e quidem fuisset grad. 168.30'.55''. e a vero grad. 27.54'.20''. tum basis esset gr. 150. nempe reliquus peripheriæ gr. 30 de 180. per 8 e 5. Eodem modo deinceps accommodabis exempla ad trianguli crura latera majora, minora quadrante: ad angulos obliquos. jam bis monuisse satis sit. Et

5. *Radius est ad sinum complementi cruris, ut sinus anguli dicto cruri non oppositi ad sinum complementi reliqui anguli.* 2. *theor. Gebri.* 18. *p.* 4. *Reg.*

Sic in triangulo nostro proportionales sunt

Rad.	sin.comp. i e.	sin. ang. i.	sin. comp. a.
Rad.	sin.comp. e a.	sin. ang. a.	sin. comp. i.

Manente enim trianguli nostri fabrica polo i describatur maximus circulus s t. jam i a per polũ secet s t, erit angulus s rectus. Sed & angulus u ex præmissa fabrica rectus est. quare si u y concurrat cum s t in t. erit t polus circuli



s i a.

s i a.& t s,t u fient quadrantes. Et cum i y fecet etiam circulum s t per polum: fient anguli ad r recti & r i quadrans.

jam per 2 e.ut finus arcus r i hoc eft radius ad finum arcus i y complementi cruris i e fic finus s r,hoc eft anguli ad i. ex fabrica & 4 e 13.ad finum arcus y u,hoc eft complementi anguli a.

Eodé modo res patebit de crure e a & angulis in bafi obliquis.

Exempli jam loco detur crus a e grad.27.54'.20''. & angulus a. gr.23.28'. Quæritur angulus i.

Sinus complementi cruris ex canone datur 8,837,203.

Sinus anguli a eft 3,982,155.

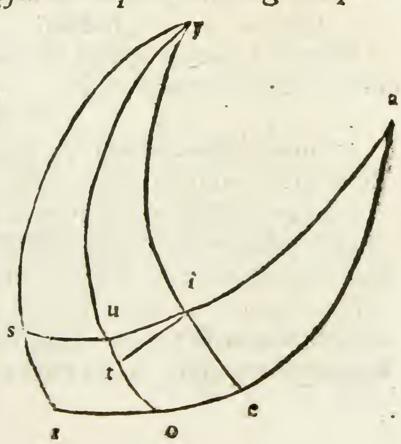
Ergo finus complementi anguli i invenietur 3,519,111.

	Com. a e.	fin. ang. a.	fin. com.ang. i.
10,000,000.	8,837,203.	3,982,155.	3,519,111

Sinui invento ex canone debetur peripheria grad.20.36'.15''. Ejus ergo complementum gr.69.23'.45''.eft pro angulo quæfito.

Et in rectangulis

6. *Si æquentur angulis duobus acutis, fed bafibus rectorum fint inæqualia: erit ut finus differentiæ crurum æqualium & rectorum ad finum differentiæ bafium, fic planus finuum complementorum crurum æqualibus oppofitorum ad planum radii atq; finus complementi anguli æqualis. 1.p.5. Regiom.*



Sint enim duo triangula rectangula a e i. a o u æqualia angulo acuto ad a. Sed bafis recti u a major bafi i a. Concurrant jam crura perpendicularia i e,u o in y.ubi per 18 e 12 polus eft circuli a o.

Rurfus per polos maximus circulus y s r fecet & a s & a r rectè fecabit per 16 e 12. & s r determinabit quãtitatem an-

 guli

guli a. & cum r y sit quadrans erit s y complementum anguli a. in utroq; triangulo æqualis: jam sit arcus perpendicularis i t.

Erit per 2 e. primo: ut sinus y u complementi u o cruris oppositi angulo æquali in uno triangulo ad sinum arcus y s complementi anguli æquales: sic sinus i u differentię basiũ ad sinũ i t.

Secundo: ut sinus y i complementi cruris oppositi angulo æquali in altero triangulo ad sinum arcus y e, hoc est ad radium. Est enim y e quadrans: sic sinus arcus i t ad sinum arcus o e differentiæ crurum rectorum angulorum & æqualium.

ut hic oculis subjecta vides.

Sin. y u.	sin. y s.	sin. i u.	sin. i t.
Sin. y i.	Rad.	sin. i t.	sin. o e.

Ergo ex arithmetica proportionum multiplicatione, cum termini secundarum rationum heterologi sint æquales: erit ut factus ab sinu y u per sinum y i ad factum ab sinu y s per radium sic sinus i u ad sinum o e. Quod propositio continebat.

Hinc in triangulis rectangulis calculus quidam est inprimis facilis: pro inventione unius quinq; istorum terminorum præter radium.

Nam ex terminis hic positis apparent hi termini proportionales.

Sin. y i.	sin. y u.	sin. y s.	Quart.
Quart.	radius	sin. i u.	sin. o e.

Ergo si exempli gratia sit quærendus angulus ille ad a. in utroq; triangulo æqualis.

Erit

ut sinus differentiæ crurum rectorum ac æqualium ad sinum differentiæ basium rectorum sic radius ad quartum aliquem.

Rursus ut sinus complementi majoris cruris oppositi æquali angulo ad sinum complementi minoris sic quartus inventus ad sinum complementi anguli quæsiti.

Exempla hujus inventionis invenies apud Astronomos: qui ex observatis phænomeni lõgitudinis ac latitudinis locis duobus angulum obliquitatis circuli phænomeni ad Zodiacum inquirunt.

quirunt.ut ſi ſit notata latitudo cometæ & i e & u o,& ex longitudinibus ſit nota differentia hoc eſt o e. Ex calculo etiam nota differentia i u (nempe ex angulo u y i & complementis latitudinum ut inferior calculus docebit) invenietur hoc ratiocinio angulus a. qui eſt dictæ obliquitatis.

Regiomontanus plura hinc deducit theoremata libro quinto. quæ illic legi poſſunt. Quæ inde eſt inventio: eam arbitror ex hoc calculo eſſe faciliorem. Quod ſi ejuſmodi theoremata colligenda ſint: collectionis finis nunquam fuerit. Quis enim eorum ultimum inveniet?

Et in obliquangulo.

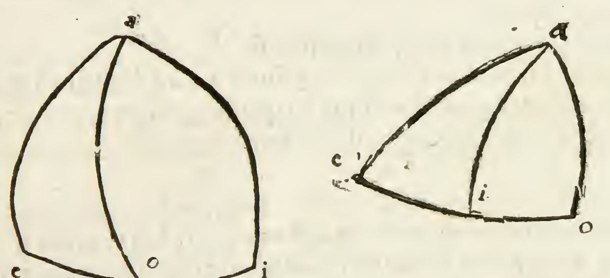
7. *Sinus angulorum ſinibus oppoſitorum laterum directè ſunt proportionales. 17.p.4.Regiom.*

Eſto triangulum obliquangulum a e i.

Erunt proportionales

Sin.an. e.	ſin.an. i.	ſin. a i.	ſin. a e.
Sin.an. e.	ſin.an. a.	ſin. a i.	ſin. e i.

Deſcendat enim perpendicularis ab angulo a in baſin e i. fient duo triangula e o a, i o a rectangula ad o. jam per 3 e. Radius, hoc eſt ſinus anguli o. (Anguli enim o menſura quadrans eſt, & quadrantis ſinus radius eſt) eſt ad ſinum e a ut ſinus anguli e ad ſinum a o.



Et rurſus ut ſinus anguli o ad ſinum a i, ſic ſinus anguli a i o ad ſinum a o.

Sin.an. o. ſin. a e. ſin an. e. ſin. a o.
Sin.an. o. ſin. a i. ſin an. i. ſin. a o.

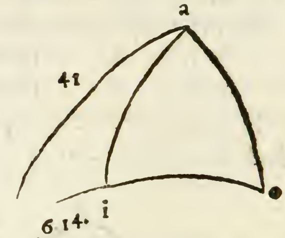
jam ex proprietate proportionis factus ex ſinu anguli o in a o ęquatur facto à ſinu a e per ſinum ang. e. itemq; facto à ſinu a i per ſinum anguli i. Et proinde facti hi ſibi æquantur itaque per eandem proprietatem ut ſinus a e ad ſinum a i, ſic ſinus anguli i, ad ſinum anguli e. Eadem res in reliquis angulis fuerit.

Et licet hinc

8. Dati anguli dato crure ac baſi, angulum cruri oppoſitum invenire ſi modo obtuſus ne ſit futurus an acutus præſciveris. 29.30.p.4. Regiom.

Exempli gratia ſumamus exemplũ Thaddæi Hagecii ex dialexi ſua de nova ętheria ſtella: habet aũt id capite iſtius ſcripti 13.

Sit angulus e a i gr. 8.50'. & e i ſit gr. 6.14'. atq; a e ſit gr. 41. Hæc enim data illic ſunt. & apud Tychonem inſcripto de eadem re. Quæritur angulus e i a qui obtuſus erit. id enim ex prima inſpectione cœleſtis illius civitatis patere potuit.



Ex illo elemento itaq; jam dicto ſinus e i eſt ad ſinum anguli e a i, ut ſinus a e ad ſinum anguli a i e. Dantur autem ex canone ſinus e i quidẽ 1,085,778. e a i 1,535,608. & a e 6,560,590. Hinc ergo datur ſinus anguli e i a 9,278,595.

e i	a e	a	i
1,085,778.	6,560,590.	1.535,608.	9,278,595.

Sinui invento competit ex canone peripheria gr. 68 6'. & paulo ultra. At angulus datur obtuſus: quare ſinus inventus non erit anguli acuti, ſed obtuſi nempe reliqui 68.6. de 180, hoc eſt 111.54'. ferè. Erit ergo angulus e i a gr 111.54'. Vides ergo neceſſe eſſe ut conſtet ſit ne angulus obtuſus aut acutus quem inveniendum proponis. Anceps enim eſt & dubium triangulum prout in planis quoq; diximus: & eadem hic ratio eſt.

Atq;

Atq; illud eſt quod Regiomontanus ad 29.p.4.admonet: Et à Copernico omiſſum P.Nonius in obſervationum regulis & inſtrumentis ægrè fert.

Et

9. Datorum angulorum data unius baſi reliqui baſin invenire: ſi quadrante major nec ne ea ſit conſtiterit. 32.p.4.Regiom.

ut retenta præmiſſa trianguli menſura detur angulus a. detur angulus i. & detur a e. Quæritur e i. Ejus ſinus invenietur 1,085,778.ut mutatis terminis præmiſsi ſic patet.

Ang. i	a	a e	e i
9,278,595.	1,535,608.	6,560,590.	1,085,778.

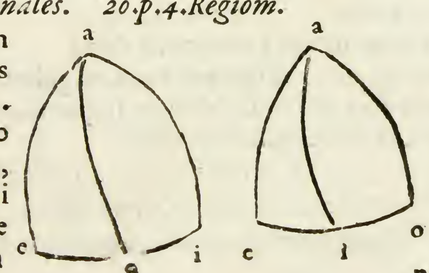
jam ſinui invento reſpondet arcus gr.6,14′. Sed & arcus grad. 173.46′.per 8 e 5.at utro baſin definiam? Certe id una dari oportet,quia calculus id non invenit:quemadmodū ex iis patet quæ ſupra de triangulis rectangulis monui: ſi obliquangulum ad rectangula per perpendicularem reducas. Et id eſt quod P.Nonius in 12.theor.14.capitis Copernici deſiderat: quod tamen in Regiom.ad 32.p.4.tulit.

Et hinc etiam,ut obiter hoc addam, pendet praxis 5 & 6. p.5. Regiom.nempe ut datis angulis cum ſumma aut differentia baſium, baſes ipſæ inveniantur: ſi poſtulata planorum in conſilium adhibueris.

Et

10. Si perpendicularis ſecat baſin: ſinus angulorum verticalium ſinibus complementorum angulorum in baſi directè ſunt proportionales. 20.p.4.Regiom.

Eſto enim triangulum a e i. jam perpendicularis ex a. ſecet baſin e i in o. Dico ſinum anguli e a o eſſe ad ſinum anguli o a i, ut eſt ſinus complementi anguli e ad ſinum complementi anguli i. Eſt enim



per

per 5 e. ut sinus anguli e a o ad sinum complementi anguli e, sic radius ad sinum complementi a o.

At ut radius ad sinum complementi a o, sic sinus anguli o a i ad sinum complementi anguli i.

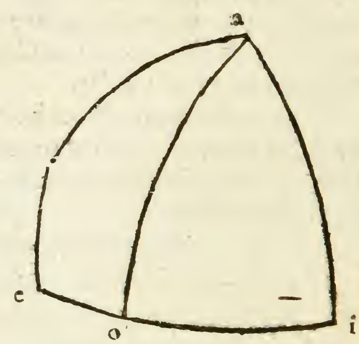
Ergo, per æquatam rationem, ut sinus anguli e a o ad sinum complementi anguli e, sic sinus anguli o a i ad sinum complementi anguli i.

Hinc patet

11. *Discretio unius anguli, in triangulo datorum angulorum, in verticales perpendicularem è dicto angulo in basin factos.* *33.p.4.Regiom.*

Datur enim hic summa duorum arcuum, nempe duorum verticalium angulorum itemque ratio sinuum ipsorum vel sinuum complementorũ angulorũ in basi ratio. unde per 8 e 10. utrumq; angulorum verticalium secernet.

Esto enim trianguli e a i angulus e a i gr. 111.54. at angulus a e i gr. 8.50. angulus demum a i e gr. 61.18′. jam cadat perpendicularis a o. ratio sinus anguli e a o ad sinum anguli o a i est ratio sinus compl. anguli e ad sinum comp. anguli i. Quare ex canone sinuũ dabitur ut 98,814 ad 48,022. resectis scilicet posteriorib. duabus notis.



Datur autem e a i summa duorum gr. 111.54′. Ergo per 8 e 10. angulus e a o erit gr. 83 & 3′. & proinde o a i gr. 28.51. His jam sequens rectangulorum calculus latera dati trianguli inveniet. Et

12. *Si arcus maximi circuli ab angulo bisecet angulum: in sinus anguli crurum sinibus conterminorum basis segmentorum directè sunt proportionales.* *7.p.5.Reg.*

ut in

ut in præmiſſa figura ſi a o biſecet angulum e a i. Erunt per 6 e. termini proportionales

Sin. a e. ſin. e o. ſin.ang. o. ſin.ang. a.
Sin.ang. o. ſin.ang. a. ſin. a i. ſin. o i.

Ergo ex æqua ratione ſunt proportionales

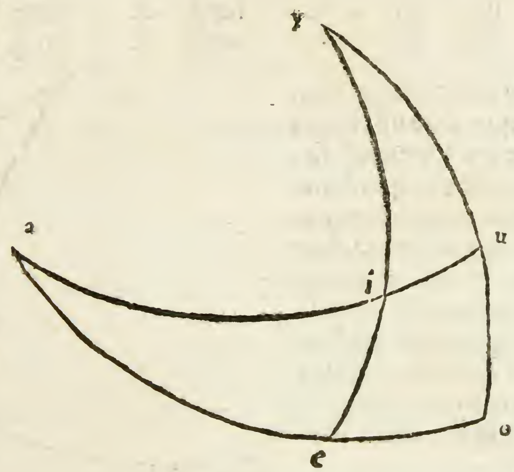
Sin. a e. ſin. e o. ſin. a i. ſin. o i.

Hinc licet angulum verticis biſecare.

Atq; ſic conſectarium unum Ptolemaici theorematis valde fœcundum habuimus: ſequitur alterum.

13. Secundum ſic eſt: Si maximus ſphæræ circulus maximum ſecet, & à ſecante duo arcus reliquo ſint perpendiculares: ſinus ſegmentorum ſecti à ſectionis puncto in perpendiculares tangentibus perpendicularium directè ſunt proportionales.

Sit enim circulus ſecans u a ſectus o a. perpendiculares u o. i e. cócurrant ergo u o, & e i in y, ubi polus erit circuli o a p 18 e 12. & proinde o y, e y ſūt quadrantes maximorum circulorum.



Jam per 1 e ratio ſinus o a ad ſinū arcus a e, fit è ratione ſinus arcus o u ad ſinum u y, hoc eſt per 24 e 5 è ratione tangentis o u ad radium: per rationem ſinus arcus y i ad

ſinum arcus i e, hoc eſt, per 24 e 5, per rationem radii ad tangentem i e.

At hæ facientes rationes multiplicatæ inter ſe, ſunt ratio tangentis u o ad tangentem i e. Quare ut tangens u o ad tangentem i e ſic ſinus a o ad ſinum a e.

Tang. o u	Rad.	Sin. o a. ſin. a e.
Rad.	Tang. i e.	

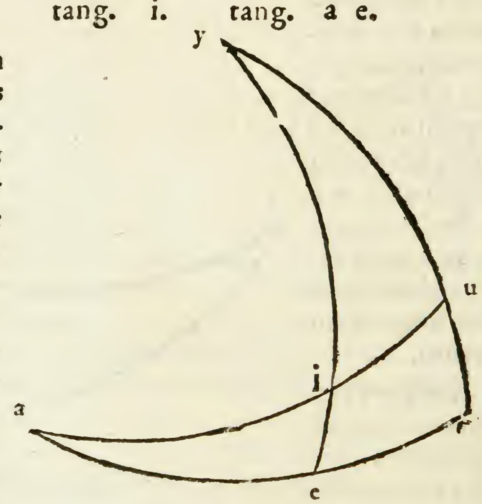
Atq; illud eſt quod Eraſmus Rheinholdus in tabulis ſuis directionum p. 8 uſum canonis fœcundi etiam conſpici in tractatione ſphæricorum triangulorum ait: quoties nimirum accidat ut duo concurrant ſinus, quorum arcus ſimul uni quadranti æquales ſint.

Itaq; in triangulo rectangulo.

14. *Radius eſt ad ſinum cruris, ut tangens anguli ad tangentem cruris ſubtenſi.*

Hic proportionales ſunt termini

Rad. ſin. a e.	tang. a.	tang. i e.
Rad. ſin. i e.	tang. i.	tang. a e.



Eſto enim triangulum i e a. quo ante uſi ſumus manente ſuperiori fabrica: erit a o quadrans maximi circuli. jam per 13 e. ſinus arcus o a, hoc eſt radius eſt ad ſinum a e cruris recti unius. ut tangẽs arcus u o, hoc eſt ex fabrica, anguli a ad tangentem i e cruris recti angulo dicto ſubtenſi.

Quamobrem dantur hic præter radium crus

recti

recti & angulus adjacens: quæritur crus recti angulo dato oppositum.

Sit exempli gratia datum crus e a gr. 27.54′.20″.& angulus a gr.23.28′. Quæritur crus i e. Eſt ergo ut radius ad ſinum e a, ſic tangens anguli a ad tangentem cruris i e.

Et cum ex canone detur ſinus e a 4,680,155. & tangens anguli a.4,341,209: hinc dabitur tangens i e.2,031,753.

	Sin. e a.	tan. a.	tan. i e.
10,000,000.	4,680,155.	4,341,209.	2,031,753.

Tangenti inventæ in canone reſpondet arcus gr. 11.29′.5″. pro arcu i e. Quadrante enim crus minus eſt per 13 e 13.

Et

15. *Radius eſt ad ſinum complementi anguli, ut tangens baſis ad tangentem cruris dicto angulo adjacentis.*

Sic in triangulo noſtro proportionales termini ſunt

Rad.	ſin. com. an. a.	tan. i a.	tan. a e.
Rad.	ſin. com. an. i.	tan. i a.	tan. i e.

Manente enim fabrica trianguli patuit o y. u a, o a eſſe quadrantes maximorum circulorum: & u o eſſe anguli a menſuram proindeq; u y menſuram ejus complementi. per 13.e. ergo ut ſinus o y, hoc eſt radius ad ſinum y u complementi anguli a. ſic tangens e o ad i u tangentem.

At ut tangens arcus o e ad tangentem i u, ſic tangens i a baſis ad tangentem a e cruris adjacentis angulo a. per 26 e 5. Sunt enim ex fabrica i a, a e complementa arcuum i u, o e.

Ergo ex æquatione rationum ut radius ad ſinum compl. anguli a. ſic tangens baſis i a ad tangentem e a cruris adjacentis.

Eodem modo de angulo i. i a & i e probabis.

Dantur ergo hic præter radium angulus & baſis. Quæritur crus angulo adjacens.

Exempli gratia ſit data baſis i a gr. 30. & angulus a. & cupias inveſtigare e a.

Sinus compl. anguli a. eſt 9,172,920. tangens aūt i a 5,773,502.

Hinc ergo invenitur tangens e a 5,295,987.

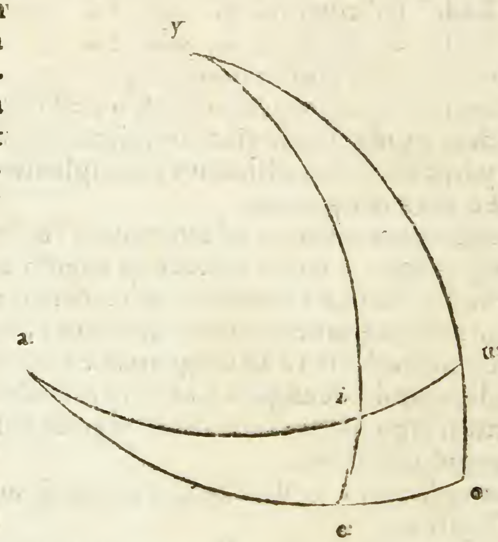
	Sin.com. a.	tan. i a.	tan. e a.
10,000,000.	9,172,920.	5,773,502.	5,295,987.

Ergo ex canone tangentiū crus ipſum invenitur gr.27.54'.20". minus ſcilicet quadrante. Quia enim baſis datur minor quadrante per 14 e 13. crura erunt ſimiliter inæqualia quadranti. at unum i e eſt minus quadrante. Nam ex theſi angulus oppoſitus a eſt acutus. Ergo per 13 e 13 i e minus quadrante. Et proinde e a minus quadrante. Et

16. Radius eſt ad ſinum complementi anguli, ut tangens complementi cruris adjacentis ad tangentem complementi baſis.

Hic ergo termini ſunt proportionales:

Rad.	ſin.compl.an. a.	tang.com. e a.	tang.com. i a.
Rad.	ſin.compl.an. i.	tang.com. i e.	tang.com. i a.



Manente enim diagrammate & fabrica noſtri trianguli o y, u a, o a ſunt quadrantes et o u menſura anguli a. Ergo u y erit complementū anguli a.& o e complementum cruris e a.itemq; u i complementū baſis i a. Sed cum ex fabrica i u, e o ſint in y o perpendiculares p 13 e. eſt ut ſinus o y hoc eſt radius ad ſinum y u ſic tan-

gens

gens o e ad tangentem i u. Sic de angulo i & i e res conſtabit.

Data ergo hujus elementi ſunt angulus & crus adjacens: quęritur baſis.

Exempli gratia detur e a gr. 27.54′.20″. & angulus a. gr. 23.28′. quæritur baſis i a.

Ex canone ergo ſinuum dabitur ſinus comp. ang. a. 9,172,920. & ex canone tangentium tangens compl. e a. 18,882,289.

Ergo hinc dabitur tangens compl. i a baſis 17,320,572.

Sin.compl.	a.	tang.comp. e a.	tang.com. i a.
10,000,000.	9,172,920.	18,882,289.	17,320,572.

Ergo ex canone tangentium datur complementum baſis gr. 60. & proin baſis ipſa i a gr. 30. Nam cum e a ſit minor quadrante erit ang. i. acutus & proinde per 15 e 13 baſis minor quadrante.

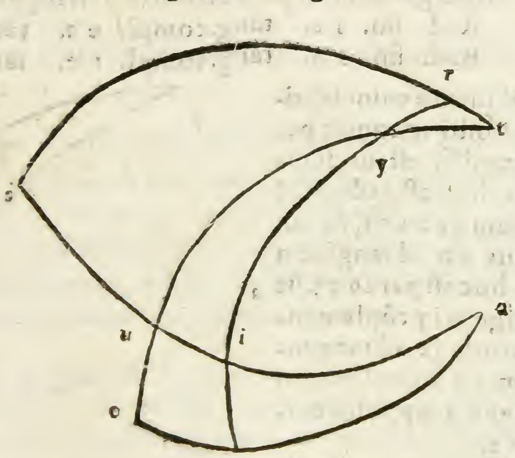
Et

17. *Radius eſt ad ſinum complementi baſis, ut tangens anguli ad tangentem complementi reliqui anguli.*

Hic ergo in noſtro triangulo termini proportionales ſunt iſti.

Rad.	ſin.compl. i a.	tang. a.	tang.compl. i.
Rad.	ſin.compl. i a.	tang. i.	tang.compl. a.

Repetita .n. fabrica trianguli quæ 5 e fuit. patuit illic i s, a u, u t, s t eſſe quadrantes, & u i eſſe complementum baſis. s r menſuram anguli i. & u y complementum anguli a. & r s, y u in s i eſſe perpendiculares. itaq; per 13 e. ſinus s i, hoc eſt ra-



dius eſt ad ſinum i u ut tangens r s ad tangentem arcus y u.

Data ergo elementi hujus præter radium ſunt baſis, & angulus in baſi unus. Quæritur reliquus angulus.

Exempli gratia detur baſis i a gr.30. detur angulus a gr.23.28′. Quæritur angulus i.

Ex canone itaq; ſinuum datur ſinus compl. baſis 8,660,254.

Et ex canone tangentium datur tangens anguli a. 4,341,209.

Hinc ergo dabitur tangens complementi anguli i. 3,759,597.

	Sin.comp. i a.	tang. a.	tang.comp. i.
10,000,000.	8,660,254.	4,341,209.	3,759,597.

Ergo ex canone tangentium datur complementum anguli i gr.20.36′.15″. & proinde angulus ipſe grad. 69.23′.45″. minor ſcilicet quadrante. Quia enim baſis ex theſi minor eſt quadrante: anguli in baſi ſimiliter inæquales erunt quadranti per 15 e 13. at ex theſi unus datur acutus. Ergo & reliquus acutus erit.

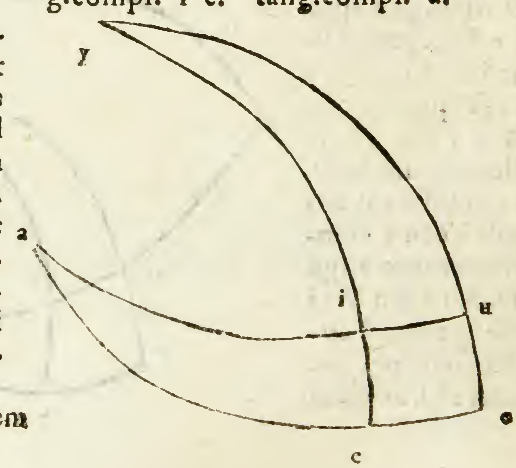
Et

18. *Radius eſt ad ſinum cruris, ut tangens complementi reliqui cruris ad tangentem complementi anguli oppoſiti.*

Hic ergo termini proportionales trianguli noſtri ſunt.

Rad.	ſin. i e.	tang.compl. e a.	tang.compl. i.
Rad.	ſin. e a.	tang.compl. i e.	tang.compl. a.

Manente enim fabrica noſtri trianguli per præmiſſa eſt ut ſinus o a, hoc eſt radius ad ſinum a e cruris, ſic tangens u o ad tangẽtem i e, hoc eſt per 26 e 5. ſic tangens i y cõplementi cruris i e ad tangentem y u complementi anguli a. oppoſiti cruri i e.



Eodem

Eodem modo de angulo i conſtabit ex i e & e a.

Data itaq; hujus theorematis ſunt præter primum aureæ regulæ terminum crura duo. & quæritur angulus alteri crurum oppoſitus.

Exempli gratia. Detur i e gr.11.29′.5″. at e a gr.27.54′.20″.

Quæratur angulus a.

Ex canone itaq; ſinuum datur ſinus e a. 4,680,155.

Et ex canone tangentium depromitur tangens complementi i e. 49,218,748.

Hinc ex præmiſſo elemento dabitur tangens complementi anguli a. 23,035,137.

	Sin. e a.	tan.com. i e.	tan.com a.
10,000,000.	4,680,155.	49,218,748.	23,035,137.

Ergo complementum anguli ex canone tangentium datur gr.66.32′. & proinde angulus ipſe a gr.23.28′. minor ſcilicet quadrante per 14 e 13. crura enim recti minora ſigillatim quadrante dantur.

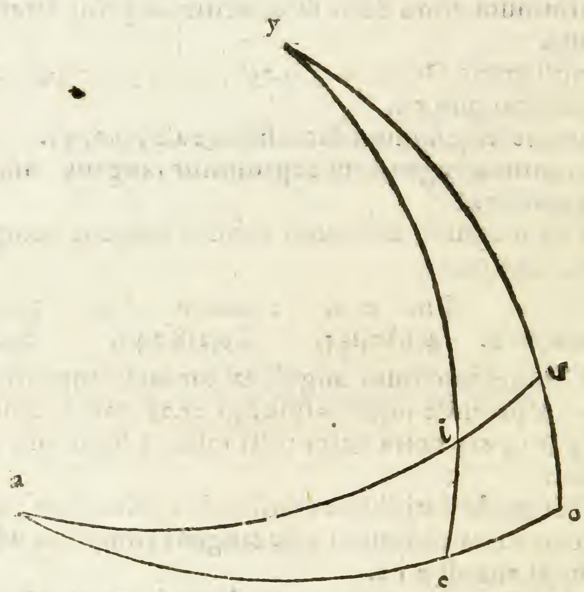
Eodem modo datis hiſce cruribus invenies angulum i.

Nam ut radius ad ſinum i e ſic tangens compl. e a ad tangentem compl. anguli e i a.

Atq; ſic è Ptolemæi theoremate ſecundum conſectarium fuit: Sequitur tertium.

19. *Tertium ſic eſt: Si maximus ſphæræ circulus maximum ſecet, atq; à ſecante duo ſint arcus reliquo perpendiculares, quorum alter ex interſectis quadrantes ſecet: erit ut radius ad tangentem minoris perpendicularis, ſic tangens complementi majoris ad ſinum ſegmenti circuli ſecti à puncto ſectionis in minorem perpendicularem.*

Sit enim maximus ſphæræ circulus ſecans u a, ſectus vero o a: atque à ſecante ſint perpendiculares arcus i e, u o: quorum alter u o ſecet ex interſectis quadrantes u a, o a. jam concurrant



currant perpendiculares erit in y polus circuli a o & y o. y e quadrantes, ut in prioribus fabricis etiam innotuit. jam sic absoluta fabrica per 1 e. ratio sinus y i ad sinum arcus i e, hoc est per 24 e 5. ratio radii ad tangentem i e fit è ratione sinus arcus y u ad sinum arcus u o, hoc est per dictum 24 e 5. ratione tangentis y u ad radium per rationem sinus arcus o a, hoc est ex thesi, radii ad sinum arcus a e. Atqui hæc multiplicata est ratio tangentis y u ad sinum a e. ut hic vides.

Facientes.			Facta.		
Sin. y u.	tan. y u.	Rad.	Sin. y i.	ad sin.	i e.
	hoc est			id est	
Sin. u o.	Rad.	sin. a e.	Rad.	ad tang.	i e.

Ergo

Ergo radius est ad tangentem i e minoris perpendicularis ut tangens y u complementi majoris ad sinum a e arcus nimirum secti circuli ab a pũcto sectionis in minorem perpendicularem.

Itaq; in triangulo rectangulo.

20. *Radius est ad tangentem cruris, ut tangens complementi anguli oppositi ad sinum reliqui cruris.*

Ergo in nostro triangulo termini proportionales sunt

Rad. tang. i e. tang. comp. a. sin. e a.
Rad. tang. e a. tang.comp. i. sin. i e.

Manente enim rursus fabrica superioris trianguli erunt y o. y e quadrantes & u y complementum anguli a. & a u, a o sunt quadrantes. Cum ergo perpendiculares sint i e,u o & u o secet quadrantes u a,o a erit per præmissum elementum, Radius ad tangentem i e ut tangens u y complementi majoris perpendicularis,hoc est ex fabrica compl.anguli a.ad sinũ a e cruris reliqui. Eodem modo de e a,& angulo i res constat.

Data itaq; hujus propositionis sunt præter radium crus & angulus oppositus:quæritur crus reliquum.

Exempli causa: sit datum crus i e gr. 11.29'.5".itemq; angulus a gr.23.28'. Quæritur crus reliquum e a.

Ex tangentiũ itaq; canone datur tangens cruris i e 2,031,745. & tangens compl.anguli a. 23,035,062.

Ergo ex præmisso elemento invenietur sinus cruris e a. 4,680,137.

	Tan. i e.	tan.com. a.	sin. e a.
10,000,000.	2,031,745.	23,035,062.	4,680,137.

Itaq; ex canone sinuum datur e a gr.27.54'.20". Sed hic dari necesse est sit'ne crus inveniendum quadrante majus aut minus. Nam fieri potest ut crure minore quadrante crus alterum majus sit:& rursus quadrante minus sit:pro magnitudine basis.

Elemento hoc Regiomont. ut alibi sic & 4 p. tabul. direct. Et Erasmus Rheinholdus in suis tabulis direct. 21.23 24.præcepto utitur in inquirenda fixarum ascensione recta & differentia ascensionali.

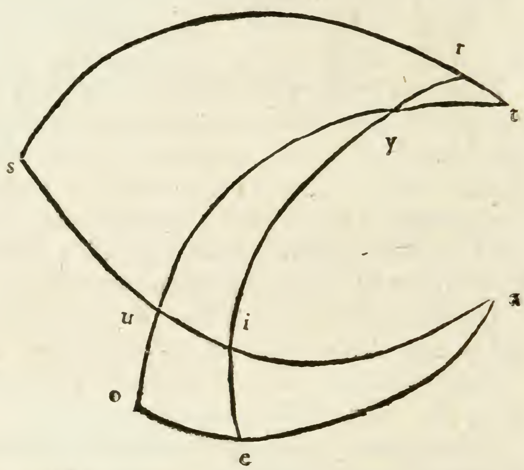
Et

21. *Radius eſt ad tangentem complementi anguli, ut tangens complementi reliqui ad ſinum complementi baſis.*

Hic termini proportionales ſunt in triangulo.

Rad. tan.com. a. tan.com. i. ſin.com. i a.

Repetita fabrica trianguli ſuperiori s e fiãt quadrãtes ut ante dictum. Et quia r i, ſi ſe ſecant & perpẽdiculares ſunt y u & r s ſecans quadrantes r i, s i. erit ut radius ad tangentẽ y u complementi u o, hoc eſt anguli a, Sic tangens r t complementi s r, hoc eſt anguli i ad ſinum i u complementi baſis.



Data ergo hic duo anguli. Quæritur baſis.

Exempli loco detur angulus a gr. 23.28'. & ang. i. gr. 69.23'.45''. Quæritur i a.

Ex canone ergo tangentium datur tangens complementi anguli a. 23,035,062.

Et tangens complementi anguli i. 3,759,583.

Ergo ex canone hujus elementi ſinus complementi i a invenietur 8,660,223.

Tan.com. a.	tan.com. i.	ſin.com. i a.
10,000,000. 23,035,062.	3,759,583.	8,660,223.

Et proinde ex canone ſinuum invenitur complementum baſis

ſis grad.60.& ita baſis gr.30.minor quadrante per 15 e 13.

Hinc jam ſequetur quartum generale conſectarium.

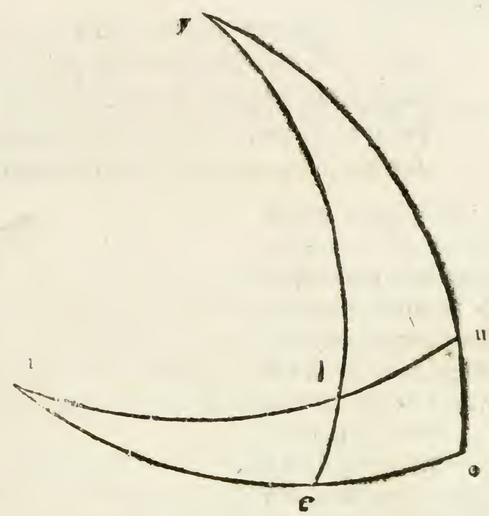
Et

22. *Radius eſt ad tangentem complementi baſis,ut tangens cruris ad ſinum complementi anguli adjacentis.*

Termini proportionales in triangulo ſunt

Rad.	tan.comp. i a.	tan. i e.	ſin.com. i.
Rad.	tan.comp. i a.	tan. e a.	ſin.com. a.

Repetita enim trianguli propoſiti fabrica erit per 19 e. ut radius ad tangẽtem i u complementi baſis ſic tangens a e cruris ad ſinum u y complementi anguli a.adjacentis cruri e a.



Sunt enim à ſecante e y. perpendiculares i u,e o.& e o ſecat ab interſectis quadrantes. & e a eſt complementum o e. ut ex fabrica hactenus ſæpe allata patet.

Eodẽ modo res patet de i e & i.

Data ergo hic ſunt cum radio baſis & crus. Quæritur angulus cruri dato adjacens.

Exemplũ rurſus ſit illud.detur i a baſis gr.30.& i e gr.11.29′.5″. Quæritur hinc angulus i.

Ex canone itaq; tangentium datur tangens i e 2,031,745.

Itemq; tangens complementi baſis 17,320,508.

Atq; hinc per expositum elementum sinus complementi anguli i invenietur 3,519,079.

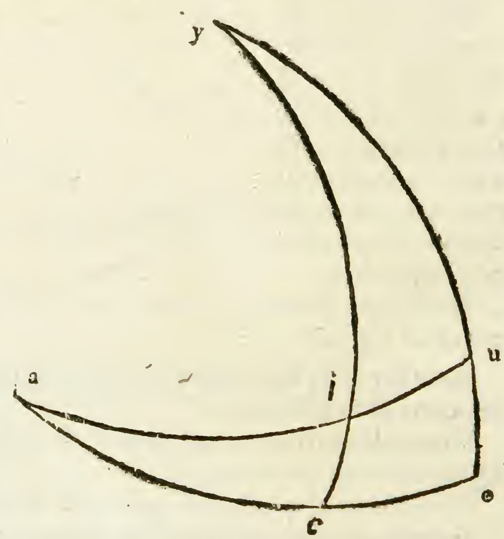
Tan.com.	i a.	tan. i e.	sin.com. i.
10,000,000.	17,320,508.	2,031,745.	3,519,079.

Ergo ex canone sinuum dabitur complementum anguli i. gr. 20.36'.15''. & proinde ipse angulus i.gr.69.23'.45''. Et est quadrante minor. Quia enim basis est minor quadrante: per 14 e 13 crus utrumq; similiter inæquale est quadranti. jam vero ex thesi crus unum i e est minus quadrante. Ergo & reliquum. & proinde per 13 e 13 angulus ab e a subtensus acutus est, quia crus est minus quadrante.

23. *Quartum ita est: Si circulus maximus maximum secet, & à secante duo arcus perpendiculares reliquo sint: sinus segmentorum secantis à puncto sectionis in perpendiculares secantibus complementorum arcuum perpendicularium sunt reciprocè proportionales.*

Maximus enim circulus u a secet maximũ o a. atque à secante demissi sint arcus perpendiculares in sectũ u o, i e. qui continuati cõcurrant in y polo circuli o a per 18 e 12. & proinde fient quadrantes o y, e y.

jam per 1 e. Ratio sinus arcus u a ad sinum arcus a i fit è ratione sinus arcus u o ad sinum arcus



u y, hoc

u y, hoc eſt per 28 e 5 è ratione radii ad ſecantem u y. per rationem ſinus arcus y e ad ſinum arcus e i, hoc eſt, ex dicto elemento, per rationem ſecantis y i ad radium.

Ac hæ multiplicatæ ſunt ratio ſecantis y i ad ſecantem u y.

Rad. Sec. y i. ſin. u a. ſin. a i.
Sec. u y. Rad.

Ergo ut ſecans y i ad ſecantem y u, ſic ſinus u a ad ſinum a i.

Poſſet quoque elementum hoc ex 2 e deduci. Nam ut ſinus u a ad ſinum a i ſic ſinus u o ad ſinum i e.

At ut ſinus u o ad ſinũ i e ſic ſecans y i ad ſecantem y u. p 29 e 5.

Ergo ut ſinus u a ad ſinum a i ſic ſecans y i ad ſecantem u y. hoc eſt ſinus ſegmentorum circuli ſecantis ſunt reciprocè proportionales complementorum i e, u o ſecantibus.

Itaq; in triangulo rectangulo.

24. *Radius eſt ad ſinum complementi cruris, ut ſecans baſis ad ſecantem reliqui cruris.*

Ergo termini proportionales hic ſunt

Rad. ſin.com. i e. ſec. i a. ſec. e a.
Rad. ſin.com. e a. ſec. i a. ſec. i e.

Repetita enim fabrica trianguli eſt y e quadrans, i u & e o perpendiculares ad o y. Quare per præmiſſum elementum: Sinus e y, hoc eſt radius eſt ad ſinum i y complementi cruris i e: ut ſecans i a baſis ad ſecantem e a reliqui cruris. Sunt enim ex fabrica u a, o a quadrantes. Ergo perpendicularium i u, e o complementa ſunt i a, e a.

Eodem modo res conſtabit de crure e a & baſi i e.

Dantur ergo hic baſis cum crure uno: Quæritur crus alterum.

Exempli jam loco ſit in triangulo baſis i a grad. 30. & crus i e gr. 11.29′.5″.

Ex canone ergo ſecantiũ dabitur ſecans i a quidē 11,547,004. at ſinus com. i e 9,799,778.

Hinc per dictum elementum ſecans cruris e a. 11,315,807.

	Sin.com. i e.	ſec. i a.	ſec. e a.
10,000,000.	9,799,778.	11,547,004.	11,315,807.

Ergo ex canone ſecantium arcus reſpondens inventæ ſecanti nempe gr.27,54′,20″.eſt pro crure e a.

Quod minus eſt quadrante: quia cū baſis eo minor ſit & crus i e etiam:per 14 e 13 crus reliquum e a eo minus erit.

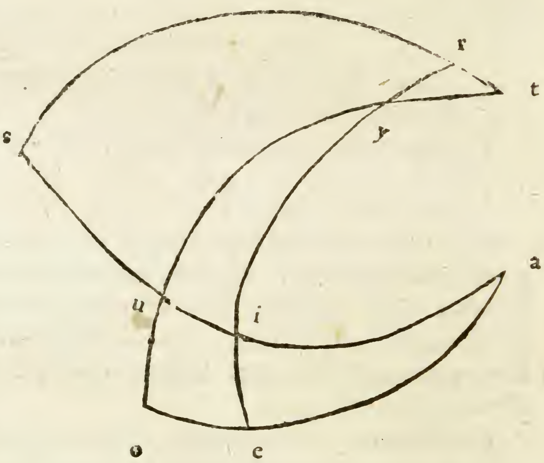
Et

25. Radius eſt ad ſinum anguli, ut ſecans anguli reliqui ad ſecantem cruris oppoſiti.

Hic termini proportionales ſunt:

Rad.	ſin.ang. a.	ſecans an. i.	ſecans e a.
Rad.	ſin.ang. i.	ſecans an. a.	ſecans i e.

Repetito enim triangulo & fabrica s e circulus maximus t o maximum r e ſecat, & à ſecante arcus perpendiculares reliquo ſunt t r & o e. Nam quia ex fabrica polus circuli s t eſt in i,& proinde i r ſecat circulum s t per polum & rectè ſecat per 16 e 12. Et anguli ad e ex theſi recti



ſunt. Ergo ex fonte horum conſectariorum, ut ſinus o y, hoc eſt radius (Eſt enim o y quadrans) ad ſinum y t, Sic ſecans s r complementi ſcilicet minoris perpendicularis r t. (Nam ex fabrica & 17 e 12 s t eſt quadrans) ad ſecantem e a.complementi majoris perpendicularis.

At y t æquatur ipſi u o.cum enim o y,& u t ſint quadrantes maximi circuli & proinde ęquales, ablato communi arcu u y.relinquentur

quentur æquales arcus.Et u o eſt menſura anguli a.Et s r eſt menſura anguli i.atq; e a crus angulo i oppoſitum.

Quare ut radius ad ſinum anguli a ſic ſecans anguli i ad ſecantem e a cruris oppoſiti. Simili modo res de reliquo crure i e patebit.

Dantur ergo in hoc elemento præter radium duo anguli: quæritur crus recti alteri datorum oppoſitum.

Exempli loco ſit iterum angulus a.grad.23.28′.angulus vero i gr.69.23′.45″. Quæritur jam crus verbi gratia i e.

Ex canone ergo ſinuum datur ſinus anguli i. 9,360,337.

Et ex canone ſecantium innoteſcit ſecãs anguli a. 10,901,656.

Hinc elementũ jam dictum inveniet ſecantem i e. 10,204.317.

	Sin. i.	ſec. a.	ſec. i e.
10,000,000.	9,360,337.	10,901,656.	10,204,317.

Cui ex canone ſecantium reſpondet arcus grad. 11.29′.5″.quadrante minor per 15 e 13 pro crure i e.

Et

26. Radius eſt ad ſinum anguli ut ſecans complementi cruris oppoſiti ad ſecantem complementi baſis.

Termini proportionales hujus elementi ſic ſunt:

Rad.	ſin.an. a.	ſec.com. i e.	ſec.com. i a.
Rad.	ſin.an. i.	ſec.com. e a.	ſec.com. i a.

Manente enim præmiſsi elementi diagrammate. Quia maximus circulus u t maximum s t ſecat, atque à ſecante ſunt perpendiculares ex fabrica s u, y r. Erit ut ſinus u t,hoc eſt radius ad ſinum y t, hoc eſt u o, id eſt anguli a,ut ſecans y i, hoc eſt complementi cruris i e. ad ſecantem i u, hoc eſt complementi baſis.

Eodem quoque modo res ſe habet in angulo i, & crure oppoſito e a.

Data ergo hujus elementi ſunt præter primum terminum aureæ regulæ angulus cum crure oppoſito: quæritur baſis. quæ ſit nec'ne quadrante major ſciri aut dari oportet ut ſupra monui.

Exemplo

Exemplo ſuperiori retento ſit i e gr.11.29′.5″. & angulus a.gr. 23.28′. Quæritur baſis eaq; quadrante minor.

Ex canone itaq; dicti elementi ſecans compl. baſis i a inuenitur 20,000,115.

Sin. a.	ſec.com. i e.	ſec.com. i a.
10,000,000. 3,982,155.	50,224,350.	20,000,115.

Cui in canone reſpondet peripheria gr.60. Ergo i a erit gr.30.

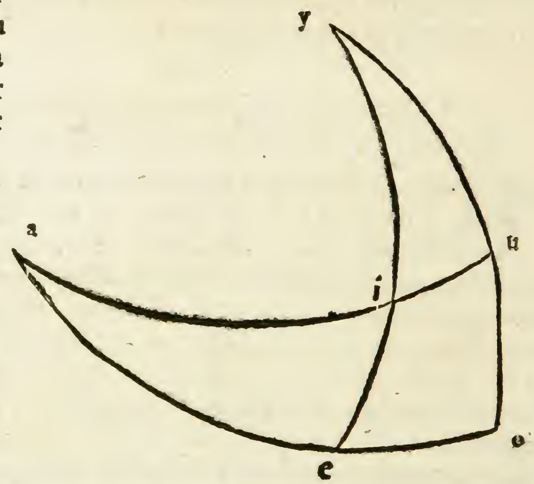
Et

27. Radius eſt ad ſinum baſis, ut ſecans complementi cruris ad ſecantem complementi anguli oppoſiti.

Termini analogias hujus elementi ſic ſunt:

Rad.	ſin. i a.	ſec.com. i e.	ſec.com. a.
Rad.	ſin. i a.	ſec.com. e a.	ſec.com. i.

Maneat enim præmiſſa triãguli fabrica. Quia hic maximus circulus u a ſecat maximum o a. atque à ſecante ſunt arcus perpẽdiculares i e, u o quorũ cõplementa ſunt y i, y u. Erit ut ſinus arcus u a, hoc eſt radius ad ſinum a i baſis: ſic ſecans i y complementi cruris i e ad ſecantẽ y u complementi u o, hoc eſt anguli a.



Simili modo res patet de e a & angulo i.

Dantur ergo in hoc elemento præter radium crus unum & baſis: quæritur angulus cruri oppoſitus.

Sic

Sic si deturi e gr. 11.29'.5". & basis i a.30.
Dabitur ex canone sinuum sinus basis 5,000,000.
Et ex canone secantium secans compl.i e. 50,224,350.
Hinc regula hujus elementi invenit secantem compl.anguli a.25,112,175.

Sin. i a.	sec.com. i e.	sec.com. a.
10,000,000. 5,000,000.	50,224,350.	25,112,175.

Quare angulus ipse ex canone dabitur grad. 23.28'. quadrante minor per 13 e 13.

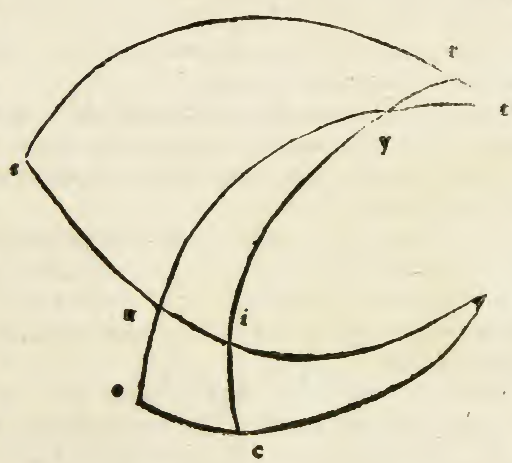
Et

28. Radius est ad sinum complementi cruris, ut secans anguli oppositi ad secantem complementi reliqui anguli.

Termini proportionis hujus theorematis sunt isti:

Rad.	sin.com. i e.	sec.ang. a.	sec.com. i.
Rad.	sin.com. e a.	sec.ang. i.	sec.com. a.

Repetito enim superiori schemate, quia ex fabrica maximus circulus i r maximum i s secat. atq; à secante sunt perpendiculares r s, y u arcus, quorum complemẽta sunt r t, y t. Erit ut sinus r i, hoc est radius ad sinum i y complemẽti cruris i e, sic secans y t, hoc est u o (sunt enim reliqui ęqualis u y de ęqualibus quadrantibus) hoc est anguli a ad secantem r t complementi s r hoc est anguli i.



Eodem modo veritas probatur in e a & angulo i.

Data ergo hujus elementi ſunt crus & angulus oppoſitus: quæritur angulus obliquorum alter. Si conſtiterit acutus ne an obtuſus exiſtat.id enim calculus non judicabit ex ſuperioribus: niſi conſtet de quantitate baſis, aut cruris oppoſiti. Nam crure uno & proinde angulo ei oppoſito acuto fieri poteſt ut alter acutus & etiam obtuſus ſit.per 13 & 14 e 13.

Eſto exempli gratia crus e i gr.11.29'.5''. & angulus oppoſitus a.gr.23.28'. Quæritur angulus i acutus.

Ex canone ſinuum datur ſinus compl. i e.9,799,778.

& ex canone ſecantium ſecans anguli a. 10,901,656.

Ergo elementum præſens inveniet ſecantem complementi anguli i. 10,683,382.

	Sin.com. i e.	ſec. a.	ſec.com. i.
10,000,000.	9,799,778.	10,901,656.	10,683,382.

Cui in canone ſecantiũ reſpondet gr.20.36'.15''.pro cõplemento. Ergo angulus erit gr.69.23'.45''.Nam ex theſi acutus datur.

Atque hoc modo conſectaria è Ptolemaico elemento habuimus:totumque inde calculum trianguli rectanguli:etiam quendam obliquanguli.Supereſt tantum ut in eo diligenter ſe Mathematum candidatus exerceat.

Elementa ſunt pro inventione alicujus tertii termini datis duobus quibuslibet:idq; per numerationem conjunctam. Via tamen multiplicationis expeditior eſt, & ad eam accommodata ſunt elementa.

Etſi autem ita plane eſſe ac perſpicuè expoſitũ hunc calculum arbitror: ut ulteriori additione non opus ſit: Quo tamen promptior ſit & expeditior inventio:datorum expoſitionem & ex iis inventionem quæſiti proponens elementum ſequens diatypoſis monſtrabit.

Inveniri autem debent aut baſis aut crus aut angulus. Angulus & crus adjacẽs aut oppoſitũ hic intelligatur ratione quæſiti.

Inventio cruris recti anguli è datis.

1. Baſi & angulo oppoſito. 3 e.
2. Baſi & angulo adjacente. 15 e.

3. Baſi

3.	Basi & crure reliquo.	24 e.
4.	Crure & angulo opposito.	14 e.
5.	Crure & angulo adjacente.	20 e.
6.	Duobus angulis in basi.	25 e.
	Inventio basis recti è datis.	
1.	Recti cruribus.	4 e.
2.	Angulo & huic crure adjacente.	16 e.
3.	Angulo & huic crure opposito.	26 e.
4.	Duobus angulis.	21 e.
	Inventio anguli in basi ex datis.	
1.	Crure opposito & angulo.	5 e.
2.	Crure adjacente & angulo	18 e.
3.	Basi & reliquo angulo.	17 e.
4.	Basi & crure adjacente.	22 e.
5.	Basi & crure opposito.	27 e.
6.	Cruribus.	18 e.

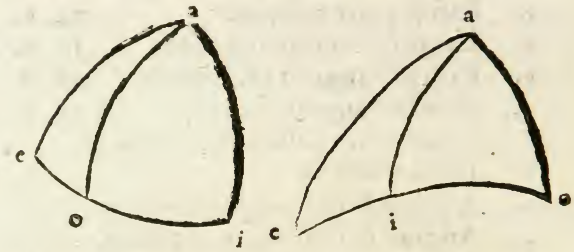
Hinc etiam deductus est quidam obliquangulorum calculus ad 8 & 9 e. sed reliquus obliquangulorum etiam inde dependet.

Itaq; hinc licet.

29. Reliquum obliquanguli calculum ita in rectangula duo perpendiculari arcu divisi, ne è tribus datis terminis duorum turbetur noticia, absolvere.

Hoc jure nobis optimo esse postulatum potest: nec quicquam habet obscuritatis aut difficultatis cognita calculi doctrina ea quam modo exposuimus. Requirimus tria data: & totidem requirenda ipse calculus admonet: Triangulum oblatum in rectangula est deducendum idq; fit perpendiculari arcu ex angulo aliquo dimisso: ita tamen dimisso ne duorum datorum offendatur noticia. Secus enim nec calculus rectanguli trianguli adhiberi potest. Hinc ergo

1. Datis duobus angulis una cum latere utriq; adjacente reliqua inveniemus. puta latera reliqua & angulum. 51.p.4.Regiom.



Esto enim trianguli a e i, cujus anguli duo sint dati nempe a & e. cum adjacente latere e a. Sitq; verbi gratia a e gr.38.20'. angulus a.gr.111.54'.& angulus e gr.8.50'. Ducatur jam ab a perpendicularis arcus: qui sive intra cadat, sive extra calculus perinde procedit: & calculus hoc docet.

Fiunt ergo hic duo rectangula e a o, o a i. quorũ calculus quæsita in obliquangulo patefacient.

In triangulo ergo rectangulo e a o datur angulus e. datur recti basis e a. Ergo per 3 e. dabitur perpendicularis a o gr.5.28' ferè.

	Sin. e a.	sin.ang. e.	sin. a o.
10,000,000.	6,202,356.	1,535,608.	952,439.

Et segmentum basis e o.gr.38.0'.4''.per 15 e.

	Sin.com.an. e.	tan. e a.	tan. e o.
10,000,000.	9,881,392.	7,906,973.	7,813,190.

Et per 5 e vel aliud aliquod, cum plura hic sint data, angulus e a o.gr.83,3'.

	Sin.com. e o.	sin.ang. e.	sin.com. a.
10,000,000.	7,879,988.	1,535,608.	1,210,031.

Quod si jam angulus e a o inventus major sit angulo dato e a i: perpendicularis cecidit extra triangulum: quod si minor sit cecidit intra. Atque ut illic subductio dati e a i ab invento relinquit angulum o a i rectanguli trianguli reliqui: sic hic cõtra subductio inventi à dato reliquum facit angulum i a o.

Et cum in præsenti exemplo inventus minor sit eo subducto de angulo e a i gr.111.54'. relinquitur angulus i a o gr.28.51.

Jam in triangulo i a o rectangulo cum detur a o perpendicularis nempe, itemque angulus i a o per 14 e invenietur crus io grad. 3.

Sin.

	Sin. a o.	tan.an. a.	tang. i o.
100,000.	952,439.	5,508,916.	524,690.

Et hinc ex duobus cruribus per 4 e basis invenitur gr.6.14'.

	Sin.com. i o.	ſin.com. a o.	ſin.com. i a.
1,000,000.	998,630.	995,452.	994,088.

Vel per 16 e.eodem modo invenietur.

	Sin.com. a.	tan.com. a o.	tan.com. i a.
10,000,000.	8,758,860.	104,491,055.	91,522,302.

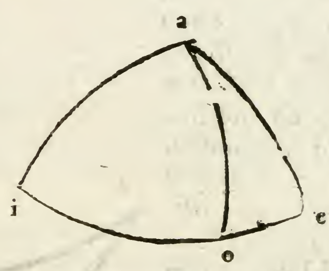
jam per 5 e angulus a i o invenietur gr.61.18'.

	Sin.com. a o.	ſin.an. a.	ſin.com. i.
1,000,000.	995,452.	482,518.	480,324.

jam itaq; ut reſpondeatur quæſtioni:habemus latus i a ex hoc calculo gr.6.14. & angulum a i e invenimus gr.61.18'.

Quantum autem erit latus e i? Habemus inventa ſegmenta e o,o i.jam ſi perpendicularis intra triangulum cecidit tum ſumma e o,o i.ſui extra differentia eſt latus e i. Hic quia perpendicularis intra cecidit ſumma gr.38.& grad.3.hoc eſt gr.41. eſt pro latere quæſito. Et.

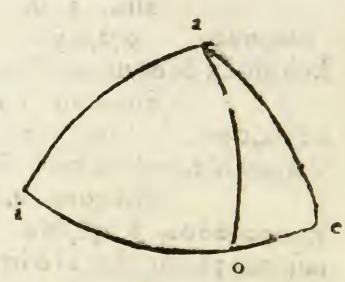
2. *Datis angulis inveniuntur latera.* 33.p.4.Regiom.



ut ſi trianguli modo dato noti ſint anguli. Deſcendat ab a perpendicularis a o.jam per 11 e. angulus e a o,dabitur itemq; o a i.Quare in triangulo rectangulo e a o. dabitur ex ſuperiori calculo a e, & e o. Rurſus in triangulo o a i eadem illa ſuperiora invenient a i & o i. Atque ita duo latera inventa ſunt. tertium e i eſt ſumma laterum e o,o i.ſi perpendicularis intra cecidit. Quæ via eſt faciliма ut ſemper intra cadat:quod fiet ſi anguli duo in baſi aſſumantur iidem ſpecie per 17 e 13. Et

3. *Quæ calculus obliquangulorum ſuperior non invenit hinc ſcrutabimur.*

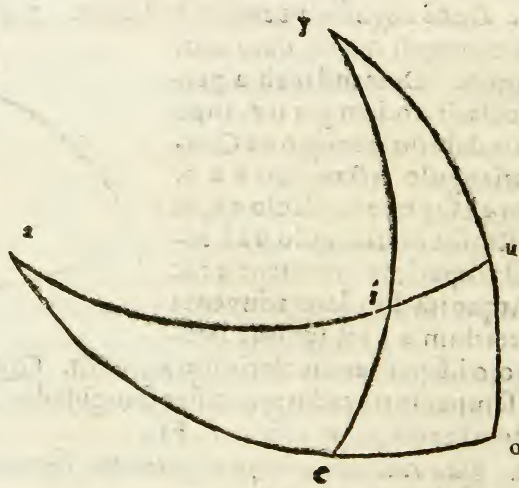
ut ſi detur a e & a i itemq; angulus e.8 e inveniet angulum i. Deſcendat jam perpendicularis a o. fient rectangula in quibus cum detur illic a e & angulus e. dabitur angulus e a o & e o. hic vero cum nota ſint a i & angulus i. invenietur angulus i a o & o i. jam ut propoſiti trianguli angulum e a i,& e i definias: ſcias caſum perpendiculi neceſſe eſt ex angulis e & i. & inde ſumma particularium quæſitorum,ſi intra cecidit: differentia,ſi extra,pro quæſito eſt.



Eodem modi ſi dentur anguli e & i cum latere e a per 9 e dabitur a i.& hinc reliqua ut jam dictum eſt.

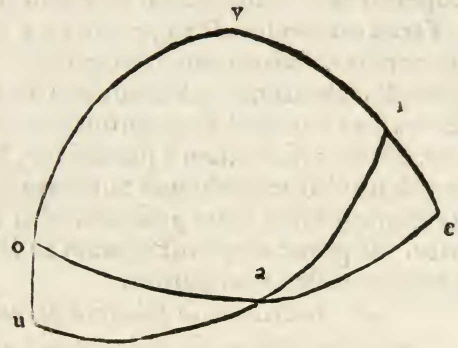
Et ſic reliquis caſibus hoc præſens poſtulatum ſufficiet. Et ſic jo.Regiomontanus 28.p.4.datis cruribus dati anguli reliqua inveſtigat.

At vero ut datorum laterum trianguli angulos inveſtigemus triplicem ab eo methodũ accepimus. Prima hujus eſt poſtulati artificioſa ſatis & cõpẽdioſa quam 4 p.5 exponit. ut ſi detur triangulum y u i datorũ laterum.



Sitq;

Sicque verbi gratia inveniendus angulus y.



jam facto polo in y describatur peripheria maximi circuli o e secans i u ad a. & y u, y i secent hūc maximum, recte secabūt per 16 e 12 ac erunt y o, y e quadrantes. & proinde u o, i e complementa laterum y u, y i.

jam quia circulus u a secat maximum o a. & ab u a sunt perpendiculares u o, i e: erit per 2 e sinus u a ad sinum a i, ut sinus u o ad sinum arcus i e. Quare cum ex thesi dentur u o, i e complementa scilicet laterum y u, y i dabuntur eorum sinus, & proinde ratio u a ad a i. Atq; hæc in utraque figura conveniunt. jam si sint latera y u, y i minora quadrante vel majora sigillatim, ut in prima figura: datur ex thesi differentia arcuum u a, a i nempe i u latus id cujus tanquam basis angulus quæritur. Sin latus unum verbi gratia y i quadrante minus sit, reliquum majus: datur ex thesi summa arcuum i a, a u. Sed jam eorum ratio innotuit. Ergo per 8 e 10. invenietur u a, & a i. jam in triangulo a i, i e rectangulo data basi ac crure i a & i e: dabitur angulus e i a. æqualis trianguli dati angulo i. in prima figura. at in secunda reliquus de duobus rectis est pro quæsito. Et invenietur crus a e.

Secundo in triangulo a u o rectangulo nota basi u a ac crure u o invenietur angulus ad u. cujus complementum ad duos rectos est pro angulo ad u trianguli dati in prima figura. Sed in secunda est ipse angulus u. & invenitur etiam a o.

jam in prima figura differentia a o, a e est angulus ad y.

at in secunda summa dictorum arcuum angulū ad y subtendit.

Secunda hujus inventionis methodus est 34. p. 4. ubi angulum in centro investigat equalem sphærico. Eam retinuit quoq;

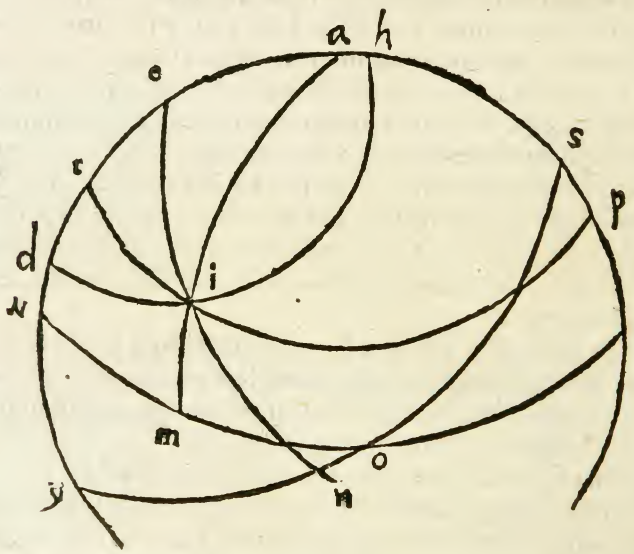
Copernicus

Copernicus 13.theor.14.cap.l.1. Apud eos hæc via legi potest.

Tertia methodus est ad 3 p 5 ex 2 p 5 theoremate certè in Mathematicis ac Astronomicis magnæ utilitatis. Ac certè in maximis & creberrimis in Astronomia inventionibus adhiberi videmus non tantum à Regiomontano atq; Copernico, & id quidem obscure: sed etiam à junioribus, Nonio Cardano & aliis. itaq; & nos hic retinebimus tanquam secundum calculi fundamentum. primum enim positum nobis fuit in Ptolemaico lemmate. Atque ut ante Ptolemæum audivimus: sic modo Regiomontano nostro attendamus.

30. In triangulo sphærico potentia radii est ad planum crurum, ut sinus secundus anguli dictorum crurum ad differentiam sinuum secundorum basis & differētiæ crurum.

Esto triangulum a e i sphæricum: ex cujus angulis a & e tanquam polis describantur maximi circuli quorum peripheriæ se-



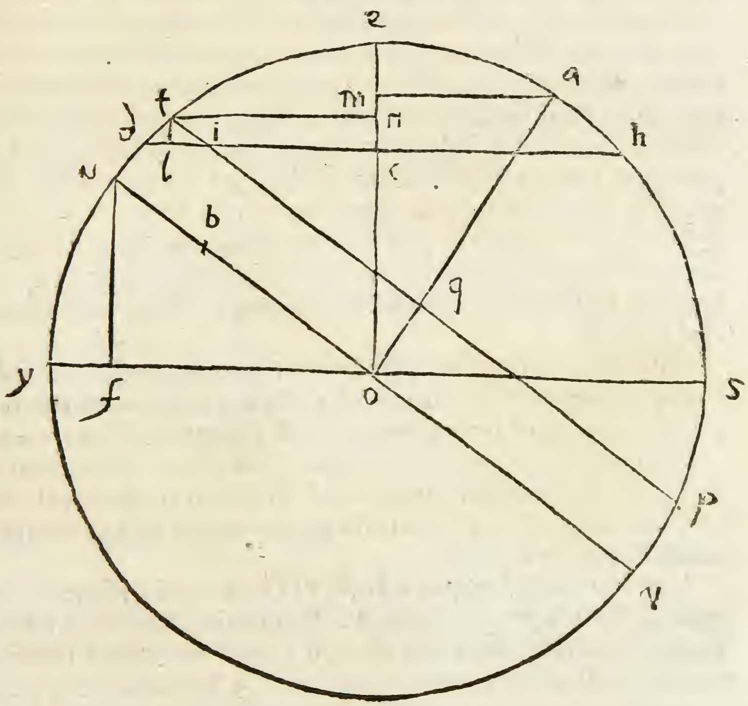
cent

cent ſe in o. tum & circulus a e ſecet deſcriptos in u & y: & quidem rectè per 16 e 12. eruntque a u, e y quadrantes, & proinde u y æqualis cruri e a. & o u, o y ſunt quadrantes. ſed & a i & e i ſecent circulos u r, y s per polos in m n. Erit u m menſura anguli a per 4 e 13. Et ſint paralleli t i p quidem ipſi u o r. at d i h ipſi y o s.

Erit ergo t e differentia crurum & e d æqualis baſi e i.

Dico jam ſinum ſecundum arcus u m eſſe ad differentiam ſinuum ſecundorum e d & e t arcuum. ut eſt potentia radii ad planum ſinuum crurum, nempe ſinuum a e, hoc eſt u y & a i.

Hoc ut plenius demonſtretur: deſcribatur circulus y e a s. in



quo ſint communes ſectiones circulorum y a s & y o s recta y s: circulorum vero y a r & y o r recta u r. rurſus communis ſectio circulorum i e h & d i h ſit recta d h. itemq; circulorum y t p & t i p recta t p. jam radius ſphæræ ſecet t p in q. itemq; radius o e ſecet d h in c. Et ſit in radium ſinus arcus a e recta a m, hoc eſt u f. Nam arcus a e & u o æquantur. Ergo ſinus ejus ſecundus erit e m. Et baſeos e d ſinus ſecundus eſt e c. Et differentiæ crurum e t ſinus ſecundus eſt e n. Quare differentia ſinuum ſecundorum baſis & differentiæ crurum eſt n c, hoc eſt per 2.c.6.e.10.R. eſt t l.

Rurſus ex ſuperioribus illud conſtat, cum circulus maximus y a p ſecet circulos t i p & d i h per polos, biſecare & proinde rectè ſecare ideoq; per centrum ſecare quare communes ſectiones t y p, d c h ſunt ſectorum diametri & proinde q & c ſunt eorum centra. Hinc & illud ſequitur t i eſſe ſinum ſecundum arcus t i. cum enim uterq; circulorum t i p & d i h perpendicularis ſit circulo y e a p. ut jam patuit: erit eorum communis ſectio, quæ per punctum i eſt, perpendicularis circulo y e a p per 10. e.21.R. & proinde perpendicularis diametro circuli t i p per 3. e. 21. R. Quare hanc ſectionem communem diameter t q p biſecabit: eritque biſegmentum ſinus arcus t i: & proinde i t ſinus ejus ſecundus. Quemadmodum hæc in ſphæra armillari plenius concipi poſſunt.

Abſcindatur etiam peripheriæ u m ſinus ſecundus u b, è diametro ſcilicet circuli u o r. jam cũ a u & a m maximi circuli ſecét parallelos per polos: ſegmenta u m & t i ſunt ſimilia per 24 e 12.

Ergo per 3 e 5. ut radius circuli u o r, hoc eſt u o ad radium circuli t i p, hoc eſt ut jam patuit t q. id eſt ſinum arcus a t vel cruris a i dati trianguli, ſic u b ſinus ſecundus anguli a ad t i ſinum ſecundum arcus t i.

Secundo cum triangula u f o, & t l i rectangula, æquentur angulo l t i & f u o per 4.c.12.e.5.R. Quia enim circuli u o r & t i p paralleli, ex theſi, ſecantur circulo y a p: communes ſectiones per 14.e.21.R. erunt parallelæ: & per 12.e.5.R. t l & u f ſunt parallelæ. Quare anguli l t i, f u o ſunt anguli crurum alternè parallelarum.

larum. Et proinde per c.3. e.7.R. triangula dicta sunt æquiangula: ac per 9. e.7. R. ut u o radius ad u f sinum cruris e a sic t i ad t l differentiam sinuum versorum basis ac differentiæ crurum. Quare termini proportionales sic sunt in triangulo proposito.

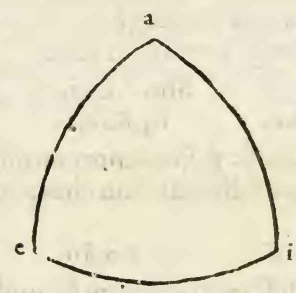
Rad. sin. a i.	sin.sec.an. a.	sin.sec. t i.	
Rad. sin. a e.	sin.sec. t i.	Differentia &c.	

Itaq; per proportionum multiplicationem ut planus radii ad planum sinuum crurum. Sic sinus secundus anguli ad differentiam sinuum secundorum basis ac differentiæ crurum.

Itaq;

31. *In triangulo sphærico radius est ad quartum radio sinibus cruris unius itemq; cruris reliqui proportionalem ut sinus secundus anguli crurum ad differentiam sinuum secundorum basis ac differentiæ crurum.*

Esto enim triangulum a e i, erit per præmissum elementum ratio sinus secundi anguli a ad differentiam sinuum secundorum basis e i, & differentiæ crurum facta ut hic vides.



An. Con.	Rad. sin. a i.	Rad. sin. a e.	Sin.sec. a.	Diff.sin.&c.

Ergo si sit ut radius ad sinum a e sic sinus a i ad quartum: Erunt facientes radius ad sinum a i & a i ad quartum. Ergo, cum heterologi æquentur, ut radius ad quartum inventum, sic sinus secundus anguli a. ad differentiam dictam.

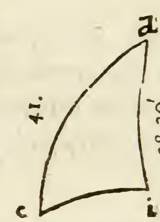
Quare termini proportionales sic sunt:

Radius.	ſin.cruris.	ſin.cruris.	Quartus.
Radius.	Quartus.	ſin.ſec. a.	Diff.ſin.ſecund. Baſ.& diff.cru.

Et hinc licet.

32. *Datis anguli dati cruribus, baſin ejus invenire. 28.p.4.Reg.*

Quod ſi crura ſunt æqualia, anguli in baſi æquantur per 7. e 13. ideo per 9 e 13. perpendicularis biſecat baſin. hinc poſtulatum ſuperius citius tibi ſatisfaciet.



Sint ergo crura inæqualia: & detur a e gr.41. a i vero gr.38.20'. cum angulo a. grad. 8.50'. quæ data ſunt exempli iſtius ſupra allati ex Tychone & Thaddæo.

Quæritur hinc e i diſtantia nempe duarum ſtellarum Caſsiopeæ.

Primo ergo termini ſic ſunt:

	Sin. a e.	ſin. a i.	
100,000.	65,606.	62,023.	40,811.

Secundo ſic diſponentur termini invento ſinu ſecundo anguli a. nempe ſubducto ſinu complementi de radio.

		Sin.ſec. a.	
100,000.	40,811.	1,186.	484.

Ergo differentia ſinuum ſecundorum baſis ac differentiæ crurum eſt 484.

Hæc ergo differentia addita ad ſinum ſecundum differentiæ crurum eſt ſinus ſecundus baſis. ut in hoc exemplo ſinus ſecundus differentiæ crurum gr.2.40. eſt 108. Ergo ſumma 484 & 108. hoc eſt 592. eſt pro ſinu ſecundo baſis. i s de ſinu toto ſeu radio relinquit 99,408. cui competit gr.83.46'. Ergo ejus complementum gr.6.14'. eſt pro baſi quæſita.

Et

Et ex hoc elemento, ut cæteros jam usus taceam, magna Geographiæ auxilia parantur: in quærendis terrestrium locorum distantiis: ubi locorum diversa & latitudo & longitudo existit: sic enim Geographi nominant. Et sic inquirit P. Nonius libro secundo de observat. Regul. ac instrument. c. 20. Et Cardanus etiam eam ad rem citat 2 p 5 Regiomontani in libris de Varietate rerum. Et nuper in edita à se Astronomia Mæstlinus amicus noster eandem viam ingressus est. Plura tamen hujus theorematis exempla luculentissima Nonius in libro de crepusculis suppeditabit.

Et

33. *Datis trianguli lateribus, cujusvis basis angulum invenire.* *3.p.5.Regiom.*

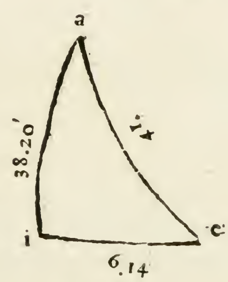
Inventio ex hactenus dictis non est obscura. Sunt enim in triangulo dato termini huic rei accommodati proportionales hoc modo:

Rad.	sin cruris.	sin cruris reliqui.	Quartus.
Quart.	Radius.	Differentia sinuum Sec. basis & differentiæ crurum	sinus secund. anguli oppositi basi.

ut si ex datis tribus lateribus

a e.	41.	
a i.	38.	20.
e i.	6.	14.

Quæratur angulus e a i.



Termini ergo proportionales sic sunt:

	I. Sin. a e.	sin. a i.	
100,000.	65,606.	62,023.	40,811.
	II.		Sin. sec. ang. a.
40,811.	100,000.	484.	1,186.

 Tertius

Tertius terminus 484. est differentia sinuum secundorum basis e i & differentiæ crurum. Hunc si basis minor quadrante sit compendio invenies si differentiam assumas sinuum complementorum basis ac differentiæ crurum. Quod Regiomontanus ad 2 p 5 admonet. Sed tamen si basis quadrante major sit. perinde non invenietur.

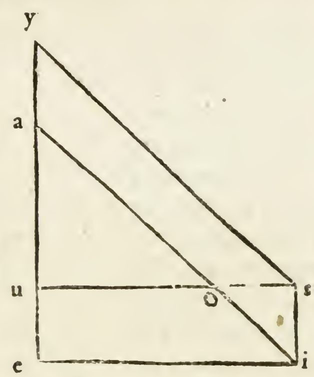
Et sic calculum triangulorum exposuimus. collegimus ea quæ usum habere videbantur: & quibus cognitis omnia ea quæ specialia ab aliis colliguntur tractari & expediri commode possint: nec tantum ea tractari quæ collecta sunt, sed quæ quotidie Astronomis, Geographis aliisque occurrunt & inveniuntur. Nec enim adhuc tot tamq; varii casus collecti sunt ut ultimum viderimus. Tu iis candide Lector fruere in suo quisq; proposito exemplo: & studiis nostris fave.

FINIS.

ELEMENTA INSE-
renda ad quintum Rami ſic ſunt reſtituta.

14. *Si duæ rectæ ſe ſecantes parallelis pluribus interſecentur: parallelæ conterminis ſegmentis ſunt proportionales.*

EST è 13 e conſectarium. Sint enim rectæ a e, a i interſectæ ad a. Eas ſecent parallelæ u o, e i. Erit ut a u ad u o ſic a e ad e i. Erigatur enim à termino i parallela i s ad a e. & u o in eam continuetur. Tum à termino s ſit ad a i parallela s y. in quam continuetur recta a e. Erit a y æqualis rectæ i s, hoc eſt u e per 5 c 12 e. Erit tandem per 13 e ut u a ad u o ſic a y, hoc eſt u e ad o s.



Ergo per compoſitionem proportionum ut u a ad u o ſic u a & u e ad u o & o s, hoc eſt e i p 6 c 12 e.

Eadem fuerit demonſtratio ſi rectæ ſint verticaliter interſectæ, ut in eodẽ diagrammate patet. Si enim datæ a i & u s interſectæ verticaliter in o ſecentur parallelis a u, s i. Completa præmiſſa fabrica erit per 13 e ut a u ad a o conterminum ſegmentum, ſic a y hoc eſt i s ad o s conterminum ſegmentum. Elementum 13 invenit tertiam & quartam proportionalem: hoc continue mediam proportionalem ſimplicem aut duplicem ſuppeditabit.

Itaq;

1. *Si datæ duæ rectæ in unam continuentur: perpendicularis à puncto continuationis in angulum normæ continuatam*

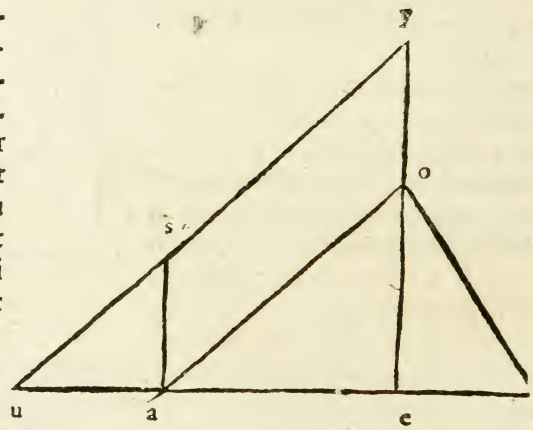
nuatam cum continuatione includentis eſt media propor- tionalis inter datas.

Norma ſive gnomon, canon, eſt inſtrumentum crurum anguli recti. de qua ſupra fuit ad 8 e. Ejus opera media datis proportionalis invenitur. unde & Meſalabum ſeu Meſographus ſimplex dici poſſet. Sint datæ duæ rectæ a e, e i.

a———————e
e————i

Quęritur his media ꝓportionalis: ad quā ſit ut a e ſic e a ad e i.

Continuetur ergo a e in i ut continuatio e i æquetur reliquæ datæ. Dehinc è puncto continuationis e erigatur perpendicularis infinita e o. jam circa hanc perpendicularem ſurſum deorſum, horſum illorſum moveatur norma a o e ut angulo ſuo comprehendat e o. cruribus vero includat rectam a i.



Dico ſegmentum perpēdicularis e o eſſe mediū proportionale inter datas.

Continuetur enim e a in u, ut continuatio a u æquetur e o. Et ad punctū continuationis a. fiat angulus u a s angulo o e i æqualis & æquicrurus: hoc eſt crus a s fiat æquale cruri e i. Quare connexis u & s. rectæ u s & o i æquabuntur, & anguli e o i, a u s per 1. c. 6. e. 3. & per 12 e. s a, o e ſunt parallelæ, & angulus s a o. æqualis angulo a o e. Sed & anguli s a e & a o i ſunt recti ex fabrica & theſi: & proinde æquales. quare

quare reliqui o a e & e o i, hoc est s u a sunt æquales: & proinde per 12 e u s & a o parallelæ: & u s ac e o continuatæ concurrent. ut hic in y: & per 5.c.12.e. o y & a s æquantur.

jam per 14 e. ut u e ad u a sic e y, ad a s. Ergo per subductionem proportionum: ut e a ad u a sic e o, hoc est u a ad o y, hoc est a s.

Et

2. *Si datæ duæ rectæ rectè conterminæ verticaliter continuentur: norma crure suo eiq; parallelo mobili incidẽs in terminos datarum, angulis in continuatas, intersecabit è continuatis duas medias continuè proportionales datis.*

Præmissum consectariũ Mesolabii simplicis fuit: hoc duplicis est. cujus usus in fabricandis optatis corporibus est perspicuus.

Sint datæ duæ rectæ a e, e i.

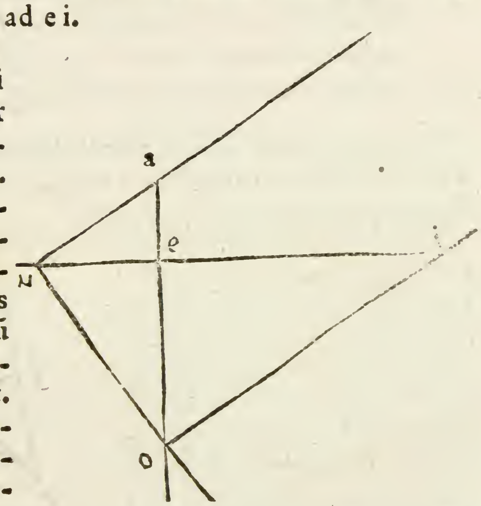
e——————i

Sintq; inveniendę rectæ mediæ continuè proportionales,

a————e

nempe ut sit a e ad inventam sic inventa ad inventam secundam & ut illa ad hanc sic hæc ad e i.

Contermine̅tur itaq; datæ rectè ad e. & infinite continuentur sed verticaliter: nempe i e versus u. at a e versus o. jam ad reliquam fabricam Platonis fuit mesographus norma scilicet oppositis cruribus parallela: Ejus crus unũ per cavum cõtermino-rum laterũ mobile est. Ergo mesographum tibi comparabis: si normæ addas unicum latus mobile, sed oppo-



fito cruri utcunq; motum parallelum: quæ duplex quasi norma est, jam si hæc norma adhibeatur: & tamq; diu latus mobile volvatur ut cum opposito contineat terminos datarum, sed anguli in continuatas incidant: rectæ à continuationis puncto in angulos normæ sunt quæsitæ. ut si Mesographi a u o i latus o i mobile sit: hac illac volves donec anguli u & o incidant in infinitas, & simul u a, o i attingant terminos a, i. Erit per consectarium præmissum

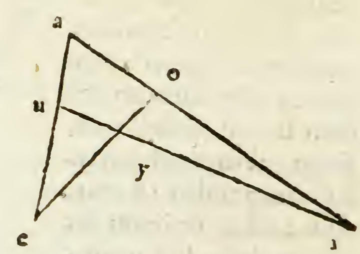
ut e i ad e o sic e o ad e u.
ut e o ad e u sic e u ad e a.

Et sic Mesolabium tam simplex quam duplex expositum fuit: cujus usus fabricis lineati optati est accommodatissimus.

15. Si quatuor rectarum duæ faciant angulum reliquæ ab harum terminis in se reflexæ priores secent: ratio unius ad segmentum suum vel segmentorum inter se fit è ratione ita conterminarum ut prima facientium contermine tur principio antecedentis factæ, secunda hujus consequentis fini contermina terminetur in finem cōsequentis factæ.

Elementi hujus duo exempla specialia Ptolemæus habet: Theonsis quatuor addidit in c 1 μεγάλ. συντάξ. de alterutra angulum continentium & reflexarum.

Sint ergo duæ rectæ a e, a i & ab harum terminis reflexæ aliæ duę i u, e o secantes se in y & datas priores in u & o. Erit ratio particularium rectarum facta: ut sequens typus ostendit: in quo antecedentes facientium superiori loco sunt: cōsequentes suis antecedentibus subiiciuntur.



1. Est

Facientes.		Facta.	
	I.	Eſt Ptolemæi & Theonis I.	
i u.	y e.	i a.	a o.
u y.	e o.		
	II.	Eſt Theonis VI.	
a u.	e y.	a i.	i o.
u e.	y o.		
	III.	Eſt Theonis III.	
e a.	u i.	e o.	o y.
a u.	i y.		
	IIII.	Eſt Theonis II.	
o a.	i u.	o e.	e y.
a i.	u y.		
	V.	Eſt Ptolem. II. Theonis IIII.	
i y.	u e.	i o.	o a.
y u.	e a.		
	VI.	Eſt Theonis V.	
e u.	a i.	e y.	y o.
u a.	i o.		

Eodem modo in reliquis duabus ſe res habet: ſive terminos alternes ſive invertas.

Pro demonſtratione foris à Ptolemæo quædam aſſumitur: quæ multiplicans duas faciendis proportionales iis proportionales alias inveniat. De hac aſſumpta Theon hanc tradit regulam: ut ſit parallela alteri duarum quibus interponitur vel adjungitur, vel pars alterius earundem, vel è contra ut altera harum ſit pars aſſumptæ deforis. Quæ tamen in ſpeciali exemplo determinate aſſumendam deforis non monſtrat. Quare relicta hac Theonis regula, iſta demonſtrationis nobis fabrica ſit: ut à prin-

cipio antecedentis factæ parallela conſequenti ſecundę facientium ducatur in aliquam datarum infinitè continuatam.

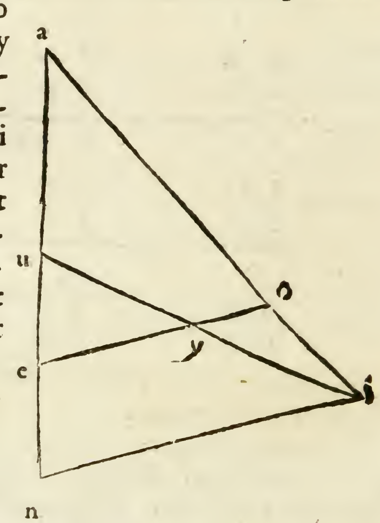
Erunt tum multiplicatæ proportiones.

Anteced. Conſeq. Anteced. Conſeq. ſecundæ facientium : utrobiq; æqualitatis ratio.

Anteced. Conſeq. primæ facientium, parallela, antecedens ſecundæ facientium per 14 e.

Ergò per multiplicationem proportionum, ratio parallelę ad conſequentem ſecundæ facientium, hoc eſt, ex fabrica & 14 e, ratio antecedentis factæ ad conſequentem fit è ratione &c. dicto modo.

Exempli gratia demonſtretur exemplum primum ſpeciale.



Dico ergo rationem i a ad a o eſſe factam è ratione i u ad u y per rationem y e ad e o. Ducatur enim à principio antecedentis factæ nempe puncto i parallela rectæ e y. quæ concurrat cum a e continuata in n. Erit ergo per 14 e. ut i a ad a o ſic parallela ducta ad e o conſequentem ſecundæ facientium. Sunt ergo jam multiplicatæ proportiones.

i u. u y. i n. e y. per 14. e.
y e. e o. e y. e o.

Ergo ut factus ab i u per y e ad factum ab u y ad e o ſic i n ad e o, hoc eſt i a ad a o.

Sic fit ſecundum Ptolemæi docendum quod in typo præmiſſo quintum eſt.

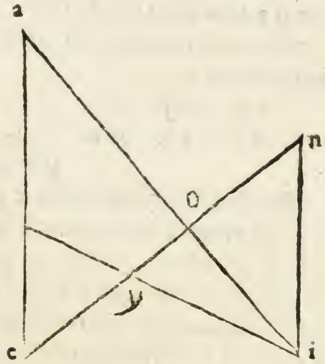
Dico ergo rationem i o ad o a eſſe factam è ratione i y ad y u, & ratione u e ad e a. Rurſus enim à principio antecedentis factæ i ſit parallela ad conſequentem ſecundæ facientium e a. quę concurrat

currat

currat cum e o in n continuata. Erit per 14 e. ut i o ad o a ſic i n ad e a. Sunt ergo jam rurſus multiplicatæ proportiones

u e e a. u e. e a.
i y. y u. i n. u e. per 14. e.

Ergo per multiplicationẽ proportionum ratio i n ad e a, hoc eſt i o ad o a fit è ratione i y ad y u per rationem u e ad e a.



Hæc duo exempla Ptolemæus paulo aliter cum Theone demonſtrat: nec parallelam eodem & communi modo ducit. ut enim in primo exemplo à puncto o ſeu fine conſequentis factæ parallela antecedenti primæ facientium ducitur: ſic in ſecundo exemplo parallela quartæ e o è fine conſequentis factæ egreditur.

Theon quoq; in ſuis exemplis ſpeciatim demonſtrandis generali via caret. Nam in ſecundo noſtri typi exemplo parallela eſt à principio antecedentis factæ ad conſequentem ſecundę facientium: at in tertio à fine conſequentis factæ parallela conſequenti primę facientium educitur. Quare ut de generali elemento ſollicitus fui: ſic generalem elementi demonſtrationem, demonſtrationisq; fabricam quærere conatus ſum. Et ſane talis eſt ea quam propoſui: & in ſpecialibus Ptolemæi exemplis exercui. Sed præſtat in Theonis exemplis idem docere. Sumemus itaq; rationem reflexæ ad ſegmentum & ſegmentorum inter ſe: nempe 4 & 6 exemplum noſtri typi.

Dico ergo rationem o e ad e y eſſe factam è ratione o a ad a i per rationem i u ad u y.

Ducatur à termino o nempe principio antecedentis factæ parallela ad u y recta n o.

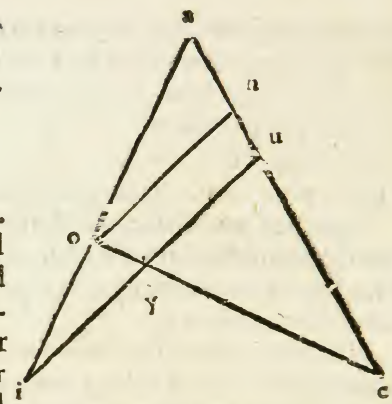
Erit per 14 e. ut o e ad e y ſic parallela n o ad u y.

· At ratio n o ad u y fit è ratione o a ad a i, & i u ad u y.

Sunt enim multiplicatę proportiones:

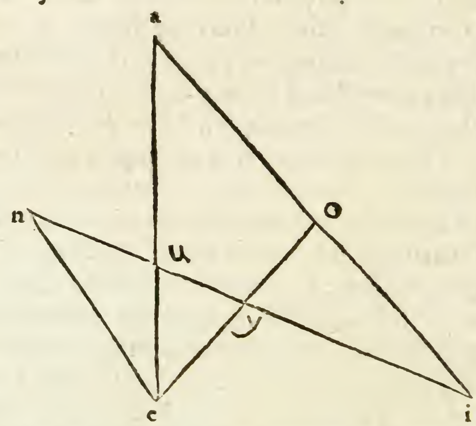
i u. u y. i u. u y.
o a. a i. o s. i u.

per 14 e.



Rursus dico rationem e y ad y o componi ex ratione e u ad u a, & a i ad i o. Theon parallelam ducit ex o. ad u i. Sed per generalem fabricam ducetur ex e ad o i. Erit ergo ut e y ad y o. sic e n ad o i. jam sunt multiplicatę proportiones istæ:

a i. i o. a i. i o.
e u. u a. e n. a i.



Ergo ratio e n ad i o, hoc est e y ad y o erit ex dictis rationibus facta.

De segmentis rectarum diversarum multa tractant Arabes sub nomine regulę sex quantitatum. Et Alchindi hac de re modi sive theoremata circumferuntur. Repetit etiam in Algorithmo suo Regiomontanus: & Maurolycus ad 1 p 3 Menelai. Verum nihil habent quod non protinus è proportionum multiplicatione Arithmeticæ peritis occurrat. Sunt enim modi illi istius multiplicationis exempla specialia.

F I N I S.

BASILEÆ,

PER SEBASTIANVM HENRIC-
PETRI, ANNO SALVTIS HV-
MANAE M. D. LXXXIII.
MENSE AVGV-
STO.


SEBASTIAN · HENRIC
PETRI ·